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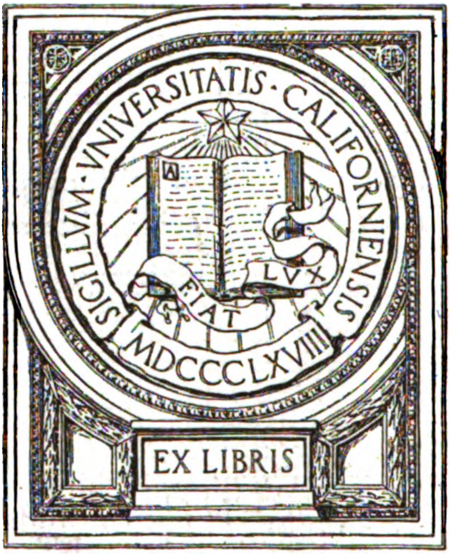

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The Elements of Electrical Engineering

VOLUME I

DIRECT AND ALTERNATING CURRENT MACHINES AND SYSTEMS

(The book entitled ***Electric Lighting and Miscellaneous Applications of Electricity***, The Macmillan Co., 1912, will be known henceforth as Volume II of Wm. S. Franklin's ***Elements of Electrical Engineering***.)

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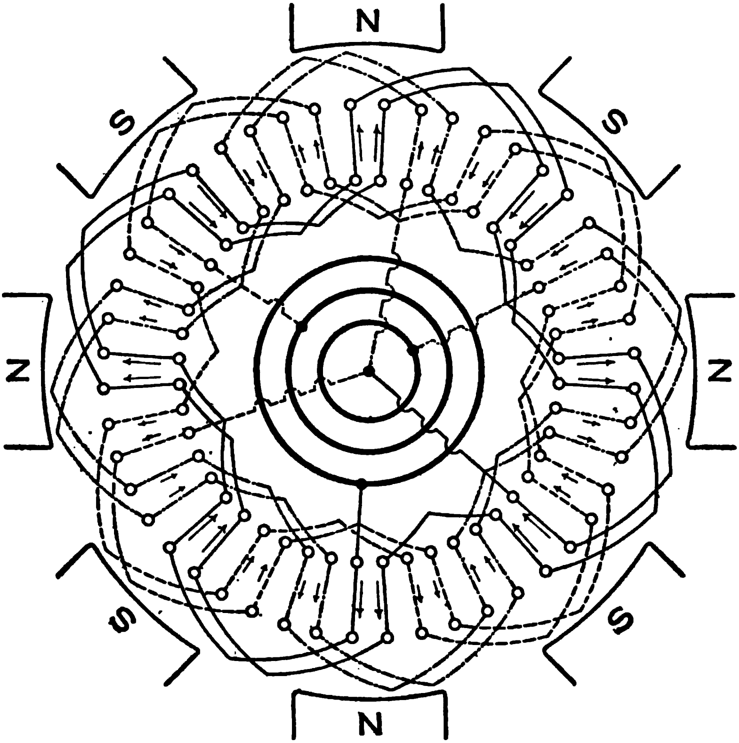

Winding diagram of a Y-connected, 3-ring, 3-phase, 8-pole alternator armature. Large arrows represent maximum-value currents, small arrows represent half-value currents. The central point is the neutral junction and it is evident that total current flowing towards neutral junction at the given instant is zero.

UNIV. OF CALIFORNIA

A TREATISE ON

The Elements of Electrical Engineering

A TEXT BOOK FOR COLLEGES
AND TECHNICAL SCHOOLS

BY

WILLIAM S. FRANKLIN

VOLUME I

DIRECT AND ALTERNATING CURRENT MACHINES AND SYSTEMS

(The book entitled ***Electric Lighting and Miscellaneous Applications of Electricity***, The Macmillan Co., 1912, will be known henceforth as Volume II of Wm. S. Franklin's ***Elements of Electrical Engineering***.)

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PREFACE.

THIS treatise on the *Elements of Electrical Engineering* has been written to encourage a first course in Electrical Engineering covering both direct and alternating current machines and systems. The emphasis throughout is on the simple physics of the subject and on operating engineering.

W. S. Franklin's *Electric Lighting and Miscellaneous Applications of Electricity* (The Macmillan Co., 1912) will henceforth be known as Volume II of Franklin's *Elements of Electrical Engineering.*

Franklin and Esty's *Elements of Electrical Engineering* (Volume I, *Direct Currents*, Volume II, *Alternating Currents*) remains in print as heretofore.

The author is under obligations to Professor Barry MacNutt for freedom to use material from Franklin and MacNutt's *General Physics.*

WM. S. FRANKLIN.

July 14, 1917.

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NATIONAL ORGANIZATIONS AND SOCIETIES RELATING TO ENGINEERING.

Every student of engineering should know about the *United States Bureau of Standards*, Washington, D. C., and about the following national societies:*

Name.	Field.	Headquarters Office.
1. American Society of Civil Engineers.	Civil Engineering.	220 W. 57th St., New York City.
2. American Society of Mechanical Engineers.	Mechanical Engineering.	29 W. 39th St., New York City.
3. American Institute of Electrical Engineers.	Electrical Engineering.	29 W. 39th St., New York City.
4. American Institute of Mining Engineers.	Mining Engineering and Metallurgy.	29 W. 39th St., New York City.
5. American Chemical Society.	Chemistry and Chemical Industry.	Chas. L. Parsons, Sec'y, Box 505, Washington, D. C.
6. National Electric Light Association.	Central Station Operation and Management.	29 W. 39th St., New York City.
7. Illuminating Engineering Society.	Illumination.	29 W. 39th St., New York City.
8. Institute of Radio Engineers.	Radio telegraphy and telephony.	Mr. Davis Sarnoff, Sec'y, 111 Broadway, New York City.
9. Electrochemical Society.	Electrochemistry and Metallurgy.	South Bethlehem, Pa.

The United States Bureau of Standards† is very helpful to the practicing engineer. A letter addressed to the Director, Dr. S. W. Stratton, will bring full information concerning the activities of the Bureau and a list of its publications. The scientific and technological papers of the Bureau are published chiefly in the *Bulletin of the Bureau of Standards*. The *Bureau Circulars* are occasional publications, and some of them are very important. For example, Circular No. 56 on *Standards for Electrical Service* presents a survey of the general field of State and Municipal Regulations Concerning Electrical Service. Equally important are Circular No. 23 on *Standards for Gas Service* and a forthcoming circular on the scientific, engineering and economic aspects of *Street Lighting*. Another important circular is No. 54, entitled *A National Electrical Safety Code.*

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* This list does not include the more highly specialized societies outside of electrical engineering; nor the engineering societies of Great Britain.

† The corresponding establishment in Great Britain is The National Physical Laboratory.

The engineering societies 1 to 6 in the above list are very large in membership and each society is broadly concerned with the scientific, engineering, economic and ethical aspects of its particular field. Every engineer should belong to one or more of these societies. They are helpful to their members, and they tend to bring the much needed counsel of the engineer effectively into municipal, state and national affairs.

The National Electric Light Association also has a very large membership and it has done and is doing a most important work in connection, chiefly, with the business and administrative problems of central station practice. Societies 7, 8 and 9 are highly specialized and they much resemble our national scientific societies in the character of their work.

All of these societies issue important publications, and all, or nearly all, of them make special provision for student membership. A letter addressed to the headquarters office of any of the societies will bring full information concerning the society.

Another organization which touches electrical engineering is the National Board of Fire Underwriters, General Office, 34 Nassau St., New York City. The *National Electrical Code* is a collection of rules and regulations adopted by the National Board for governing the installation of all kinds of electrical apparatus, the object being to minimize fire risks.

The notation used in this text is in conformity with the *Standardization Rules* of the American Institute of Electrical Engineers.

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ELEMENTS OF ELECTRICAL ENGINEERING.

CHAPTER I.

FERROMAGNETISM AND ELECTROMAGNETISM.

1. Poles of a magnet.—The familiar property of a magnet, namely, its attraction for iron, is possessed only by certain parts of the magnet. These parts of a magnet are called the *poles of the magnet.* The poles of a straight bar-magnet are usually at the ends of the bar.

When a bar-magnet is suspended in a horizontal position by a fine thread, it places itself approximately north and south like a compass needle. The north pointing end of the magnet is called its *north pole*, and the south pointing end of the magnet is called its *south pole.*

The north poles of two magnets repel each other, the south poles of two magnets repel each other, and the north pole of one magnet attracts the south pole of another magnet; that is to say, *like magnetic poles repel each other, and unlike magnetic poles attract each other.*

The mutual force action of two magnets is, in general, resolvable into four parts, namely, the forces with which the respective poles of one magnet attract or repel the respective poles of the other magnet. *In the following discussion we consider only the force with which one pole of a magnet acts upon one pole of another magnet*, not the forces with which one complete magnet acts on another complete magnet.

2. Distributed poles and concentrated poles.—The poles of a bar magnet are always distributed over considerable portions of the bar. This is especially the case with short thick bars. In the case of a long slim bar magnet, however, the poles are ordinarily approximately concentrated at the ends of the bar. The forces of attraction and repulsion of concentrated magnet poles are easily formulated, therefore *the following discussion applies to ideally concentrated poles at the ends of ideally slim bar magnets.*

3. **Definition of unit pole.**—Consider a large number of pairs of magnets a, b, c, d, etc., as shown in Fig. 1, the two magnets of each pair being exactly alike.* From such a set it would be possible to select a pair of magnets such that the north pole of one magnet would repel the north pole of the other with a force of one dyne when they (the two north poles) are one centimeter apart; each pole of such a pair is called a *unit pole.* That is, *a unit pole is a pole which will exert a force of one dyne upon another unit pole at a distance of one centimeter.*

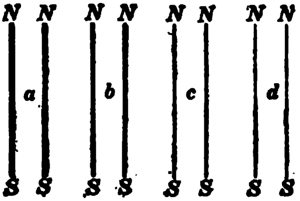

Fig. 1.

Pairs of exactly similar magnets.

4. **Strength of pole.**—Let us choose a slim magnet with unit poles, and let us use one of these unit poles as a " test pole." Any given magnet pole is said to have more or less *strength* according as it exerts more or less force on our " test pole " at a given distance. And the force m (in dynes) with which the

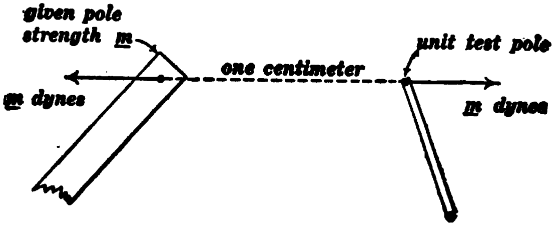

Fig. 2.

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* That is, the magnets of each pair are made of identically the same kind of steel, subjected to the same heat treatment and magnetized by the same means.

given pole attracts or repels (or is attracted or repelled by) the unit test pole at a distance of one centimeter is taken as the measure of the strength of the given pole. That is, *a given pole has* m *units of strength when it will exert a force of* m *dynes on a unit pole at a distance of one centimeter*, as indicated in Fig. 2.

5. Attraction and repulsion of magnet poles.—Unlike poles attract and like poles repel each other, as stated in Art. 1. When the two attracting or repelling poles are *unit poles* their attraction or repulsion is equal to one dyne when they are one centimeter apart, and the attraction or repulsion of two poles whose respective strengths are m' and m'' is equal to $m'm''$ dynes when the poles are one centimeter apart. *One may think of each* **unit** *of* m' *as exerting a force of one dyne on each* **unit** *of* m''. Thus if $m' = 3$ units and $m'' = 2$ units, the force of attraction or repulsion will be six dynes, as indicated in Fig. 3, where each dotted line represents one dyne.

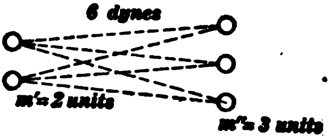

Fig. 3.

6. Coulomb's law. Complete expression for the force of attraction or repulsion of two magnet poles.—Coulomb discovered in 1800 that the force of attraction or repulsion of two magnet poles is inversely proportional to the square of the distance between them. But the force of attraction or repulsion of two magnet poles when they are one centimeter apart is $m'm''$ dynes as explained in Art. 5. Therefore, according to Coulomb's law, the force of attraction or repulsion is $\frac{m'm''}{r^2}$ dynes when the poles are r centimeters apart. That is:

$$F = \frac{m'm''}{r^2} \qquad (1)$$

in which m' and m'' are the respective strengths of two magnet poles, r is their distance apart in centimeters, and F is the force in dynes with which the poles attract or repel each other.

7. Magnetic figures. The magnetic field.—When iron filings are dusted over a pane of glass which is placed over a magnet, the filings arrange themselves in filaments if the glass is jarred slightly. Thus Fig. 4 shows the filaments of filings between the unlike poles of two large magnets. The region between and surrounding the poles is called a *magnetic field*, and the filaments of iron filings show the trend of what are called the *lines of force* of the magnetic field. *Indeed any region is a magnetic field in which a suspended* magnetic needle (like a compass needle) points in a definite direction*, and the direction in which the north pole of the needle points is called the *direction of the magnetic field* at the place where the needle is suspended.

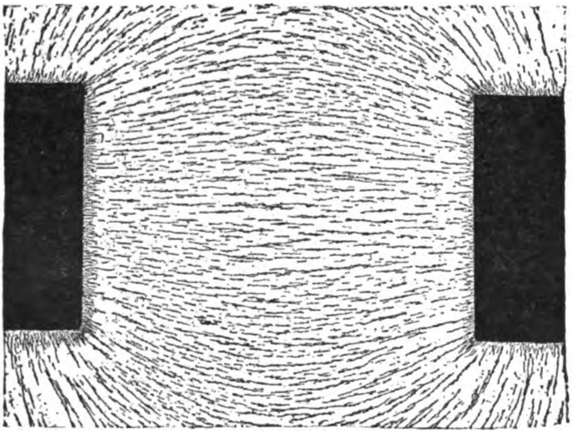

Fig. 4.

8. Intensity of a magnetic field at a point.—When a magnet is placed in a magnetic field a force is exerted on each pole of the magnet by the field. Thus the two arrows in Fig. 5 represent the forces which are exerted on the poles of a small magnet which is placed in the magnetic field between two large magnet poles (see Fig. 4).

The force H in dynes which a magnetic field exerts on a "unit

---

* The needle is supposed to be suspended at its center of mass or center of gravity.

test pole " is used as a measure of the strength or intensity of the field, and this **force-per-unit-pole** *is hereafter spoken of simply as the* **intensity** *of the field.* The unit of magnetic field intensity (one dyne-per-unit-pole) is called the *gauss.* That is to say, a magnetic field has an intensity of one gauss when it will exert a force of one dyne upon a unit pole.

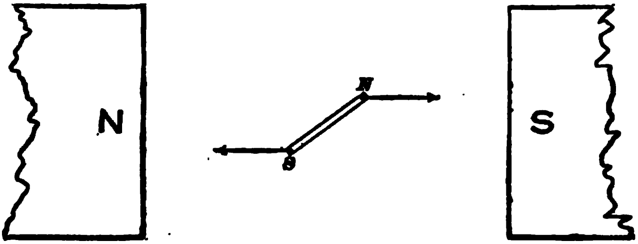

Fig. 5.

The arrows show the forces which act upon the poles of the small magnet.

Complete expression for the force exerted on a magnet pole by a magnetic field.—A magnetic field of which the intensity is H gausses exerts a force of H dynes upon a unit pole as above explained, and it exerts a force of mH dynes upon a pole of which the strength is m units. That is:

$$F = mH \tag{2}$$

in which F is the force in dynes which is exerted on a pole of strength m by a field of intensity H.

Uniform and non-uniform fields.—A magnetic field is said to be *uniform* when it has everywhere the same direction and the same intensity, otherwise the field is said to be *non-uniform.* The earth's magnetic field is sensibly uniform throughout a room. The magnetic field surrounding a magnet is non-uniform. The magnetic field surrounding an electric wire is non-uniform.

9. Direction and intensity of the magnetic field surrounding an " isolated " magnet pole of strength M.—By an " isolated " magnet pole is meant one pole of a very long slim magnet—the other pole being so far away as to be negligible in its action.

The magnetic field in the neighborhood of an isolated north pole is everywhere directed away from the pole as shown by the radiating straight lines (lines of force) in Fig. 6. The magnetic field in the neighborhood of an isolated south pole is everywhere directed towards the pole as indicated in Fig. 7.

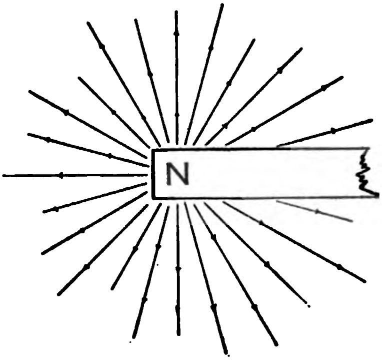

Fig. 6.

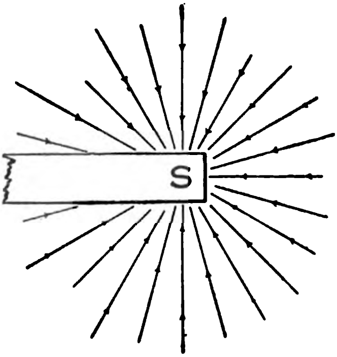

Fig. 7.

Consider two magnet poles M and m which are at a distance of r centimeters apart as shown in Fig. 8. The force F with which M repels m is equal to $\frac{Mm}{r^2}$, according to Art. 6; but the force exerted on m can also be expressed as equal to mH *where* H *is the intensity at* m *of the magnetic field which is due to* M. Therefore $mH = \frac{Mm}{r^2}$, whence we have:

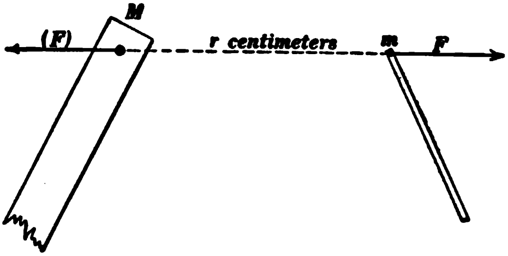

Fig. 8.

$$H = \frac{M}{r^2} \tag{3}$$

in which H is the intensity of the magnetic field produced by the pole M at a place which is r centimeters from M.

10. Magnetic flux.—A conception of great importance in the consideration of induced electromotive force is the conception of magnetic flux. The lines of force in Fig. 4, as visualized by the filaments of iron filings, suggest the idea that something "flows out of" one magnet pole, "traverses" the surrounding region in smoothly curved paths, and "flows into" the other magnet pole.

Definition of magnetic flux.—Figure 9 represents a plane area of a square centimeters at right angles to a uniform magnetic field of which the intensity is H gausses. The product aH is called the *magnetic flux* across the area. That is

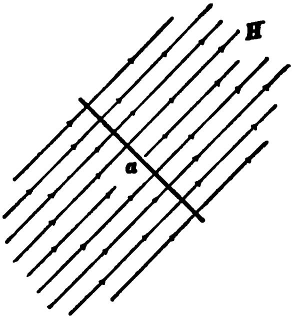

Fig. 9.

$$\Phi = aH \tag{4}$$

where Φ is the magnetic flux across an area of a square centimeters at right angles to a magnetic field of which the intensity is H gausses.

The unit of magnetic flux is the flux across an area of one square centimeter $(a = 1)$ when the area is at right angles to a magnetic field of which the intensity is one gauss $(H = 1)$, or it is the flux across n square centimeters of area when the area is at right angles to a magnetic field of which the intensity is $1/n$th of a gauss. The unit of magnetic flux is properly called the *maxwell;* the common usage however is to call the unit of flux a *line of force* for the following reason:

Consider any magnetic field whatever, and imagine a surface or shell BB, as shown in Fig. 10. Imagine the surface BB to be divided up into small squares such that one unit of magnetic flux crosses each square, and imagine lines of force to be drawn so that one line of force passes through each square. *Then the* *magnetic flux across any other surface or shell AA placed anywhere in the field will be equal to the number of these* **lines of force** *which cross AA.*

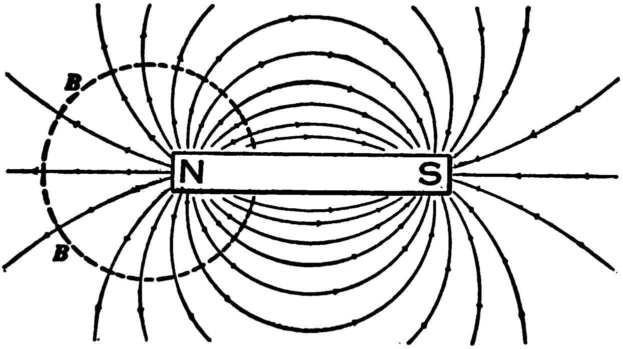

Fig. 10.

11. **Proposition.**—*The amount of magnetic flux Φ which emanates from a north magnet pole is equal to $4\pi M$, where M is the strength of the pole.* Imagine a spherical surface of radius r drawn with its center at the pole M as shown in Fig. 11. The area of this spherical surface is $4\pi r^2$. The magnetic field at the spherical surface due to M is everywhere at right angles to the surface, and its intensity at the surface is $H = \frac{M}{r^2}$ according to Art. 9. Therefore, (neglecting the small portion of the sphere which falls inside of the slim magnet at b) the magnetic

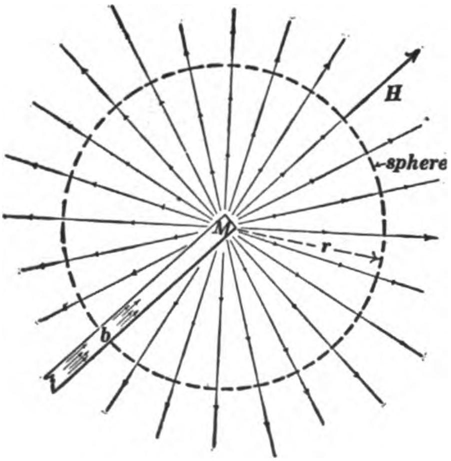

Fig. 11.

flux across the spherical surface is equal to $4\pi r^2 \times \frac{M}{r^2}$ or $4\pi M$, according to Art. 10. But the flux across the spherical surface is the flux that emanates from the pole. Therefore the flux that emanates from the pole is $4\pi M$.

12. Magnetization.—When a piece of iron or other magnetic substance is placed in a magnetic field it becomes a magnet. Thus a bar of iron or steel placed in the intense magnetic field inside of a coil of wire through which an electric current is flowing, becomes strongly magnetized. This action of a magnetic field upon iron and steel is called *magnetization.*

When a piece of iron is placed in a magnetic field the original trend of the lines of force in the field is greatly altered. Thus Fig. 12 shows the effect of a small piece of iron upon the magnetic field between two flat-ended magnet poles. In the absence of the small piece of iron the field is as shown in Fig. 4.

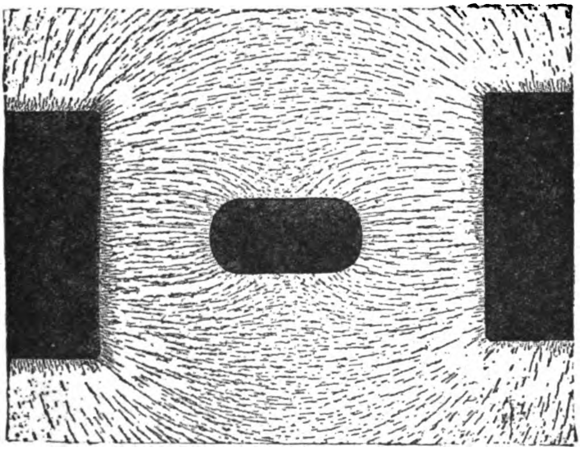

Fig. 12.

The effect of a piece of iron in a magnetic field is always such as to suggest that " iron is a better carrier of lines of force than air." The lines of force tend to converge into the piece of iron and pass through it.

13. The magnetic field surrounding a straight electric wire. Direction of current.—Figure 13 is a photograph of the filaments of iron filings on a horizontal glass plate, the black circle is a hole through the plate, and a straight electric wire passes vertically through this hole. *The lines of force of the magnetic field near a straight electric wire encircle the wire* as shown by the filaments of iron filings in Fig. 13. The north pole of a magnet would be everywhere pulled along these lines of force in one direction (a south pole in the opposite direction). Therefore it is evident from Fig. 13 that the force exerted on a magnet pole by a straight electric wire tends to carry the pole round and round the wire, and *it has been agreed to think of the current in the wire as flowing in the direction in which a right-handed nut would travel*

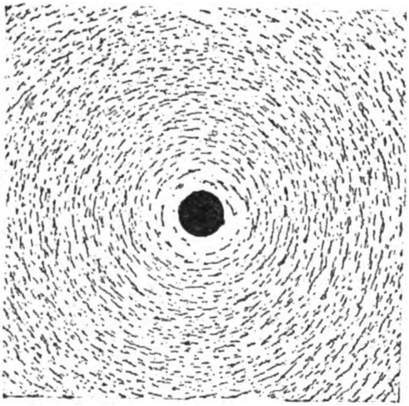

Fig. 13.

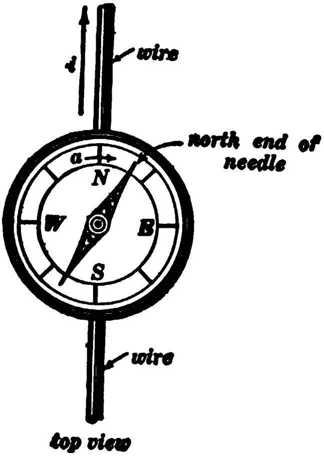

Fig. 14.

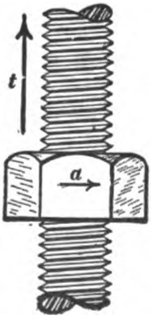

Fig. 15.

if turned in the direction in which a **north pole** *tends to circle round the wire.* This may be understood with the help of Figs. 14 and 15. Figure 14 is a top view of a compass placed over a straight electric wire, and the arrow *a* in Fig. 14 shows the direction in which the north pole of the compass needle starts to move. Now if the nut in Fig. 15 is turned in the direction indicated by the arrow *a*, the nut will travel in the direction of the arrow *t*; and this is the direction of " flow " of the current *i* in Fig. 14.

When it is desired to show an end-view of a wire through which current is flowing, the section of the wire is represented by a small circle, *current flowing towards the reader is represented by a dot in the circle*, as if one were looking endwise at the point of an arrow; and *current flowing away from the reader is represented by a cross in the circle*, as if one were looking at the feathered end of an arrow, as shown in Fig. 16.

Fig. 16.

⊙ Current flowing towards reader.

⊕ Current flowing away from reader.

When an iron rod is magnetized by the flow of current round it, *the north pole of the rod is at the end towards which a nut would travel (on a right-handed screw) if the nut were turned in the direction in which the current flows round the rod* as shown in Fig. 17.

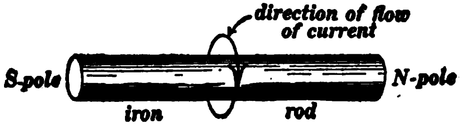

Fig. 17.

14. Side push of a magnetic field on an electric wire.—One aspect of the magnetic effect of the electric current is embodied in the electromagnet, which is an iron rod with a winding of insulated wire. When electric current flows through the winding the rod is magnetized. Another aspect of the magnetic effect of the electric current is the production of magnetic field in the region surrounding an electric wire as shown in Fig. 13. Still another aspect of the magnetic effect of the electric current is shown in Fig. 18. A wire *AB* through which an electric current is flowing is stretched across the end of a magnet, and the wire is pushed sidewise by the magnet as stated in the legend under the figure. If the current is reversed or if the magnet is turned end for end the side push on the wire is reversed.

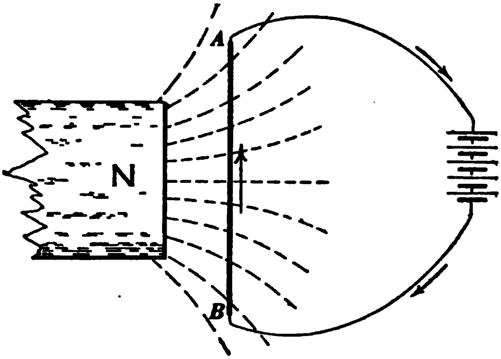

Fig. 18.

The wire *AB* is pushed away from the reader.

The side force on the wire in Fig. 18 is exerted by the magnet, and this force is no doubt transmitted by something which connects the magnet and the wire together, namely, the *magnetic lines of force* which emanate from the magnet. These magnetic lines of force are indicated by the dotted lines in Fig. 18.

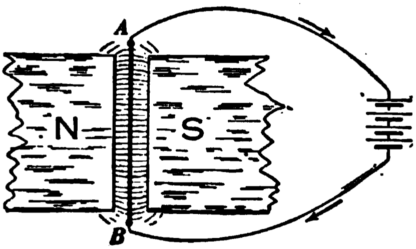

Fig. 19.

The wire *AB* is pushed away from the reader.

Figure 19 shows a straight wire *AB* placed in a narrow air gap between two opposite magnet poles. The fine lines across the gap represent the magnetic lines of force in the air gap, and these lines of force push the wire sidewise (away from the reader in Fig. 19).

When an electric wire is placed in a magnetic field at right angles to the lines of force of the field, a force is exerted on the wire (a side push on the wire) at right angles to the lines of force and at right angles to the wire.

15. **Explanation of the side push exerted upon an electric wire by a magnetic field.**—Figure 4 represents the magnetic lines of force between two opposite magnetic poles, and the attraction of the two opposite poles for each other may be thought of as due to a state of tension in the lines of force. That is, the lines of force may be thought of as stretched rubber-like filaments leading from pole to pole in Fig. 4, and the attraction of the two opposite poles may be thought of as the tendency of these stretched filaments to shorten.

Figure 20 shows how the magnetic field between the two opposite poles in Fig. 4 is modified by the presence of an electric wire, and it is evident from the trend of the lines of force in Fig. 20

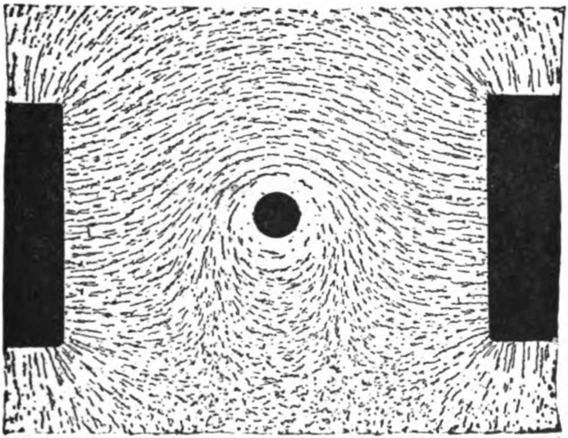

Fig. 20.

The black circle represents a section of the straight electric wire which stands perpendicular to the plane of the paper. The wire is pushed towards the bottom of the page.

that the side push on the wire may be attributed to a state of tension in the lines of force (tendency for the lines of force to shorten like stretched rubber filaments).

16. Side push of magnetic field on an electric wire exemplified in the direct-current ammeter.—Figure 21 shows the steel and iron parts of a direct-current ammeter. There is a strong magnetic field in the air spaces between the soft iron cylinder *C*

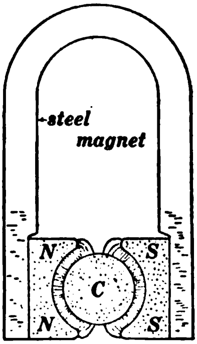

Fig. 21.

Dotted portions are soft iron.

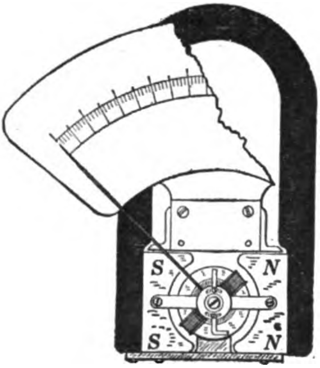

Fig. 22.

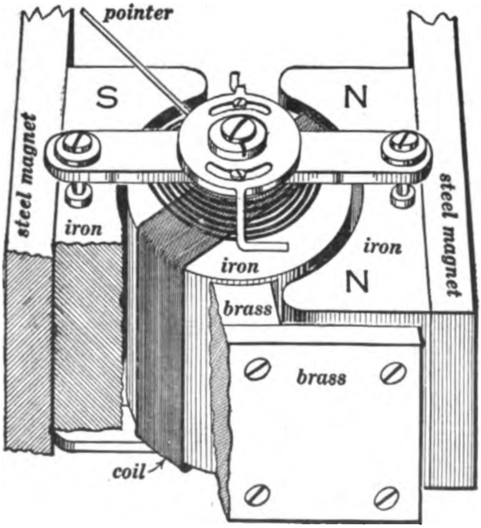

Fig. 23.

and the soft iron pole-pieces *NN* and *SS*, and the lines of force are radial as indicated in the figure. The essential parts of the complete ammeter are shown in Fig. 22; and Fig. 23 shows the moving coil on an enlarged scale. The moving coil is supported on pivots; current is led into and out of the moving coil through the two hair springs, one at each end of the pivot-axis; and the magnetic field pushes sidewise on the limbs of the coil thus turning the coil and moving the pointer over the scale. The limbs of the coil are the portions which lie between *C* and *NN* and *SS*.

17. Side push of magnetic field exemplified by the magnetic blow-out.—When an electric switch is opened the current continues for a short time to flow across the switch gap, forming what is called an electric arc, as shown in Fig. 24. This arc melts the contact parts of the switch, and the switch is soon spoiled. This difficulty may be obviated to some extent by always opening the switch quickly and unhesitatingly, but where the switch is to be opened and closed hundreds of times per day, as in the control of a street car motor, it is necessary to blow out the arc so as to avoid the rapid wear of the switch contacts by fusion. This blowing out of the arc is accomplished by a magnet placed as shown in Fig. 25. This magnet pushes sidewise on the arc (towards or away from the reader in Fig. 25), and this sidewise push on the arc lengthens it very quickly and breaks the circuit.

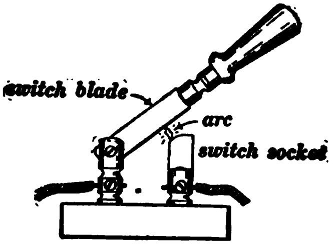

Fig. 24.

Showing arc when switch is opened.

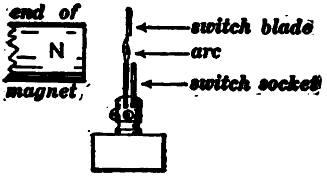

Fig. 25.

The arc is pushed towards or away from the reader.

18. Side push of magnetic field exemplified in the electric motor.—Figure 26 shows a soft iron cylinder *AA** between the two pole pieces of a large electromagnet. There is an intense magnetic field in the air gaps between *A* and the pole pieces *N* and *S*, as indicated by the fine lines (lines of force). Imagine straight pieces of insulated wire to be laid on and fastened to the cylinder *A*, these wires being parallel to the axis of *A*; and imagine electric current to flow in these wires as indicated by the dots and crosses in Fig. 27. Then all of the wires in the gap spaces (where there is a strong magnetic field as indicated

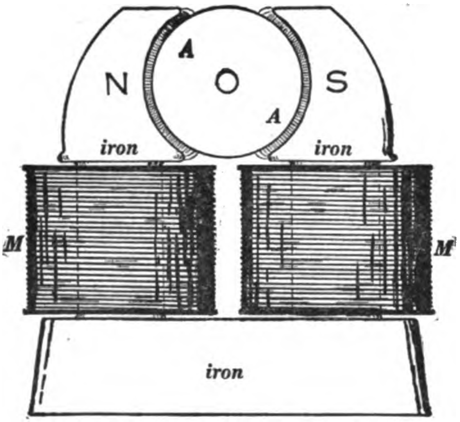

Fig. 26.

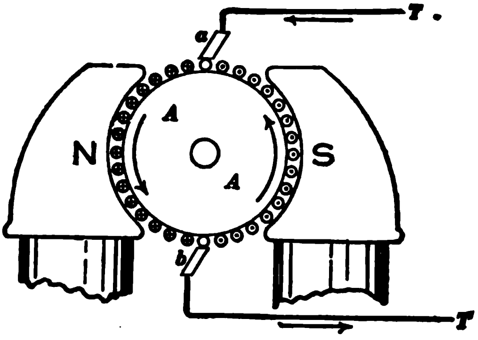

Fig. 27.

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* This cylinder is built up of thin sheet iron disks for reasons which are explained in Art. 47.

in Fig. 26) will be pushed sidewise, and these side pushes will all tend to turn the cylinder as indicated by the curved arrows in Fig. 27.

The arrangement in Fig. 27 is called an *electric motor.* The electromagnet NS is called the *field magnet,* and the magnetizing coils MM are called the *field coils* or *field windings.* The rotating cylinder AA with its winding of wire is called the *armature.*

The arrangement of the wires on the armature and the method of leading current into and out of them so that the current may flow as indicated by the dots and crosses in Fig. 27 can be most easily understood by considering the simplest type of armature winding, namely, the so-called *ring-winding,* the essential features of which are shown in Fig. 28. An iron ring AA is wound uniformly with insulated wire as shown, the ends of the wire being spliced together and soldered so as to make the winding endless. Imagine the insulation to be removed from the outward faces of the wire windings on the ring so that two stationary metal or carbon blocks (*brushes*) a and b can make good electrical contact with the wires as the ring rotates. Then if current is led into the windings through brush a and out through brush $b,$ the current will flow towards the reader in the wires which lie under the south pole $S,$ and away from the reader in the wires which lie under the north pole $N,$ as shown by the dots and crosses in Fig. 27.*

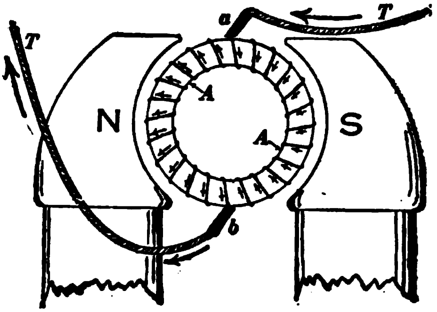

Fig. 28.

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* Let the reader carefully trace the flow of current in Fig. 28. The current which enters at brush a divides, and half of the current flows through the windings on each side of the armature.

In practice, short lengths of wire are attached to the various turns of wire on the ring and led to copper bars near the axis of rotation, as shown in Fig. 29. These copper bars are insulated from each other, and sliding contact is made with these copper bars as indicated in Fig. 29, instead of being made as indicated in Fig. 28. The set of insulated copper bars is called the *commutator.*

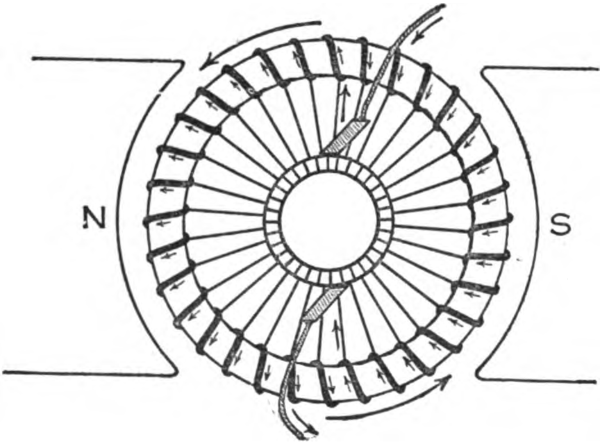

Fig. 29.

19. **Strength of electric current magnetically defined.**—Consider a straight electric wire stretched across a uniform magnetic field, the wire being at right angles to the field as shown in Fig. 30. Let us suppose, for a moment, that the field is of unit

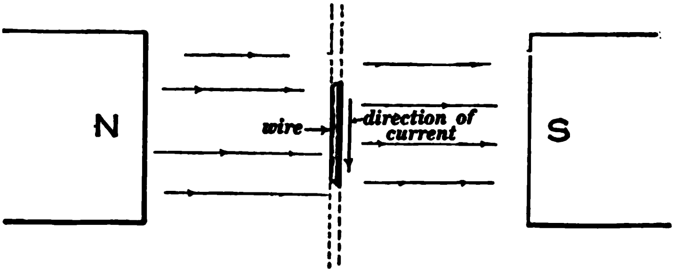

Fig. 30.

Wire is pushed towards reader.

intensity. The force in dynes with which this unit field pushes sidewise on one centimeter of the electric wire has been adopted as the fundamental measure of the strength of the current in the wire. This **force-per-unit-length-of-wire-per-unit-field-intensity** is called simply *the strength of the current in the wire*, and it is represented by the letter I. The force pushing sidewise on l centimeters of the wire is lI dynes; and if the field intensity is H gausses instead of one gauss, then the force is H times as great, or lIH dynes. That is

$$F = lIH, \tag{5}$$

in which F is the force in dynes pushing sidewise on l centimeters of wire at right angles to a uniform magnetic field of which the intensity is H gausses, and I is the strength of the current in the wire.

Note. When an electric wire is not at right angles to a magnetic field the side push in dynes is $F = IlH \sin \theta$, where θ is the angle between the wire and the lines of force of the field.

Definition of the abampere.—A wire is said to carry a current of one abampere when one centimeter of the wire is pushed sidewise with a force of one dyne, when the wire is stretched across a magnetic field of which the intensity is one gauss, the wire being at right angles to the field. The current I in equation (5) is expressed in abamperes when F is expressed in dynes, l in centimeters and H in gausses. The abampere is the c.g.s. unit of current.

Definition of the ampere.—The ampere is defined as *one tenth of an abampere.*

The electrical units now almost universally employed, namely, the ampere, the volt, the ohm, the coulomb, the farad, and so forth, are not the c.g.s. units but convenient multiples or submultiples of them. The c.g.s. units as a rule have no names, therefore it is convenient to call the c.g.s. unit of current the *abampere*, the c.g.s. unit of resistance the *abohm*, the c.g.s. unit of electromotive force the *abvolt*, the c.g.s. unit of capacity the *abfarad*, and so forth.

The c.g.s. units here referred to are the so-called "electromagnetic" c.g.s. units. The c.g.s. units of the "electrostatic system" are entirely ignored in this text.

The international standard ampere.—Very careful measurements have shown that one ampere deposits 0.001118 gram of silver per second from a solution of silver nitrate in water; and, inasmuch as it is very difficult to measure a current accurately in terms of its magnetic effect so as to get the value of the current directly in amperes, the ampere has been legally defined as the current which will deposit *exactly* 0.001118 gram of silver in one second from a solution of pure silver nitrate in water.

PROBLEMS.

1. The accompanying diagram Fig. 31 shows a lamp L with connections arranged so that the lamp can be turned on or off at switch A (or B) regardless of how switch B (or A) stands. Make four diagrams like Fig. 31 showing the four possible combinations of switch-positions, and indicate the flow of current, if any, by arrows.

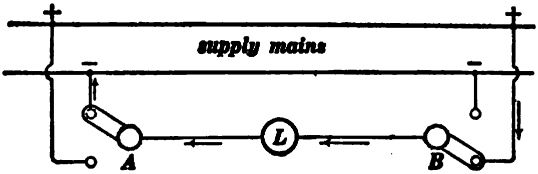

Fig. 31.

2. The six small circles in Fig. 32 represent the contact posts on a double-pole double-throw switch, and the dotted lines represent the switch blades. The diagram shows the lamp L taking current from the direct-current mains. Make a diagram

showing the lamp taking current from the alternating current mains.

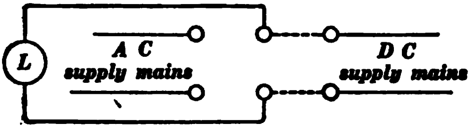

Fig. 32.

3. The six small circles in Fig. 33 represent the contact posts on a double-pole double-throw switch with crossed connections adapting it for use as a reversing switch, and the dotted lines represent the switch blades. Make a diagram showing a reversed flow of current through the receiving circuit *R*.

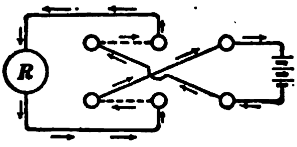

Fig. 33.

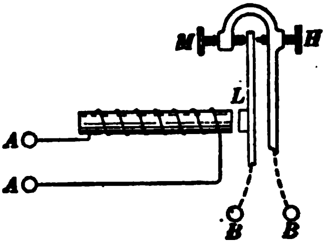

Fig. 34.

4. Figure 34 shows the diagram of connections of an ordinary telegraph relay. A "local" circuit connected to the binding posts *BB* is opened and closed as the lever *L* of the relay is moved back and forth by pulses of current coming over the telegraph line which is connected through the binding posts *AA* to ground; *M* is a screw with a metal tip, and *H* is a screw with a hard-rubber insulating tip. Make a diagram showing *M* and *H* interchanged, and showing *AA* and *BB* connected to each other and to a battery so that the relay will buzz like an ordinary interrupter bell.

5. It is possible to connect any number of bell circuits to one battery and have any number of push buttons arranged to close each bell circuit; but, of course, a small battery cannot operate more than one or two bells simultaneously. Make a

diagram showing two bells connected to one battery with two push buttons for closing the circuit of each bell.

6. A room 6 meters long by 5 meters wide by 3 meters high has its longest dimension magnetic north and south. The intensity of the earth's field in the room is 0.62 gauss and the dip is 72°. Find the number of lines of magnetic flux across each of the walls, the ceiling, and floor of the room and specify in each case whether the flux is passing out of the room or into the room. Ans. East wall, 0; west wall, 0; north wall, 28,740 maxwells out; south wall, 28,740 maxwells in; ceiling, 176,900 maxwells in; floor, 176,900 maxwells out.

7. The pole face of the field magnet of a dynamo has an area 20 centimeters by 30 centimeters. The magnetic field between the pole faces and the armature core is perpendicular to the pole face at each point and its intensity is 6,000 gausses. Calculate the number of lines of force which pass from the pole face into the armature core.

8. Figure 35 shows a magnet *NS* placed near a long straight electric wire. The wire exerts forces on the magnet poles *N* and *S* as indicated by the arrows *F'* and *F''*. Draw a diagram showing the total, or resultant, force exerted on the magnet by the wire.

9. The current in Fig. 35 is reversed so as to flow towards the reader. Make a diagram showing the forces exerted on the poles *N* and *S* by the wire, and make a diagram showing the total, or resultant, force exerted on the magnet.

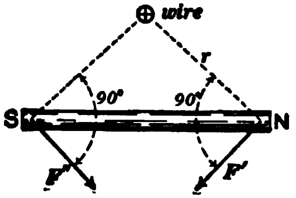

Fig. 35.

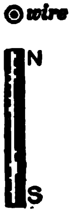

Fig. 36.

10. The small circle with a dot in Fig. 36 represents a straight

wire at right angles to the paper with current flowing towards the reader. Draw arrows showing the directions and approximate values of the forces exerted by the electric wire on the poles N and S of the magnet.

Note. The magnitude of the force exerted on a magnet pole by a long straight electric wire is inversely proportional to the distance of the pole from the wire.

11. Specify the direction of the side push exerted on the wire by the magnet pole in Fig. 37.

Note. The force exerted on a wire by a magnetic field is at right angles to the wire and at right angles to the lines of force of the field. The direction of a magnetic field at a point is the direction in which a compass needle would point if placed at that point, the *north pole* of the needle being thought of as the pointing end. Now a magnetic needle put in place of the wire in Fig. 37 would point towards S, as shown by the short arrows *ff* in Fig. 38. Therefore the wire will be pushed towards A or towards B; to determine which, the following considerations are sufficient: The lines of force of the magnetic field *due to the current in the wire* encircle the wire as indicated by the curled arrows c in Fig. 38, and the heads of the curled arrows are in the direction in which a right-handed screw would have to be turned to travel in the direction of flow of the current in the wire. Now the lines of force *ff* bend to one side of the wire so as to go *with* the curled arrows c, as shown in Fig. 39; and the tension of these bent lines of force pushes sidewise on the wire in the direction of the arrow F.

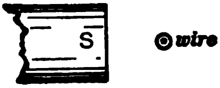

Fig. 37.

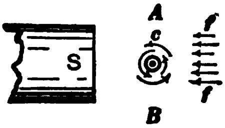

Fig. 38.

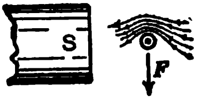

Fig. 39.

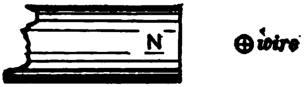

Fig. 40.

12. Specify the direction of the side push exerted on the wire by the magnet pole in Fig. 40.

13. The armature of a dynamo has a length, under the pole-face, of 30 cm. The magnetic field intensity between the pole-face and the armature core is 6,000 gausses. The surface of the armature is covered with straight wires parallel to the axis of the armature. Each of these wires carries a current of 75 amperes. Calculate the force acting on each wire. Ans. 1,350,000 dynes.

14. A horizontal electric light wire stretched due magnetic north and south carries 1,000 amperes of current flowing towards the north. The length of the wire is 250 meters, the intensity of the earth's field is 0.57 gauss and the magnetic dip is 63°. Find the value of the force pushing on the wire and specify its direction. Ans. 1,269,500 dynes west.

Note. It is only the component of the field which is at right angles to the wire that exerts side push.

15. A rectangular frame 25 × 40 cm. has 10 turns of wire wound upon it. The frame is balanced horizontally upon an axis pointing due magnetic east and west. A current of 28 amperes is sent through the wire. Required the distance from the axis at which a 10-gram (9,800-dyne) weight must be hung to balance the torque action due to the earth's magnetic field at a place where its intensity is 0.57 gauss and its dip is 63°. Ans. 0.74 cm.

Note. It is convenient to take, say, the 40-centimeter edge of the coil as being parallel to the balance axis.

CHAPTER II.

HEATING EFFECT OF THE ELECTRIC CURRENT.

20. The heating effect of the electric current.—Careful observation shows that every portion of an electric circuit is heated more or less by the current. It is important to understand, however, that the heating effect of the electric current is *not* the heating of a wire to a definite temperature, *it is the generation of heat in the wire at a definite rate*, so many calories or joules of heat per second. A given wire carrying a given current always grows hotter and hotter until heat is given off by the wire as fast as heat is generated in the wire by the current. Therefore the final temperature of an electric wire depends upon the surroundings of the wire. Thus if an electric wire is enclosed in a narrow air space its temperature may rise very considerably before it gives off heat as fast as heat is generated in it by the current, whereas the rise of temperature of the same wire in the open air would be much less with the same current flowing through it.

This heating of electric wires is an important matter because excessive heating of a wire in a building involves a risk of fire, and because even a moderate rise of temperature may cause a serious damage to the insulating material with which the wire is covered, especially if the insulating material is rubber. The larger the current the hotter the wire will become, and the accompanying table of "carrying capacities" of copper wires gives the following data:

Column 1 gives sizes of wires in Brown and Sharpe's gauge.

Column 2 gives diameters of wires in mils, a mil equals one thousandth of an inch.

Column 3 gives the current in amperes required to cause a bare copper wire of the specified size to be heated 50° F. above the temperature of the surrounding air when the bare wire is stretched across a room in which the air is still.

Column 4 gives the current in amperes which a rubber covered wire under a wooden moulding can carry without becoming hot enough to seriously damage the rubber insulation.

Column 5 gives the current in amperes which a wire under a wooden moulding and with other than rubber insulation can safely carry without damage to the insulation.

Columns 4 and 5 give the limiting carrying capacities of copper wires according to the National Board of Fire Insurance Companies.

TABLE OF CARRYING CAPACITIES OF COPPER WIRES.

(From the National Electrical Code.)

Brown and Sharpe gauge.	Diameters in mils.	Amperes to give 50° F. rise of bare wires in still air.	Carrying capacities in amperes of wires covered with rubber.	Carrying capacities in amperes of wires covered with non-rubber insulation
18	40	6.0	3	5
16	51	8.5	6	8
14	64	12.1	12	16
12	81	17.1	17	23
10	102	24.3	24	32
8	128	41.5	33	46
6	162	58.8	46	65
5	182	69.7	54	77
4	204	83.3	65	92
3	229	98.8	76	110
2	258	117.6	90	131
1	289	140.0	107	156
0	325	169.8	127	185
00	365	201.5	150	220
000	410	240.2	177	262
0000	460	286.0	210	312
——	632	462.0	330	500
——	776	631.0	450	680
——	1,000	922.0	650	1,000
——	1,225	1,250.0	850	1,360
——	1,414	1,550.0	1,050	1,670

For insulated aluminum wire the safe carrying capacity is 0.84 of that given for copper wire with the same kind of insulation.

Utilization of the heating effect of the electric current.—The heating effect of the electric current is utilized in a great variety of electric heating and cooking devices, in a variety of electric lamps, in the safety fuse which melts and opens an electric circuit in case of excessive current, and in a variety of electric furnaces.*

21. The idea of resistance.—The term *electric current* and the *idea of flow* of the electric current through a wire have been adopted because a battery maintaining an electric current in a circuit of wire is to some extent analogous to a pump forcing a stream of water through a circuit of pipe. The idea of *electrical resistance* came also from hydraulic analogy as follows:

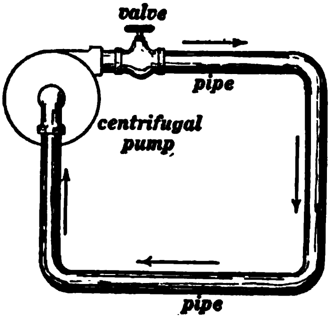

Fig. 41.

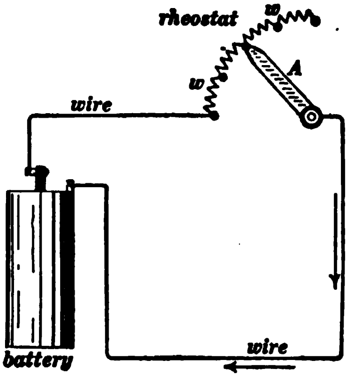

Fig. 42*b*.

A pump forces air or water through a circuit of pipe as shown in Fig. 41, and the work done in driving the pump is converted into heat in the pipe, because the flow of the water or air through the pipe is opposed by friction. Therefore we may speak of the friction or resistance of the pipe.

A dynamo (or battery) forces an electric current through a circuit of wire as shown in Fig. 42, and the work done in driving the dynamo is converted into heat in the wire. Therefore we think of the " flow " of " electricity " through the wire as being " opposed " by a kind of " friction," and we speak of the *resistance* of the wire.

The amount of water flowing per second can be increased or decreased by opening or closing the valve so as to decrease or

The strength of the electric current can be increased or decreased by turning the metal arm *A* so as to include less or increase the friction which opposes the flow.

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* See *The Electric Furnace* by Alfred Stansfield, McGraw-Hill Book Co., New York, 1914.

more of the wires *ww* thus decreasing or increasing the resistance of the circuit.

22. Joule's law.—The rate at which heat is generated in a given piece of wire is proportional to the square of the current flowing through the wire, or, in other words, the amount of heat generated is proportional to the square of the current and proportional to the elapsed time. Therefore we may write:

$$h = RI^2t \tag{6}$$

where h is the amount of heat generated in a wire in t seconds by a current whose strength is I, and R is a proportionality factor which has a definite value for the given piece of wire. This factor R is called the *resistance* of the wire.

The abohm.—A piece of wire is said to have a resistance of one abohm when one erg of heat is generated in it per second by a current of one abampere. The factor R in equation (6) is expressed in abohms when h is expressed in ergs, I in abamperes and t in seconds.

The ohm.—A piece of wire is said to have a resistance of one ohm when one joule of heat is generated in it per second by a current of one ampere. The factor R in equation (6) is expressed in ohms when h is expressed in joules, I in amperes and t in seconds. One ohm is equal to one thousand million *ab*-ohms.

23. Dependence of resistance upon the length and size of a wire. Definition of resistivity.—The resistance R of a wire of given material is directly proportional to the length l of the wire, and inversely proportional to the sectional area s of the wire; that is

$$R = k\frac{l}{s} \tag{7}$$

in which k is a constant for a given material, and it is called the *resistivity** of the material. The exact meaning of the factor

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* Sometimes called *specific resistance*. The reciprocal of the resistivity of a substance is called its *conductivity*.

k may be made apparent by considering a wire of unit length ($l = 1$) and of unit sectional area ($s = 1$). In this case R is numerically equal to k, that is to say, *the resistivity of a material is numerically equal to the resistance of a wire of that material of unit length and unit sectional area.*

Electrical engineers nearly always express lengths of wires in feet and sectional areas in circular mils.* If equation (7) is used to calculate the resistance of a wire in ohms when the length l of the wire is expressed in feet and the sectional area s in circular mils, then the value of k must be the resistance of a wire of the

TABLE.—RESISTIVITIES AND TEMPERATURE COEFFICIENTS.

	a	b	β
Aluminum wire (annealed) at 20° C.	27.4×10^{-7}	16.5	+ 0.0039
Copper wire (annealed) at 20° C.	17.24×10^{-7}	10.4	+ 0.0040
Iron wire (pure annealed) at 20° C.	95×10^{-7}	58.0	+ 0.0045
Steel telegraph wire at 20° C.	150×10^{-7}	91†	+ 0.0043†
Steel rails at 20° C.	120×10^{-7}	72†	+ 0.0035†
Mercury at 0° C.	943.4×10^{-7}	—	+ 0.00088
Platinum wire at 0° C.	89.8×10^{-7}	54.0	+ 0.00354
German-silver wire at 20° C.	212×10^{-7}	127†	+ 0.00025†
Manganin wire (Cu 84, Ni 12, Mn 4) at 20° C.	475×10^{-7}	286	
"Ia Ia" metal wire, hard (copper-nickel alloy) at 20° C.	500×10^{-7}	300†	− 0.00001†
"Climax" or "Superior" metal (nickel-steel alloy) at 20° C.	800×10^{-7}	480†	+ 0.00067†
Arc-lamp carbon at ordinary room temperature	0.005		− 0.0003†
Sulphuric acid, 5 per cent. solution at 18° C.	4.8 ohms		− 0.0120*
Ordinary glass at 0° C. (density 2.54)	10^{15} ohms†		
Ordinary glass at 60° C.	10^{12} ohms†		
Ordinary glass at 200° C.	10^{8} ohms†		

a = resistance in ohms of a bar 1 centimeter long and 1 square centimeter sectional area.

b = resistance in ohms of a wire 1 foot long and 0.001 inch in diameter.

β = temperature coefficient of resistance per degree centigrade (mean value between 0° C. and 100° C.).

Near ordinary room temperature the resistance of a manganin wire is very nearly independent of temperature.

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* One *mil* is a thousandth of an inch. One *circular mil* is the area of a circle of which the diameter is one mil. The area of any circle in circular mils is equal to the square of the diameter in mils. Thus a wire 100 mils in diameter has a sectional area of 10,000 circular mils.

† Between 18° C. and 19° C.

‡ These values differ greatly with different samples.

given material one foot long and one circular mil in sectional area. The accompanying table gives the resistivities of the more important substances together with their temperature coefficients of resistance.

24. Variation of resistance with temperature.—The electrical resistance of a conductor which forms a portion of an electrical circuit varies with temperature. Consider, for example, (*a*) an iron wire, (*b*) a copper wire, (*c*) a platinum wire, (*d*) a German-silver wire, (*e*) a carbon rod, and (*f*) a column of dilute sulphuric acid, *each of which has a resistance of 100 ohms at zero centigrade.* Then the values of the resistances of (*a*), (*b*), (*c*), (*d*), (*e*) and (*f*) at other temperatures, as determined by experiment, are shown by the ordinates of the curves in Fig. 43. As shown in this figure, iron and copper increase greatly in resistance with rise of temperature, and German silver increases only slightly

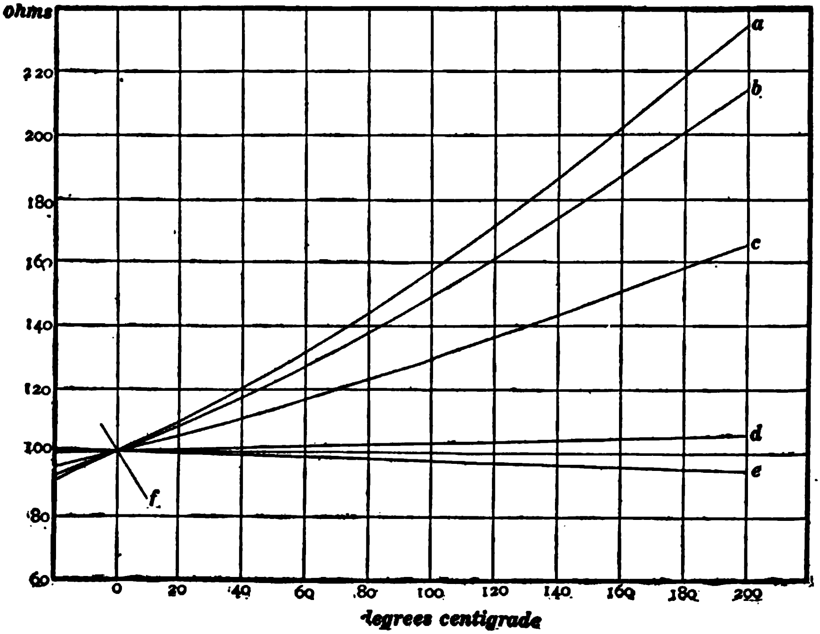

Fig. 43.

in resistance with rise of temperature. But carbon and sulphuric acid decrease in resistance with rise of temperature, carbon to a very slight extent and dilute sulphuric acid to a very great extent.

For most practical purposes the curves in Fig. 43 may be thought of as straight lines so that:

$$R_\theta = R_0(1 + \beta\theta) \tag{8}$$

where R_0 is the resistance of a wire at 0° C., R_θ is the resistance of the wire at θ° C. and β is a constant for the material of which the wire is made. This constant β is called the *temperature coefficient of resistance* of the material. The values of β for various materials are given in the table in Art. 23.

25. Power required to maintain a current in a circuit in which all of the energy reappears in the circuit in the form of heat in accordance with Joule's law.—Work must of course be done in forcing an electric current through an electric motor, but all of the work so done does not reappear in the motor wires as heat, a large portion reappears at the motor pulley and is delivered as mechanical energy to the machine which is driven by the motor.

Work also must be done in forcing an electric current backwards through an exhausted storage battery (to charge the battery), but all of the work so done does not reappear as heat in the circuit, a large portion of the work is expended in bringing about the chemical action which takes place as the battery is charged.

When a current is maintained in a simple circuit of wire, or in a circuit containing glow lamps, all of the work done in maintaining the current *does* reappear in the circuit as heat, *and the rate at which work is done in maintaining the current is equal to the rate at which heat energy appears in the wire.* Now heat energy is generated by a current of I amperes at the rate of RI^2 joules per second in a circuit of which the resistance is R ohms. Therefore, to maintain a current of I amperes in a circuit having a resistance of R ohms work must be done at the rate of RI^2 joules per second. That is:

$$P = RI^2 \tag{9}$$

where P is the power in watts (or joules per second) required to maintain a current of I amperes in a circuit of which the resistance is R ohms. *Equation (9) is true only when all of the work expended in maintaining the current reappears in the circuit as heat in accordance with Joule's law.*

26. Electromotive force.—We think of an electric generator (battery or dynamo) as a kind of "pump" "forcing" a "current of electricity" through a circuit, the "flow" of electric current being opposed by a kind of "resistance." The centrifugal pump, or fan blower, is more nearly like the electric generator (battery or dynamo) than the ordinary piston pump, and therefore the centrifugal pump is used as the basis of the following discussion.

Figure 41 shows a centrifugal pump forcing current of air or water through a circuit of pipe.

The pump in Fig. 41 exerts a propelling force on the air or water, thus causing the air or water to flow round the circuit of pipe in spite of the friction which opposes the flow.

The propelling force of the pump in Fig. 41 is the pressure-difference between the inlet and the outlet of the pump. That is to say, the water or air enters the pump at low pressure, the action of the pump is to raise the pressure, and the water leaves the pump at increased pressure.

Figure 42 shows a battery forcing a current of electricity through a circuit of wire.

The battery in Fig. 42 exerts a kind of propelling force which causes a current of electricity to flow round the circuit of wire in spite of an opposing resistance, the resistance of the wire.

The propelling force of the battery in Fig. 42 is the "electrical pressure difference" (expressed in volts, as we shall see) between the terminals of the battery. That is to say, the electric current enters the battery at low electrical pressure, the action of the battery is to raise the pressure, and the current flows out of the carbon terminal of the battery at increased pressure.

Throughout this treatise the " propelling force " or " electrical pressure-difference " developed by a battery or dynamo is called *electromotive force* or *voltage.*

A clear idea of electromotive force can be reached most easily by considering the familiar gravity cell or battery. The rate at which zinc is consumed in this battery is proportional to the current which flows through the battery.* Therefore if the current flowing through the battery is doubled the rate of consumption of zinc is doubled and the rate at which energy is developed by the chemical action in the battery is doubled. *Let us assume that all of this energy is available for battery action, that is for pushing current through the circuit to which the battery is connected,* then the power output of the battery (rate at which work is done by the battery) will be proportional to the current. That is

$$P = EI \tag{10}$$

in which P is the power in watts delivered by the battery, I is the current in amperes flowing through the battery and through the circuit to which the battery is connected, and E is a factor which has a definite value for the given battery. This factor E is called the *electromotive force* of the battery. *The electromotive force of any battery or dynamo is the factor by which the current* I *must be multiplied to give the power output* P *of the battery or dynamo.*

Note.—It may seem from equation (9) that the power P delivered to a circuit must be proportional to the square of the current, and indeed this is true if the resistance of the circuit is invariable; but to increase the current from a given battery the resistance of the circuit must be decreased as explained in Art. 21.

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* This matter is discussed at length in the chapter on Electrolysis and Batteries, and with due regard for what is called local action and polarization. The statement here made refers to ideally simple conditions.

27. Ohm's law.—The rate at which energy is delivered by a battery is EI watts, as explained above, and the rate at which heat is produced in the circuit is RI^2 watts according to equation (9). Therefore, *if all the energy supplied by the battery is converted into heat in the circuit in accordance with Joule's law* then the power developed by the battery must be equal to the rate at which heat is generated in the circuit, that is, EI must be equal to RI^2, or cancelling I, we must have:

$$E = RI \tag{11}$$

or, solving for I, we have:

$$I = \frac{E}{R} \tag{12}$$

The relation expressed by equation (11) or by equation (12) was discovered by G. S. Ohm in 1827, and it is known as *Ohm's law.*

Ohm's law is true when all of the energy delivered by an electric generator is used to heat the circuit, that is when $EI = RI^2$. *Ohm's law is not true when a portion of the energy delivered by the generator is used to drive a motor or to produce chemical action as in the charging of a storage battery.*

28. Ohm's law and Joule's law are nearly always applied to a portion of an electric circuit, not to an entire electric circuit.—Consider the electric lamp in Fig. 44. Let R be the resistance of the lamp in ohms, and let I be the current flowing in the circuit in amperes. Then

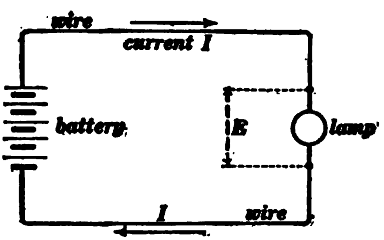

Fig. 44.

RI^2 is the rate in watts at which heat is generated *in the lamp.*

RI $(= E)$ is the electromotive force *between the terminals of the lamp.*

EI $(= RI^2)$ is the rate at which energy is delivered *to the lamp.*

29. Definition of the volt.—Consider any portion of an electric circuit, for example consider the lamp in Fig. 44, and let I be the current flowing in the circuit. Then the electromotive force E between the terminals of the lamp is equal to RI as stated in the previous article, and if E is expressed in ohms and I in amperes, then E $(= RI)$ is expressed in *volts*. That is, the product *ohms* × *amperes* gives *volts*.

One *abvolt* is the electromotive force between the terminals of one abohm of resistance when a current of one abampere is flowing through the resistance.

One *volt* is the electromotive force between the terminals of one ohm of resistance when a current of one ampere is flowing through the resistance.

One volt is equal to a hundred million abvolts.

30. The voltmeter.—Consider an ammeter (see Art. 16) of which the resistance is R ohms. When a current of I amperes flows through the ammeter the electromotive force across the terminals of the instrument is RI volts, *and the scale of the instrument can be numbered so as to give the value of* RI *in volts instead of giving the value of* I *in amperes*. An ammeter arranged in this way is called a *voltmeter*.

It would seem from the above that the only difference between an ammeter and a voltmeter would be in the numbering of the scale; but an instrument which is to be used as an ammeter must have a very low resistance in order that it may not obstruct the flow of current in the circuit **IN** which it is connected, and an instrument which is to be used as a voltmeter must have a very high resistance in order that it may not take too much current from the supply mains **BETWEEN** which it is connected.

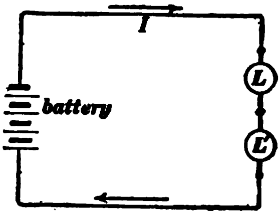

Fig. 45.

Two lamps in series.

31. Connections in series.—When two or more portions of an electric circuit are so connected that the entire current passes through each portion, then the portions are said to be *connected in* *series*. Thus Fig. 45 shows two lamps, L and L', connected in series. The ordinary arc lamps which are used to light city streets are connected in series, and the entire current delivered by the generator flows through each lamp; but the electromotive force of the generator is subdivided. That is to say *the electromotive force of a generator is subdivided among a number of lamps or other units which are connected in series.*

32. **The voltmeter multiplying coil.**—Given a voltmeter which, for example, reads up to 10 volts; one can use such a voltmeter for measuring a higher voltage by connecting an auxiliary resistance in series with it. Thus Fig. 46 shows a voltmeter V of which the resistance is R ohms, and which has an auxiliary resistance of $9R$ ohms connected in series with it. Under these conditions the voltage between the mains is found by multiplying the reading of the voltmeter by 10. This may be explained as follows: Let I be the current flowing through the circuit in Fig. 46. Then RI is the electromotive force across the terminals of the voltmeter, and $9RI$ is the electromotive force across the terminals of the auxiliary resistance. Therefore $RI + 9RI$ or $10RI$ is the electromotive force between the mains; but the voltmeter reading gives the value of the electromotive force between the voltmeter terminals, namely RI; therefore the electromotive force between the mains is ten times as great as the voltmeter reading.

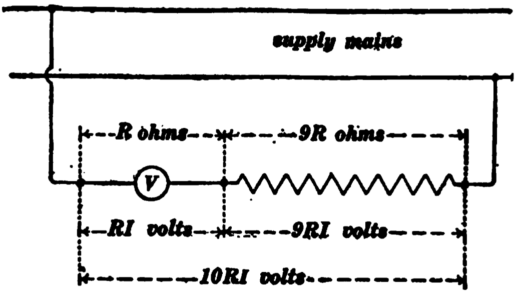

Fig. 46.

33. **Connections in parallel.**—When two or more portions of

an electric circuit are so connected that the current divides, part of it flowing through each portion, then the portions are said to be *connected in parallel.* Thus Fig. 47 shows two lamps L and L', connected in parallel. The ordinary glow lamps which are used for house lighting are connected in parallel between copper mains which lead out from the terminals of the generator; and (if the resistance of the mains is negligible) the full voltage of the generator acts on each lamp, but the current delivered by the generator is subdivided. That is to say, *the current delivered by a generator is subdivided among a number of lamps or other units which are connected in parallel.*

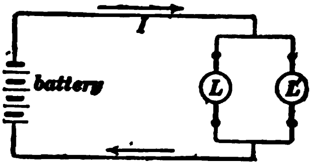

Fig. 47

Two lamps in parallel.

34. The division of current in two branches of a circuit.—Figure 48 shows a battery delivering a current to a circuit which branches at the points A and B. Let I be the current delivered by the battery, I' the current in the upper branch, I'' the current in the lower branch, R' the resistance of the upper branch, and R'' the resistance of the lower branch. The product $R'I'$ is the electromotive force between the branch points A and B, also the product $R''I''$ is the electromotive force between the branch points A and B. Therefore we have:

$$R'I' = R''I'' \qquad \text{(i)}$$

The current in the main part of the circuit is equal to the sum of the currents in the various branches into which the circuit divides. Therefore in the present case we have:

$$I = I' + I'' \qquad \text{(ii)}$$

By using equations (i) and (ii) the values of I' and I'' can both be determined in terms of I, R' and R''.

It is important to note that a definite fractional part of the total current flows through each branch; and equation (i) shows that the currents I' and I'' are inversely proportional to the respective resistances R' and R''. Thus if R' is nine times as large as R'', then I'' is nine times as large as I'.

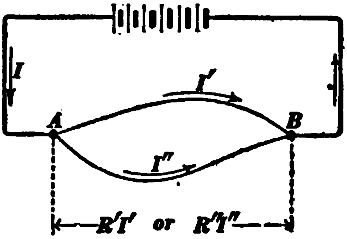

Fig. 48.

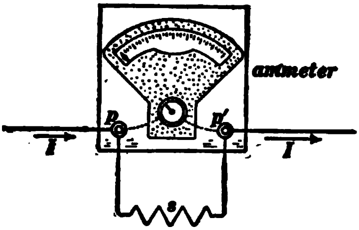

Fig. 49.
Impracticable arrangement of ammeter shunt.

35. The multiplying shunt for the direct-current* ammeter.— A low-reading voltmeter can be used to measure a higher voltage by connecting an auxiliary resistance (a multiplying coil) in series with it as explained in Art. 32. A low-reading ammeter can be used to measure a larger current by connecting an auxiliary low resistance (a multiplying shunt) in parallel with it.

It is not practicable, however, to use interchangeable shunts with a low resistance instrument (an ammeter). This may be illustrated by an example as follows: The ammeter in Fig. 49 has, let us say, a resistance of 0.01 ohm, and let us suppose that a 0.01-ohm shunt s is connected across its terminals. Under these conditions one half of the total current flows through the ammeter and one half flows through s. Therefore the value of the total current is twice the ammeter reading. The difficulty, however, is that if s is detachable there is likely to be an appreciable† unknown resistance in the contacts of s with the two binding posts p and p' so that s may be in fact 10 or 20 per cent. greater than it is supposed to be. *Any circuit in which binding-post contacts are to be made must be of fairly high resistance if the uncertain resistance at the contacts is to be negligible.*

* The alternating-current ammeter is discussed later.

† Appreciable, that is, as compared with 0.01 ohm.

Figure 50 shows an ammeter provided with a permanent shunt, *j* and *j* being soldered joints. In this case the shunt *s* may be once for all adjusted by the maker of the instrument so that the full deflection of the instrument may correspond to any desired number of amperes. In fact a manufacturer usually makes the working part, *C*, of all of his direct-current ammeters alike. The only difference between an ammeter for large current and an ammeter for small current is in the resistance of the shunt *s*.

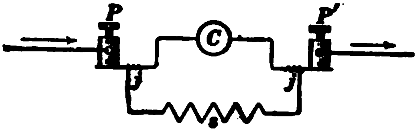

Fig. 50.

Practicable arrangement of permanent ammeter shunt.

PROBLEMS.

16. A current of 0.5 ampere flowing through a glow lamp generates 150 calories of heat in 10 seconds. What is the resistance of the lamp? Ans. 252 ohms.

Note. One calorie is equal, very nearly, to 4.2 joules.

17. The field coil of a dynamo contains 11,340 grams of copper (specific heat 0.094), weight of cotton insulation negligible. The resistance of the coil is 100 ohms. At what rate does the temperature of the coil begin to rise when a current of 0.5 ampere is started in the coil? Ans. 0.0056 centigrade degree per second.

18. A given piece of copper wire has a resistance of 5 ohms, another piece of copper wire is 1.5 times as long but it has the same weight (and volume) as the first piece. What is its resistance? Ans. 11.25 ohms.

19. A given spool wound full of copper wire 60 mils in diameter has a resistance of 3.2 ohms. An exactly similar spool is wound full of copper wire 120 mils in diameter; what is its resistance? Ans. 0.2 ohm.

Note. The spool will contain half as many layers and half as many turns in each layer of the larger wire, and the mean length of one turn of wire is the same in each case.

20. What is the resistance at 20° C. of 2 miles of commercial copper wire 300 mils in diameter? Ans. 1.22 ohms.

21. Find the resistance at 20° C. of a copper conductor 100 feet long having a rectangular section 0.5 inch by 0.25 inch. Ans. 0.00653 ohm.

Note. The area of a circle d mils in diameter is d^2 circular mils, or $\frac{\pi}{4}\left(\frac{d}{1000}\right)^2$ square inches. Therefore the sectional area of a rectangular bar in square inches must be multiplied by $\frac{4,000,000}{\pi}$ to reduce to circular mils.

22. What is the resistance at 20° C. of a wrought iron pipe 20 feet long, one inch inside diameter and 1.25 inches outside diameter. Ans. 0.00206 ohm.

Note. Use resistivity of pure annealed iron.

23. Calculate the resistance in ohms of an arc lamp carbon 0.5 inch in diameter and 12 inches long. Ans. 0.1207 ohm.

24. Calculate the resistance of a column of 5 per cent. solution of sulphuric acid at 18° C., the length of the column being 20 centimeters and the sectional area being 12 square centimeters. Ans. 8 ohms.

25. A coil of copper wire has a resistance of 5 ohms at 20° C., what is its resistance at 0° C., and what is its resistance at 90° C.? Ans. 4.63 ohms; 6.297 ohms.

Note. According to equation (8) of Art. 24 the resistance R_θ of a wire at θ° C. should be calculated from its resistance R_ϕ at ϕ° C. by using the two equations

$$R_\phi = R_0(1 + \beta\phi)$$

and

$$R_\theta = R_0(1 + \beta\theta)$$

from which we get

$$R_\theta = R_\phi \left(\frac{1 + \beta\theta}{1 + \beta\phi}\right)$$

26. A wire has a resistance of 164.8 ohms at 0° C. and a resistance of 215 2 ohms at 85° C. What is the mean temperature coefficient between 0° C. and 85° C.? Ans. 0.0036.

27. The field coil of a dynamo has a resistance of 42.6 ohms after the dynamo has stood for a long time in a room at 20° C. After running for several hours the resistance of the coil is 51.6 ohms. What is its running temperature? Ans. 77° C.

28. The curves of Fig. 43 can be expressed more accurately by an equation of the form $R_\theta = R_0(1 + a\theta + b\theta^2)$ than by the simpler equation $R_\theta = R_0(1 + \beta\theta)$. A sample of very pure annealed platinum wire has a resistance of 124.3 ohms at 0° C., 242.38 ohms at 250° C., and 338.8 ohms at 500° C. Find the values of the coefficients a and b. Ans. $a = +0.003945$; $b = -0.000000584$.

29. A coil of very pure annealed platinum wire has a resistance of 24.62 ohms at 62° C., and when the coil is placed in a furnace and protected from the furnace gases by a porcelain tube it has a resistance of 120.8 ohms. What is the temperature of the furnace? Ans. 1205° C.

30. A carbon-filament glow lamp has a resistance of 277 ohms at 0° C., and a resistance of 220 ohms at 1,000° C. What is the mean temperature coefficient of resistance of the filament between 0° C. and 1,000° C.? Ans. − 0.000206

31. A glow lamp takes 0.6 ampere when the electromotive force between its terminals is 110 volts. Find the power delivered to the lamp and express it in horse-power. Ans. 0.0884 horse-power.

Note. One horse-power equals 746 watts.

32. A motor takes 79.78 amperes of current from 110-volt mains, and the motor-belt delivers 10 horse-power. What is the efficiency of the motor? Ans. 85 per cent.

Note. Power output of motor in watts divided by power intake of motor in watts gives the efficiency of the motor.

33. A so-called "25-watt, 110-volt" tungsten lamp takes 25 watts of power when it is connected to 110-volt supply mains. How much current does the lamp take, and what is the resistance of the lamp filament while the lamp is burning? Ans. (*a*) 0.227 ampere; (*b*) 473.6 ohms.

34. A motor to deliver 10 horse-power has an efficiency of 89 per cent. The motor is supplied with current at 110 volts across its terminals. Find the full-load current of the motor and find the size of rubber insulated wire required (according to Insurance Rules) to deliver current to the motor. Ans. 76.2 amperes; number 3 wire B. & S. gauge.

35. When electrical energy costs 11 cents per kilowatt-hour how much does it cost to operate for 10 hours a lamp which takes 0.227 ampere from 110-volt supply mains? Ans. 2.75 cents.

36. Find the cost of energy for operating a 5 horse-power motor at full load for 10 hours, the efficiency of the motor being 85 per cent. and the cost of energy being 6 cents per kilowatt-hour. Ans. $2.63.

37. Assume the actual cost of electrical energy delivered to a street car to be 0.8 cent per kilowatt-hour. Find the cost of developing 100,000 British thermal units for heating the car first by an electrical heater in which all of the heat generated is available for heating, and second by burning coal costing $6 per ton (2,000 pounds) and giving 14,000 British thermal units per pound of which 30 per cent., say, is lost by incomplete combustion and by flue gas losses. Ans. (*a*) $2.35; (*b*) $1.43.

Note. **One British thermal unit is equal, very nearly, to 1056 joules.**

38. One cubic foot of good illuminating gas costing one tenth of a cent gives about 600 British thermal units when it is burned, and about 20 per cent. of the heat of a burner is taken up by the water in a tea kettle. On the other hand about 70 per cent. of the heat given off by an ordinary electrical heater is given to a tea kettle which completely covers the hot disk of the heater, and electrical energy for domestic use costs, say, 10 cents per kilowatt-hour. What is the cost of bringing 2 gallons of water from 15° C. to 100° C. by a gas burner and what is the cost by electric heater? Ans. (*a*) 2.31 cents; (*b*) 10.74 cents.

39. When a certain dynamo electric generator is delivering no current it takes 1.75 horse-power to drive it. When the

generator delivers 150 amperes it takes 25 horse-power to drive it. Calculate the electromotive force of the generator on the assumption that all of the additional power required is used to maintain the current of 150 amperes. Ans. 115.7 volts.

40. A coil of wire of which the resistance is to be determined is connected to 110-volt direct-current supply mains in series with an ammeter and a suitable rheostat, and a voltmeter is connected across the terminals of the coil. The ammeter reads 13 amperes and the voltmeter reads 80.6 volts. What is the resistance of the coil? Ans. 6.2 ohms.

41. A battery of which the electromotive force is 1.07 volts and the resistance is 2.1 ohms is connected to a wire circuit of which the resistance is 5 ohms. (*a*) What current is produced? (*b*) What is the electromotive force between the terminals of the cell? (*c*) What is the electromotive force drop in the cell? Ans. (*a*) 0.15 ampere; (*b*) 0.75 volt; (*c*) 0.32 volt.

42. A voltmeter connected across the terminals of a set of 60 storage battery cells connected in series reads 120.4 volts when the battery is delivering no current, and the voltmeter reading falls instantly to 112.25 volts when the battery begins to deliver 15 amperes of current. What is the resistance of the battery? Ans. 0.55 ohm.

Note. When a battery continues to deliver current the voltage falls off because of polarization. The sudden drop of voltage at the instant that current delivery begins is due almost entirely to the battery resistance.

43. A voltmeter connected across the terminals of a battery reads 15 volts when the battery is not delivering current (except the negligible current which flows through the voltmeter), and the voltmeter reading drops suddenly to 9 volts when a wire circuit having a resistance of 6 ohms is connected to the battery. What is the resistance of the battery? Ans. 4 ohms.

44. The voltage between the supply mains at a direct-current power station is 115 volts and it is desired to deliver 25 amperes to a group of glow lamps 500 feet distant from the station and have 110 volts between the mains at the lamps. What size of copper wire is required? Ans. 225 mils in diameter.

45. What size of copper wire is required to deliver current at 110 volts to a 10 horse-power motor of 85 per cent. efficiency; the motor being 500 feet from the generator, and the electromotive force across the generator terminals being 125 volts? Ans. 235 mils in diameter.

Note. Find the power in watts delivered to the motor, then find the current, and then proceed to find the resistance of the line from a consideration of the line drop (the voltage lost in the line), and so on.

46. A motor is to receive 100 kilowatts of power from a generator at a distance of 15 miles. A loss of 10 per cent. of generator voltage (or 10 per cent. of the generator output of power) is to be permitted in the transmission line. Find the generator voltage which must be provided for in order that copper transmission wires 200 mils in diameter may be used. Ans. 6,763 volts.

47. If a 10 per cent. line loss is allowed as in the previous problem, but if the generator voltage is doubled (making it 13,526 volts), what size of copper transmission wires would be used to deliver 100 kilowatts at a distance of 15 miles? Ans. 100 mils in diameter.

Note. It is worthy of note that by doubling the generator voltage the cost of the copper required to transmit a given amount of power over a given distance with a given percentage loss of voltage is quartered. Compare this problem and the preceding one with the next one.

48. What is the weight of 30 miles of copper wire 200 mils in diameter and what will it cost at 15 cents per pound? What is the weight of 30 miles of copper wire 100 mils in diameter and what will it cost at 15 cents per pound? Ans. (*a*) 24,420 pounds, $3,663.00; (*b*) 6,105 pounds, $915.75.

Note. The density of copper is 555 pounds per cubic foot.

49. A millivoltmeter has a resistance of 15.4 ohms. What resistance must be connected in series with the instrument so that the scale reading may give volts instead of millivolts? Ans. 15,384.6 ohms.

50. Three lamps (or other units) are connected in series to 110-volt mains, the resistances of the lamps are 10 ohms, 8 ohms and 4 ohms respectively, find the voltage across the terminals of each lamp. Ans. 50 volts, 40 volts and 20 volts.

51. Three resistances of 4, 4 and 2 ohms respectively are connected in parallel; and two resistances of 6 ohms and 3 ohms respectively are connected in parallel. The first combination is connected in series with the second combination, and to a battery of negligible resistance and of which the electromotive force is 3 volts. What is the current in the 2 ohm resistance and what is the current in the 3 ohm resistance? Ans. 0.5 ampere and 0.66 ampere respectively.

Note. The combined resistance of several branches in parallel is equal to the reciprocal of the sum of the reciprocals of the resistances of the individual branches.

52. An ammeter has a resistance of 0.05 ohm. The instrument is provided with a shunt so that the total current through instrument and shunt is 10 times the current through the ammeter itself. What is the resistance of the shunt? Ans. 0.00556 ohm.

53. The scale of a direct-reading millivoltmeter has 100 divisions, each division corresponding to one thousandth of a volt between the terminals of the instrument. The instrument is connected to the terminals of a low-resistance shunt, and each division on the instrument scale corresponds to 0.25 ampere in the shunt. What is the resistance of the shunt? Ans. 0.004 ohm.

54. A voltmeter which has a resistance of 16,000 ohms is connected in series with an unknown resistance R to 110-volt supply mains, and the reading of the voltmeter is 4.3 volts. What is the value of R? Ans. 393,300 ohms.

Note. As here used the voltmeter is to be thought of as an ammeter, the current being equal to the voltmeter reading divided by the resistance of the instrument.

55. A 40-mile telegraph line is disconnected from ground at both ends, the line is then connected to ground at one end

through a 220-volt battery and a direct-reading voltmeter of which the resistance is 16,000 ohms, and the voltmeter reads 2.9 volts. What is the insulation resistance of the 40-mile telegraph line, and what is the insulation resistance of one mile of the line? Ans. 1,197,000 ohms; 47,880,000 ohms.

Note. It is best to think of the voltmeter, as here used, as an ammeter as explained in the note to the previous problem.

The measured current flows through the battery, through the voltmeter, through the leakage path, and back to the battery through the ground; and the resistances of battery, wire and ground are negligible.

The leakage path is from the 40-mile wire through insulators and poles to the ground, and the sectional area of the leakage path on one mile of the line is one-fortieth of the sectional area of the leakage path on the whole line.

CHAPTER III.

INDUCED ELECTROMOTIVE FORCE.

36. Back electromotive force in the windings of an electric motor. Induced electromotive force.—Consider a motor armature of which the resistance is one ohm. *When the armature is standing still* it takes 10 volts to push 10 amperes through the armature, according to Ohm's law as expressed by equation (12). In this case all the power delivered to the armature is used in heating the wires in accordance with Joule's law.

To push 10 amperes through the motor when it is running requires more than 10 volts, because the motor is doing mechanical work and the power delivered to the motor must be greater than the power lost in heating the wires. Thus Fig. 52 represents 100 volts pushing 10 amperes through the running armature.

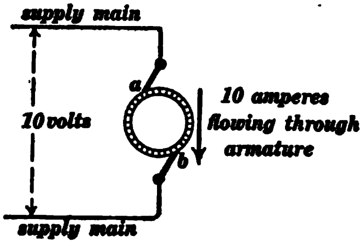

Fig. 51.

Armature not running.

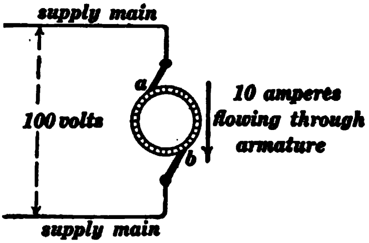

Fig. 52.

Armature running.

It is harder, as it were, to push current through a running motor armature than to push current through the same armature while it is standing still. Something besides resistance must therefore oppose the flow of current through the running armature. In fact a *back electromotive* force exists in the windings of the running armature. This back electromotive force is produced by the sidewise motion of the armature wires as they cut across the lines of force of the magnetic field in the gap spaces as shown by the fine lines in Fig. 26. An electromotive force produced in this way is called an *induced electromotive force.*

The dynamo as a motor and the dynamo as an electric generator.—The action of the direct-current dynamo as a motor is described in Art. 18, and, as explained above, there is induced in the armature windings of a motor, an electromotive force which opposes the flow of current through the windings.

When the armature of a dynamo is driven by a steam engine or water wheel, the cutting of the lines of force of the magnetic field in the gap spaces (see Fig. 26) by the armature wires as they move sidewise induces an electromotive force in the armature windings, and current is delivered to any circuit that is connected to the brushes. A dynamo operating in this way is called a *dynamo electric generator*, or simply a *generator*.

37. The starting rheostat for the direct-current motor.—When a motor is running, the flow of current through the armature windings is opposed only in small part by the resistance of the windings; the chief opposition to the flow of current is the back electromotive force in the armature windings, as explained in Art. 36. For example, a fully loaded motor takes 50 amperes from 110-volt mains, and the resistance of the armature windings is 0.3 ohm. Multiplying the resistance of the armature windings by the current we get 15 volts (0.3 ohm × 50 amperes = 15 volts), which is the portion of the supply voltage (110 volts) that is used to overcome the resistance of the armature. The remainder, namely 95 volts, is used to overcome the back electromotive force in the armature windings.

If there were no back electromotive force the supply voltage would produce a current of 110 volts ÷ 0.3 ohm, or 367 amperes through the armature windings, according to Ohm's law. Now, as a matter of fact, there is no back electromotive force when the motor armature is standing still, and, therefore, if the motor were to be started by connecting the armature terminals (the brushes) directly to the supply mains, a current of 367 amperes would flow through the armature at the beginning. This excessive flow of current through a motor armature would damage the motor, and it is therefore always necessary to connect a rheostat in series with a motor armature at starting. As the motor speeds up the rheostat resistance may be cut out more and more, and the greater and greater back electromotive force due to the greater and greater speed keeps the current down to a moderate value.

38. The fundamental equation of the direct-current dynamo.—The equation which expresses the electromotive force which is induced in the armature windings of a direct-current dynamo in terms of various data as explained below is called the fundamental equation of the dynamo. To derive this equation let us use c.g.s. units throughout, let us consider the dynamo as an electric generator, and let us think of the armature as rotating without energy losses of any kind* such, for example, as friction losses. *Then the mechanical power required to drive the dynamo armature is equal to the electrical power output* EI *of the armature, where* E is the electromotive force induced in the armature windings and I is the current delivered by the armature. Also let us limit the discussion to the simple form of dynamo with a bipolar field magnet and a ring-wound armature, as shown in Figs. 26–29.

Let r = radius of armature, measured out to the layer of wires.

L = length of armature core (AA Figs. 26 and 27) parallel to armature shaft. This is also the length of the pole faces parallel to the armature shaft.

b = breadth of each pole face measured along the circumference of the armature where the wires lie.

Z = number of wires on outside of armature. These wires are straight, they lie parallel to the armature shaft, and the length of the portion of each which lies in the gap space is L.

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* This is not necessary but it avoids tedious qualifying specifications in the discussion.

n = speed of armature in revolutions per second.

H = intensity of magnetic field in gap spaces. We assume this field to have the same intensity everywhere in the gap spaces, we assume the lines of force to be radial as shown in Fig. 26, and we ignore the fringe of the magnetic field which spreads out beyond the edges of the pole faces.

The total current I is supplied by the coming together at the brushes of $I/2$ abamperes flowing through the windings on each side of the armature. That is, the current in each armature wire is $I/2$ abamperes.

The side push of the magnetic field on each armature wire in the gap spaces opposes the motion of the armature, and is equal to $LHI/2$ dynes according to equation (5) of Art. 19; and the number of armature wires in the gap spaces at any time is $\frac{2b}{2\pi r} \times Z$. Therefore the total tangential drag (or opposing force) on the armature wires is $\frac{LHI}{2} \times \frac{2b}{2\pi r} \times Z$ dynes, and the power P required to drive the armature against this dragging force is equal to the product of the force and the velocity $(2\pi r n)$ of the armature wires, that is $P = \frac{LHI}{2} \times \frac{2b}{2\pi r} \times Z \times 2\pi r n$. But this power is equal to the power output EI as above explained. Therefore $EI = \frac{LHI}{2} \times \frac{2b}{2\pi r} \times Z \times 2\pi r n$, or

$$E = (LbH)Zn \tag{i}$$

But the area of a pole face Lb multiplied by H gives the amount of magnetic flux Φ which enters the armature core from the N-pole of the field magnet (and leaves the armature core to enter the S-pole of the field magnet), according to equation (4) of Art. 10. Therefore equation (i) becomes

$$E \text{ (in abvolts)} = \Phi Z n \tag{13}$$

or, since one volt is equal to 10^8 abvolts, we have

$$E \text{ (in volts)} = \Phi Z n \times 10^{-8}. \tag{14}$$

39. **Elementary expressions for induced electromotive force.**—Equations (13) and (14) refer to a rather complicated physical structure, and it is desirable to consider the simplest possible mechanism for the production of induced electromotive force as shown in Fig. 53. A rod or wire *bc* is made to slide sidewise at velocity *v* on two metal rails, as shown, and the moving rod or wire "cuts" the lines of force of a magnetic field which is perpendicular to the plane of the paper. Suppose a current of *I* abamperes is flowing round the circuit *abcd*, and let *F* be the side force exerted on the rod *bc* by the magnetic field. Then $F = lIH$ dynes, where l is the length of the wire *bc* in centimeters, and *H* is the intensity of the magnetic field in gausses. The force *F* opposes the motion *v*, and *Fv* ergs per second is the rate at which work is done in overcoming the opposition of *F*; *all of the work so spent reappears as work done by the induced electromotive force* E *in maintaining the current* I. Therefore $EI = Fv = lIHv$, or

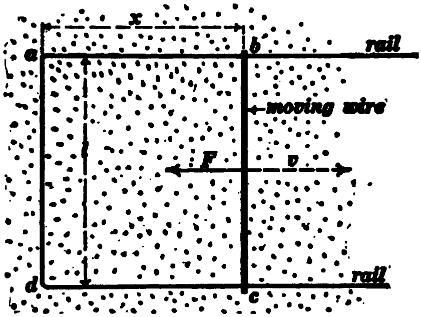

Fig. 53.

Dots represent magnetic lines of force which are perpendicular to the plane of the paper.

$$E \text{ (in abvolts)} = lHv \tag{15}$$

where *E* is the electromotive force induced in the moving rod *bc*.

The idea of cutting of magnetic flux.—The moving rod *bc* in

Fig. 53 is said to "cut" the lines of force of the magnetic field, and *the induced electromotive force* E *is equal to the rate at which the rod cuts the magnetic flux*, in *maxwells per second or lines of force per second.* Let Δx be the sidewise distance moved by the rod bc in the short interval of time Δt. Then

$$\Delta x = v \cdot \Delta t. \tag{i}$$

The area swept over by the rod during the time interval Δt is $l \cdot \Delta x$, and the amount of magnetic flux crossing this area is $\Delta\Phi = Hl \cdot \Delta x = Hlv \cdot \Delta t$ so that the rate of cutting flux, $\Delta\Phi/\Delta t$, is equal to Hlv; but Hlv is equal to the electromotive force E induced in the moving rod. Therefore the induced electromotive force is equal to the rate at which the moving rod cuts magnetic flux.

Idea of changing flux.—The area of the rectangle $abcd$ in Fig. 53 is lx square centimeters, and the amount of magnetic flux Φ which passes through the opening $abcd$ is found by multiplying H by the area lx; that is

$$\Phi = Hlx. \tag{i}$$

Now, if x changes, Φ must change Hl times as fast, so that

$$\frac{d\Phi}{dt} = Hl\frac{dx}{dt}. \tag{ii}$$

But, dx/dt is the velocity v of the moving rod, so that equation (ii) becomes

$$\frac{d\Phi}{dt} = Hlv. \tag{iii}$$

Comparing this with equation (15) we see that *the rate of change of the flux* Φ *which passes through the circuit* $abcd$ *in Fig.* 53 *is equal to the induced electromotive force* E; or

$$E \text{ (in abvolts)} = -\frac{d\Phi}{dt}. \tag{16}$$

The negative sign is explained in the following paragraph.

This result is here derived for the case which is shown in Fig. 53, *where the change of magnetic flux through a circuit is due to motion of a part of the circuit,* **but equation (16) is also true when the changing magnetic flux is due to a varying magnetic field or to the varying state of magnetism of an iron core as in the induction coil and transformer.** Equation (16), however, expresses the electromotive force which is induced in a single turn of wire. When a region of changing magnetic flux is encircled by Z turns of wire, the induced electromotive force is multiplied Z times and equation (16) becomes

$$E \text{ (in abvolts)} = -Z\frac{d\Phi}{dt}. \tag{17}$$

The negative sign is chosen in equation (16) for the following reason: Figure 54 represents a turn of wire with magnetic flux passing through it. Let it be agreed to consider this flux as positive. That is to say, the direction of the fine arrows in Fig. 54 is chosen as the positive direction through the loop or turn of wire, and it is conventional to choose the positive direction around the loop or turn in accordance with the right-handed screw relation as indicated by the bolt and nut in the figure. Therefore the arrows aa indicate the positive direction around the loop or turn of wire.

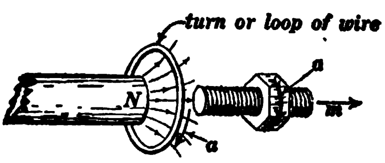

Fig. 54.

Now the flux Φ in Fig. 54 is considered positive so that an actual increase of Φ means a positive value of $\frac{d\Phi}{dt}$; but the electromotive force induced in the loop in Fig. 54 by an increase of Φ is opposite in direction to the arrow a, and therefore this electromotive force is to be considered as negative so that we must have $E = -\frac{d\Phi}{dt}$.

40. **The alternating-current dynamo.**—An alternating-current dynamo is usually called an *alternator.* An ideally simple alternator is shown in Figs. 55 and 56. A wire WW is supported by arms mm which are fixed to a rotating shaft as shown in Fig. 56, and as the shaft revolves the wire sweeps across the pole faces N and S of an electromagnet as shown in Fig. 55. The ends of the wire WW are attached to two metal rings r

and r', and metal or carbon brushes a and b rub on these rings, thus keeping the ends of the moving wire connected to an external circuit as shown in Fig. 56.

While the wire WW is sweeping across the north pole N an electromotive force is induced in the wire in one direction; and while the wire WW is sweeping across the south pole S an electromotive force is induced in the wire in the opposite direction. This rapidly reversed electromotive force is called an *alternating electromotive force*, and it produces an *alternating current* in the moving wire and in the external circuit to which the moving wire is connected.

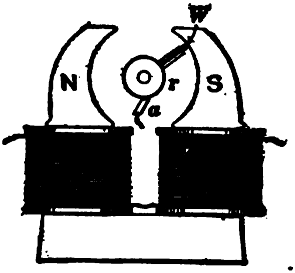

Fig. 55.

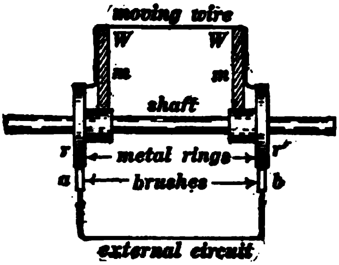

Fig. 56.

An improvement on the ideally simple alternator of Fig. 55 would be to place the moving wire WW in a slot in a rotating iron cylinder AA as shown in Fig. 57; because to do so would give a firm support for the moving wire, and the presence of the iron cylinder AA would greatly intensify the magnetic field (see lines of force in Fig. 26). The iron cylinder AA is built up of thin sheet iron disks or stampings, and it usually has a large number of slots in which many wires are placed as described later. The electromagnet NS in Figs. 55 and 57 is called the *field magnet* of the alternator. The iron cylinder AA

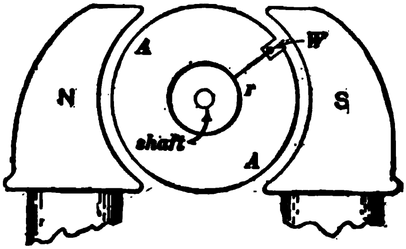

Fig. 57.

with its slots and wires is called the *armature*. The cylinder *AA* itself is called the *armature core*. The insulated metal rings *r* and *r'* are called the *collector rings*. The collector rings are usually placed side by side at one end of the armature.

The field magnet of an alternator is always *excited* (magnetized) by direct current, and this direct current is usually supplied by a small auxiliary direct-current generator which is called the *exciter*.

Commercial alternators nearly always have multipolar field magnets, whereas Figs. 55 and 57 show bipolar field magnets. Also the armature windings always consist of many wires in many slots. Thus, Figs. 58 and 59 show two possible arrangements of the armature windings of a 4-pole alternator. The dotted circles represent front and back ends of the armature core, the short, heavy, radial lines represent the wires which lie lengthwise (parallel to the armature shaft) in the armature slots, the curved lines *F* represent the cross connections on the front end of the armature, the curved lines *B* represent the cross connections on the back end of the armature, and the straight lines *C* represent the connections to the collector rings (the

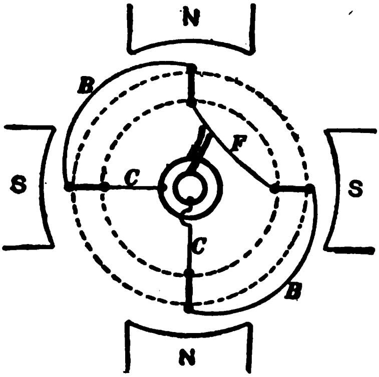

Fig. 58.

Possible armature winding diagram for 4-pole alternator.

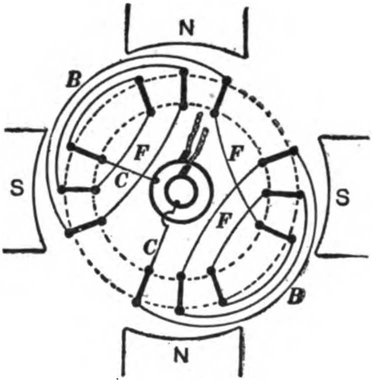

Fig. 59.

Possible armature winding diagram for 4-pole alternator.

small black circles) upon which the brushes rub. The field magnet poles are of course very close to the armature but they are shown widely separated in Figs. 58, 59 and 60, so as to give room to show the back connections *B*.

Figure 60 shows a possible arrangement of the armature windings of an 8-pole alternator.

The simple alternator above described is called a *single-phase alternator*. It has a single armature winding and two collector rings. The *two-phase alternator* has two distinct armature windings, each winding being connected to two collector rings (four collector rings in all). The *three-phase alternator* has three distinct armature windings, each winding being connected to two collector rings (six collector rings in all). Because of certain relations between the three distinct alternating currents which are delivered by the three armature windings of a three-phase alternator (delivered to three distinct receiving circuits, of course) it is possible to use only *three* collector rings; and this is the usual practice.

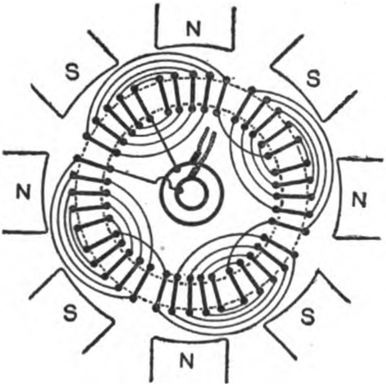

Fig. 60.

Possible armature winding diagram for 8-pole alternator.

41. Definition of cycle. Definition of frequency.—The electromotive force developed by a direct-current dynamo is nearly constant in value and it is unchanging in direction, and the current which is delivered by such a machine flows steadily in one direction through the receiving circuit. But the electromotive force of an alternator (and also the current which is delivered by the machine to a receiving circuit) is subject to rapid reversals of direction. Two successive reversals constitute what is called a *cycle*, and the number of cycles per second is called the *frequency* of the alternating electromotive force or current. The electromotive force of a two-pole alternator like Figs. 55 and 57 is reversed twice for each revolution of the armature. But two reversals constitute a cycle; therefore the frequency of the electromotive force of a two-pole alternator in cycles per second is equal to the speed of the armature in revolutions per second. A four-pole alternator gives two complete cycles of electromotive force during each revolution, an eight-pole alternator gives four complete cycles of electromotive force during each revolution of the armature, and so on.

The standard frequencies of alternating electromotive force and current in practice are 25 cycles per second (50 reversals per second) and 60 cycles per second (120 reversals per second).

42. Electromotive force induced by increase or decrease of magnetism of an iron rod.—The electromotive force induced in the wire on a dynamo armature is due to the motion of the wire across the magnetic field, or, as it is sometimes stated, the electromotive force is due to the "cutting" of the lines of force of the magnetic field in the gap space by the armature wires as they move sidewise. *An electromotive force is also induced in a winding of wire on an iron rod while the magnetism of the iron rod is being increased or decreased.* Figure 61 shows an iron core *CC* (made of strips of sheet iron) with a winding of wire *PP* upon it. The winding is connected to alternating-current supply mains, and the rapid reversals of the alternating current in the coil *PP* cause the iron core *CC* to be rapidly magnetized and demagnetized, first in one direction and then in a reversed direction; that is to say, the upper end of the core is at one instant a north pole and at the next instant a south pole. An auxiliary coil, *SS* consisting of a few turns of wire, has its terminals *TT* con-

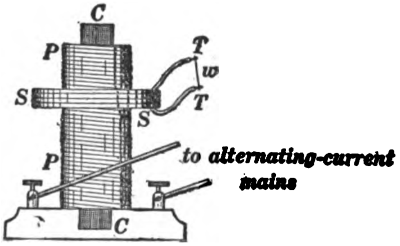

Fig. 61.

nected by a fine wire w. Under these conditions the fine wire becomes red hot. The heating of the fine wire shows the existence of an electric current in the wire and in the coil SS. This current is an alternating current, and it is produced by an alternating electromotive force which is induced in the coil SS by the magnetic reversals of the core CC.

43. **The alternating current transformer.**—The alternating-current transformer is a device essentially like the arrangement shown in Fig. 61. The winding PP which receives alternating current from some outside source is called the *primary coil* of the transformer, and the coil SS in which an alternating current is produced by the reversals of magnetism of the core is called the *secondary coil* of the transformer. In the commercial transformer the iron core forms a closed circuit (a closed " magnetic circuit ") as shown in Fig. 62. The iron core CC is built up of sheet iron stampings, and the two coils P and S are wound, one over the other. One of the coils P or S usually contains many turns of fine wire, and the other contains few turns of coarse wire. Either coil may be used as the primary coil.

Step-down transformation.—A small alternating current may be delivered at high voltage to the coil-of-many-turns, in which case the coil-of-few-turns will deliver a large alternating current at low voltage. This constitutes what is called *step-down transformation.*

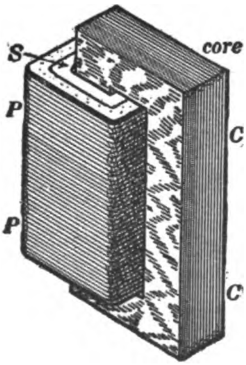

Fig. 62.
Alternating-current transformer.

Step-up transformation.—A large alternating current may be delivered at low voltage to the coil-of-few-turns, in which case the coil-of-many-turns will deliver a small alternating current at high voltage. This constitutes what is called *step-up transformation.*

In practice a transformer like Fig. 62 is placed in an iron case which is filled with mineral oil. Thus Fig. 63 shows a step-down transformer in an iron case mounted on a pole. Alternating current is delivered to the coil-of-many-turns of the transformer from high-voltage street mains *ss*, and a large alternating current at low voltage is delivered by the coil-of-few-turns and led into an adjoining house over the house wires *hh*. The small cases *FF* contain safety fuses. In large cities the high-voltage street mains are usually laid underground, and the step-down transformers are usually placed in vaults under the street.

Note. The action of the transformer is explained in the note to problem 64.

44. High voltage must be used in the long-distance transmission of power.—In order to appreciate the very great practical importance of the alternating-current transformer one must understand that *high voltage is necessary* for long-distance transmission of power, whereas *low voltage is necessary* for supplying power to lamps and motors. Thus the voltage between the street mains *ss* in Fig. 63 is usually 1,100 or 2,200 volts, and the voltage between the house wires *hh* is usually 110 volts.

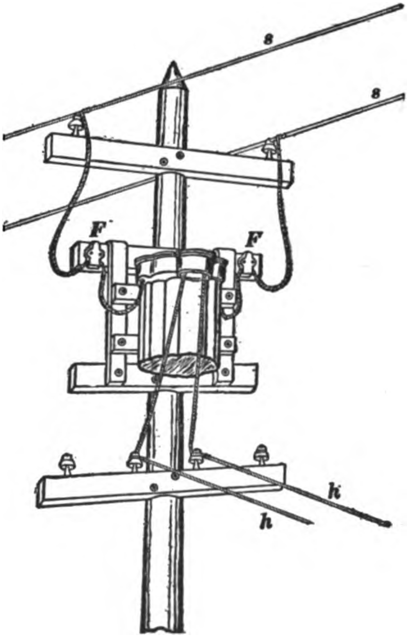

Fig. 63.

Consider the transmission of power by the pumping of water through a long pipe to a water motor. A given amount of power might be thus transmitted by using a very large pipe to carry a large volume of water per second from a low-pressure pump to a low-pressure water motor; or the same amount of power might be transmitted by a small pipe carrying a small amount of water per second from a high-pressure pump to a high-pressure water motor. If power were to be transmitted over a considerable distance in this way the cost of the pipe would be the most important item of cost in the entire installation, and it would therefore be most economical to use high-pressure water so as to be able to use a small pipe.

Consider the transmission of power by an electric current. A given amount of power can be transmitted by using a large wire to carry a large current from a low-voltage generator to a low-voltage motor; or the same amount of power can be transmitted by using a small wire to carry a small current from a high-voltage generator to a high-voltage motor. The cost of the transmission line is one of the largest items of expense in an installation for the long-distance transmission of power by the electric current, and therefore it is most economical to transmit the power by small current at high voltage so as to be able to use small wires. Therefore high voltage is always used for long-distance transmission of power. But the power must be delivered to motors and lamps at low voltage so that it is necessary to use a device for transforming the power at the receiving station from high-voltage-and-small-current to low-voltage-and-large-current.

The advantage of the alternating-current system over the direct-current system lies almost wholly in the cheapness of construction and the economy of operation of the alternating-current device which is used for this transformation, namely, the alternating-current transformer. To accomplish the same transformation in the direct-current system would require the use of a specially constructed motor operated by the high-voltage supply, and this motor would have to drive a low-voltage generator for delivering current at low voltage to the receiving apparatus. Such a combination of motor and generator is called a *motor-generator*, and the advantages of the transformer as compared with the motor-generator are as follows:

A transformer costs less than one fourth as much as a motor-generator of the same capacity.

A transformer can be placed anywhere, and it needs only to be occasionally inspected; whereas a motor-generator requires a building and the care of an attendant.

A transformer wastes only two or three per cent. of the power; whereas a motor-generator wastes 20 or 30 per cent. of the power.

The essential parts of an installation for the long-distance transmission of power are as follows: A water wheel drives an alternator which delivers alternating current at a moderately low voltage to a step-up transformer. The step-up transformer delivers alternating current to the transmission line at a very high voltage. At the other end of the transmission line the small current at high voltage is delivered to a step-down transformer which in turn delivers a large current at low voltage to the receiving apparatus. If direct current is desired at the receiving station, the step-down transformer delivers alternating current at low voltage to a machine called a *rotary converter*,* and this converter delivers direct current.

45. The induction coil.—An iron rod or core wound with insulated wire can be repeatedly magnetized and demagnetized by connecting a battery to the winding and repeatedly making and breaking the circuit; and the increase and decrease of mag-

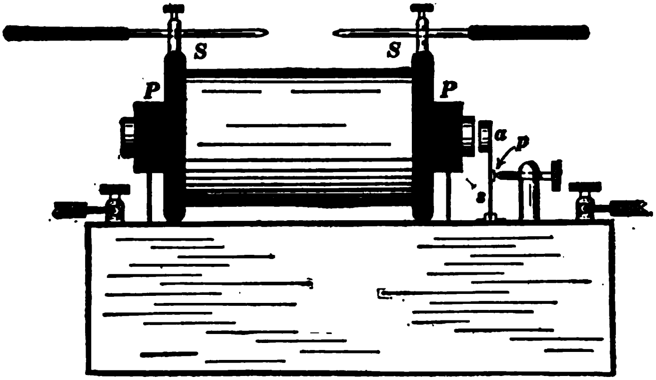

Fig. 64.

Induction coil.

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* The rotary converter or synchronous converter is described in Chapter XV.

netism of the core thus produced can be utilized to induce electromotive forces in an auxiliary coil of wire wound on the iron core. Such an arrangement is called an *induction coil.* The winding through which the magnetizing current from the battery flows is called the *primary coil*, and the auxiliary winding in which the desired electromotive forces are induced is called the *secondary coil.* The iron core is always made of a bundle of fine iron wires or strips of sheet iron.

A general view of an induction coil is shown in Fig. 64, and the diagram of connections is shown in Fig. 65. When the iron core is magnetized, the block of iron *a* is attracted, and the battery circuit is broken at the point *p*. The iron core then looses its magnetism, and the spring *s* brings the points at *p* into contact again so that the battery current again flows through the circuit and magnetizes the iron core. The iron block *a* is then attracted again, and the above action is repeated.

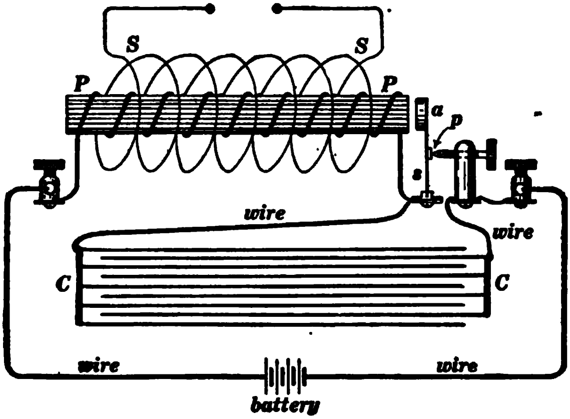

Fig. 65.

Induction coil diagram.

When the iron core of an induction coil is magnetized, a momentary pulse of electromotive force is induced in the secondary coil; and when the iron core is demagnetized a reversed momentary pulse of electromotive force is induced in the secondary coil. Electromotive forces are induced only while the core is being magnetized or demagnetized, and each pulse of electromotive force may be made very large in value (many thousand of volts) by using many turns of wire in the secondary coil and by providing for the quickest possible magnetization or demagnetization of the core. A battery cannot, however, magnetize a core very quickly when connected to a magnetizing coil; in fact a very considerable fraction of a second is required for the core to become magnetized. Therefore during the magnetization of the iron core of an induction coil the electromotive force induced in the secondary coil is a comparatively weak pulse of fairly long duration.

On the other hand the use of the condenser *CC*, Fig. 65, causes the iron core of the induction coil to be demagnetized very quickly as explained in Art. 50, and this quick demagnetization induces in the secondary coil an intense pulse of electromotive force of very short duration.

The iron core of the alternating-current transformer may be as shown in Fig. 61, but it is better if the iron core forms a complete "magnetic circuit" as shown in Fig. 62. The induction coil must have its iron core in the form of an open "magnetic circuit" as shown in Figs. 61 and 65, because after the core has been magnetized *it is the energy of this magnetism which becomes available when the primary circuit is broken*, and the greater part of the available energy of a magnet resides in the magnetic field near the poles of the magnet.

46. The telephone.—The telephone set includes a transmitter, a receiver and an arrangement for calling. The *transmitter* is a device for producing over the line a current which is reversed with each to and fro movement of a diaphragm, the diaphragm being set into vibration by a speaker's voice; and the *receiver* is a device in which a diaphragm is set into vibration by these rapidly reversed currents (which come to it over the line from the transmitter) thus reproducing the original sound.

The transmitter.—A sectional view of a telephone transmitter is shown in Fig. 66. It consists of a small quantity of granulated carbon between two corrugated carbon blocks. The black patches in Fig. 66 represent the carbon blocks. One of these blocks is supported rigidly, and the other is attached to the diaphragm DD; and as the diaphragm moves inwards or outwards it produces increased or decreased compression of the granular carbon which produces a decreased or increased electrical resistance. *A battery sends current through the granular carbon and through the primary coil of a small induction coil or transformer.* The varying resistance of the granular carbon causes the battery current to rise and fall as the diaphragm moves inwards and outwards, and this rise and fall of battery current produces in the secondary coil of the transformer a current which flows in one direction and the other alternately as the transmitter diaphragm moves to and fro.

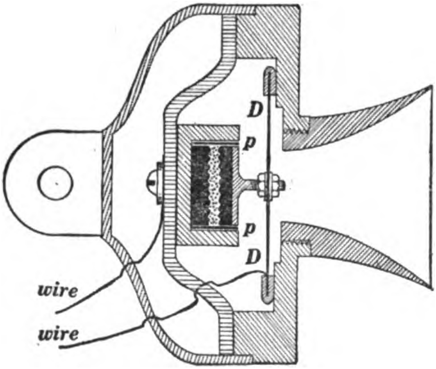

Fig. 66.

Telephone transmitter.

The telephone receiver.—The simplest type of telephone receiver is shown in Fig. 67. A coil of very fine insulated wire is wound around one end of a permanent steel magnet MM. *The reversals of current from the distant transmitter flowing through this coil alternately strengthen and weaken the steel magnet*, and these variations of strength of the steel magnet cause the iron diaphragm dd to move to and fro, thus reproducing the original sound. The most improved form of telephone receiver has a bipolar magnet.

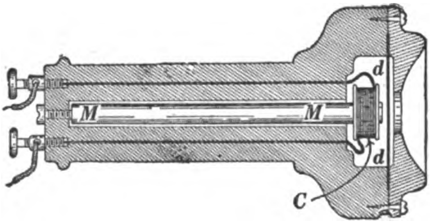

Fig. 67.

Telephone receiver (old style).

The simple telephone set.—Two telephone stations connected up for talking are shown in Fig. 68. The ground return, as shown in Fig. 68, is replaced by wire return in Figs. 69 and 70. To give a call at the distant station a small hand-operated dynamo is used to ring a bell, and the change from the connections required to operate the bell to the connections required for the operation of the transmitter and receiver is made by the movement of the hook when the telephone receiver is taken from the hook. Figure 69 shows the hooks down, and the connections, as indicated by the full lines, are proper for operating the bell at either station. Figure 70 shows the hooks up, and the

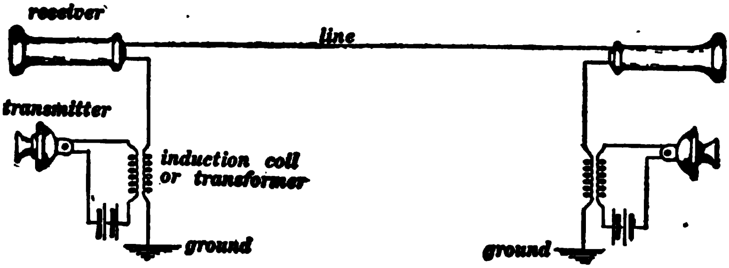

Fig. 68.

Two-station telephone set (without call bells).

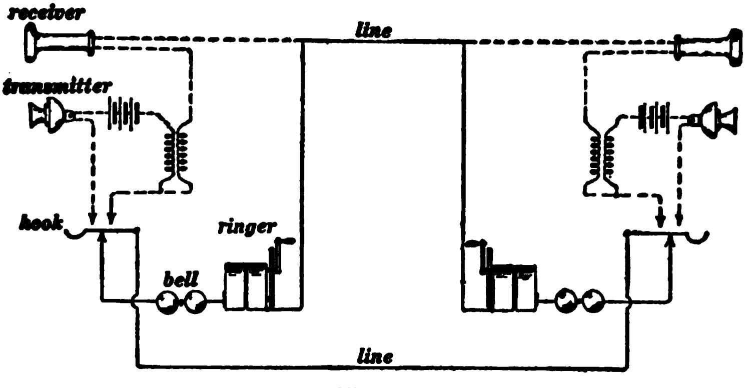

Fig. 69.

Two-station telephone set—connections for ringing.

connections are proper for operating the transmitters and receivers.

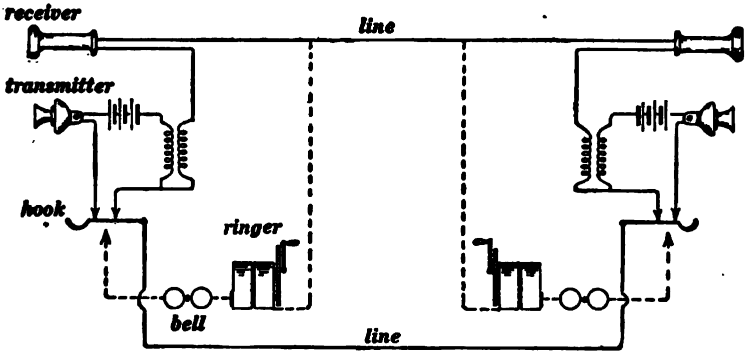

Fig. 70.

Two-station telephone set—connections for talking.

47. Eddy currents. Lamination.—Figure 71 shows an end view of an iron rod surrounded by a wire ring. While the iron rod is being magnetized or demagnetized an electromotive force is induced in the ring, according to Art. 42, and an electric current is produced in the ring in the direction of the small arrows or in the opposite direction. Figure 72 shows the end of a larger iron rod. While the rod is being magnetized or demagnetized an electric current is produced in the circular filament of iron. *The*

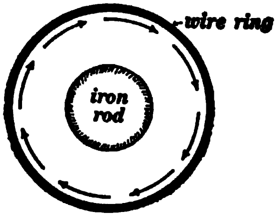

Fig. 71.

Current induced in wire ring.

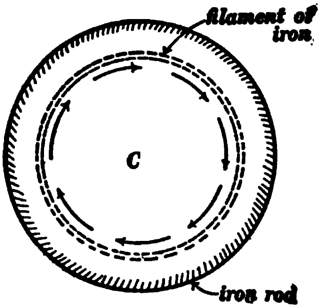

Fig. 72.

Current induced in filament of solid iron rod.

increasing or decreasing magnetism of the central portion C of the rod in Fig. 72 has the same action on the filament of iron in Fig. 72 as the increasing or decreasing magnetism of the iron rod in Fig. 71 has on the wire ring in Fig. 71.

Every circular filament in an iron rod has more or less current induced in it while the rod is being magnetized or demagnetized. Thus the currents which are induced in a solid iron rod while it is being magnetized or demagnetized are in the directions of the arrows in Fig. 73 or in the opposite directions, and these currents are called *eddy currents.* One effect of these eddy currents is to make it impossible to magnetize or demagnetize a solid iron rod quickly and another effect is to generate heat in the rod.

An iron rod which is arranged to be magnetized by a winding of wire (thus constituting an electromagnet) is called a *core;* thus we speak of the core of a transformer (the part *CC* in Figs. 61 and 62), the core of an induction coil, the core or cores of the field magnet of a dynamo, and so on.

An iron core which is to be repeatedly magnetized or demagnetized is always built up of sheet iron stampings or of small iron wires as indicated in Figs. 74 and 75. This leaves the iron continuous in the direction of magnetization, but discontinuous in the direction of flow of the eddy currents.

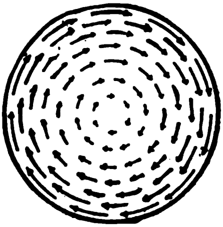

Fig. 73.

Currents induced in solid iron rod, end view.

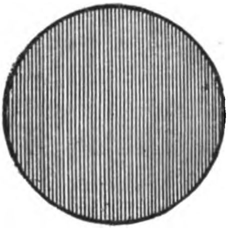

Fig. 74.

End view of rod or core made of strips of sheet iron.

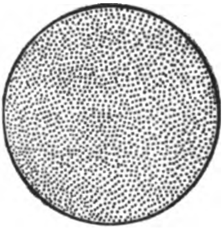

Fig. 75.

End view of rod or core made of iron wires.

Eddy currents are not only produced in a solid iron rod while it is being magnetized or demagnetized, but eddy currents are also generally produced in a piece of solid iron (or in any solid piece of metal) which moves near a magnet. Thus if the cylinder *AA*, Fig. 26, were solid and if it were set rotating as indicated by the curved arrows in Fig. 27, eddy currents would be produced in it as indicated by the circles with dots and crosses in Fig. 76.

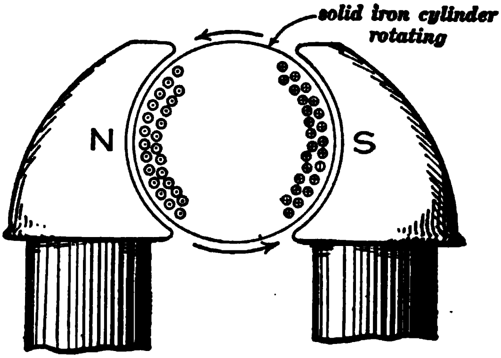

Fig. 76.

Eddy currents in rotating solid cylinder; away from reader on right side, towards reader on left side.

If the cylinder is built up of thin sheet iron disks or stampings, these eddy currents cannot flow because the disks are sufficiently insulated from each other by films of iron oxide.

An iron rod or core which is built up of stampings of thin sheet iron or of fine iron wires is said to be *laminated*. Armature cores of dynamos and transformer cores are always laminated.

48. Inductance of a coil or circuit.—A coil of wire, having Z turns of wire, has a current i flowing in it as indicated in Fig. 77. The current in the coil produces a magnetic field in the surrounding region as indicated by the lines of force in Fig. 77, and it can be shown* from the magnetic definition of current strength (see Art. 19) that the intensity of the magnetic field is everywhere doubled in value if i is doubled in value, the trend of the

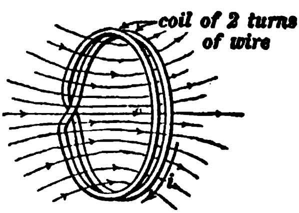

Fig. 77.

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* It is not worth while to introduce this proof here.

lines of force remaining unchanged. Therefore the amount of magnetic flux through each turn of wire or *the average flux per turn*, Φ, is doubled if i is doubled. Therefore Φ is proportional to i so that we may write

$$\Phi = bi \tag{i}$$

where b is a constant for a given coil or circuit. If i changes it is evident that Φ must change b times as fast, or

$$\frac{d\Phi}{dt} = b\frac{di}{dt} \tag{ii}$$

But the changing Φ induces an electromotive force $-\frac{d\Phi}{dt}$ in each turn of wire or a total electromotive force $-Z\frac{d\Phi}{dt}$ in the whole coil, according to equation (17), or using the value of $\frac{d\Phi}{dt}$ from (ii) we get $-bZ\frac{di}{dt}$ as an expression for the electromotive force induced in the coil by the changing current, and, if we represent the quantity bZ by the single letter L, we have

$$E' = -L\frac{di}{dt} \tag{18a}$$

where E' is the electromotive force induced in a coil or circuit by the changing of the current. The negative sign indicates that E' is *opposed* to i when $\frac{di}{dt}$ is positive, that is, when i is increasing. Therefore, to make the current increase at the rate $\frac{di}{dt}$, an outside electromotive force (the electromotive force of a battery, for example) *equal to* $L\frac{di}{dt}$ *but helping the current* must act or push on the circuit. That is

$$E = L\frac{di}{dt} \tag{18b}$$

where E is the outside electromotive force required to make the current in a coil or circuit increase at the rate $\frac{di}{dt}$, and L is a constant for the given circuit. The quantity L is called the *inductance* of the circuit.

Let R be the resistance of the circuit, and let E be the total electromotive force acting on the circuit. Then a portion, Ri, of the electromotive force is used to overcome resistance, and the remainder, $E - Ri$, causes the current to increase in accordance with equation (18*b*).

Note.—If there is an iron core in the coil in Fig. 77 the flux Φ will be nearly proportional to i for small values of i, but as the iron approaches magnetic saturation the flux Φ is no longer even approximately proportional to i.

Flux-turns.—The quantity Φ in the above discussion is the average flux per turn of wire, and the product ΦZ might properly be called the *total flux* passing through the coil (each part of the actual flux in Fig. 77 being counted n times, where n is the number of turns of wire surrounding that part). This product ΦZ is usually called the *number of linkages of lines of force and turns of wire*, and it is expressed as *flux-turns*. Multiply both members of equation (i) by Z and we get $\Phi Z = bZi$. But bZ is the inductance L of the coil, so that

$$\Phi Z = Li. \tag{iii}$$

Mechanical analogue of inductance.—Figure 78 shows a centrifugal pump (like a fan blower) maintaining a current of water

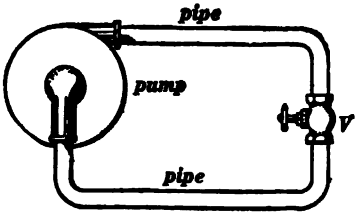

Fig. 78.

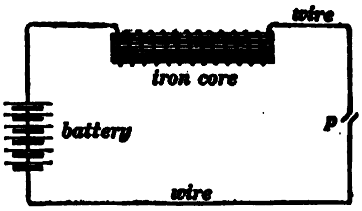

Fig. 79.

in a circuit of pipe. When the valve V is suddenly closed a momentary excessive force is exerted on the valve by the water. The moving water cannot be stopped instantly. To make this behavior of the water stream more clearly analogous to the behavior of the electric current in Fig. 79, it is best to say that the moving water builds up momentarily an excessive *pressure difference* on the two sides of the valve when the valve is suddenly closed.

Figure 79 shows a battery maintaining an electric current through a circuit. When the circuit is suddenly broken at p the stopping of the current builds up momentarily an excessive electrical pressure-difference or electromotive force across the break. The electric current cannot be stopped instantly. If the circuit contains a winding of wire on a laminated iron core, as shown in Fig. 79, the above described effect is so marked that the current continues to flow for a very short time across the break in the form of a visible spark. This effect is called the *spark at break.*

It is the *mass* and *velocity-value* of the moving water that determine the violence of the "hammer" effect in Fig. 78; it is what is called the *inductance of the circuit* and the *current-value* that determine the violence of the spark at break in Fig. 79.

The above described hydraulic analogue of the spark at break appeals to one largely because the picture of a circuit of pipe is like the picture of an electrical circuit; but one can more easily develope the mechanical analogy by considering the acceleration or deceleration of a moving body; and in the following discussion friction and electrical resistance are supposed to be non-existent, not that this supposition is necessary but that it avoids tedious and non-essential qualifications.

A force F acting on a body (which moves without friction) causes the velocity v of the body to increase at a definite	An electromotive force E acting on a circuit (of which the resistance is zero) causes the current i in the circuit to

rate $\frac{dv}{dt}$[*] such that

$$F = m \frac{dv}{dt}$$

where m is the mass of the body. This equation expresses Newton's second law of motion.

A moving body stores a certain amount of energy which it gives up when it stops. This is the kinetic energy, W, of the body and it is

$$W = \tfrac{1}{2} mv^2.$$

increase at a definite rate $\frac{di}{dt}$ such that

$$E = L \frac{di}{dt}$$

where L is the *inductance* of the circuit.

A circuit in which a current is flowing stores a certain amount of energy which it gives up when the flow of current stops. This is the kinetic energy, W, of the circuit, and it is

$$W = \tfrac{1}{2} Li^2. \qquad (19)$$

A very interesting and instructive experiment is to connect an ordinary glow lamp in series with a coil having a large inductance as indicated in Figs. 80 and 81. If the coil is made of fairly coarse wire its resistance will be negligible, and in Fig. 80 (direct-current supply) the lamp is heated to full brightness; *the only effect of the inductance in Fig. 80 is to delay the heating of the lamp (delay the establishment of current in the lamp) to a very slight extent at the beginning.* In Fig. 81, however, the lamp filament is not perceptibly heated because an alternating electro-

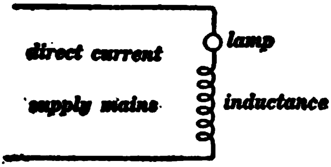

Fig. 80.

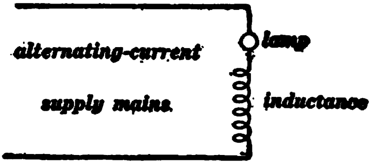

Fig. 81.

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[*] Let the reader understand that $\frac{dv}{dt}$ is a single symbol meaning *the rate of change of v.*

motive force does not produce a perceptible current through a large inductance. This effect of a large inductance in an alternating-current circuit is called a *choking effect*, and an inductance coil* is sometimes called a *choke coil.*

A very considerable force is required to produce a perceptible to and fro motion of a body of large mass; or a moderate to and fro force produces only an imperceptible to and fro motion of a body of large mass, especially if the reversals of the force are rapid (of high frequency).

A very considerable electromotive force is required to produce a perceptible to and fro current (alternating current) in a circuit of large inductance; or a moderate to and fro electromotive force (an alternating electromotive force) produces only an imperceptible alternating current in a circuit of large inductance, especially if the frequency of the alternating electromotive force is high.

Units of inductance.—The inductance of a circuit is defined by equation (18), and a circuit has unit of inductance when one unit of electromotive force will cause the current in the circuit to increase at the rate of one unit per second.

A circuit has an inductance of one *henry* when one volt will cause the current in the circuit to increase at the rate of one ampere per second.

A circuit has an inductance of one *abhenry* when one abvolt will cause the current in the circuit to increase at the rate of one abampere per second. One henry equals one thousand million abhenrys.

When L is expressed in henrys and i in amperes in equation (19), then W is expressed in joules. When L is expressed in abhenrys and i in abamperes, then, of course, W is expressed in ergs.

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* *Inductance coil*, a coil having inductance. An *induction coil* is a very different thing. These two terms must not be confused.

49. Gas-engine ignition.—The older form of gas-engine igniter, the *wipe-spark igniter*, is an arrangement of which the essential features are shown in Fig. 79. An electric circuit containing a battery and an inductance is broken at a point inside of the gas engine cylinder, and the spark at break ignites the mixture of gas and air (or the mixture of gasoline vapor and air) in the cylinder.

The more usual form of gas engine igniter, the *jump-spark igniter*, is an induction coil, the primary circuit of which is closed at the instant that ignition is desired, and the vibration of the interrupter in the primary circuit of the induction coil produces a torrent of sparks across a short spark gap (inside of the gas engine cylinder) to which the secondary coil of the induction coil is connected.

PROBLEMS.

56. The armature of a direct-current motor has a resistance of 0.064 ohm, and when the motor is running under full load a current of 81 amperes is forced through the armature from 110-volt supply mains. What is the back electromotive force in the armature? Ans. 104.8 volts.

57. How much power is delivered to the motor armature in the previous problem? How much power is spent in heating the armature wires? How much power is used in forcing the current through the armature in opposition to the back electromotive force? What becomes of the power so used?

58. A two-pole direct-current dynamo has 560 wires on the outside of its armature $(= Z)$, it runs at a speed of 900 revolutions per minute, and it gives an electromotive force of 125 volts (at zero current output). What amount of magnetic flux passes from north pole face into armature core and from armature core into south pole face?

59. The intensity of the magnetic field in the gap spaces between field pole-faces and armature core of a direct-current dynamo is 6,000 gausses and the length of iron parts of armature core and pole-face is 15 centimeters. The diameter of the arma-

ture (measured out to centers of armature wires) is 30 centimeters, and the speed of the armature is 900 revolutions per minute. What is the velocity of the armature wires as they cut the lines of force of the magnetic field in the gap spaces and what amount of electromotive force is induced in each wire?

60. The large water-wheel-driven alternators in the upper power house at Niagara Falls have a speed of 250 revolutions per minute and they give an alternating electromotive force having a frequency of 25 cycles per second. How many field poles are there in the field magnet of one of these machines?

61. The first of the large steam-driven power plants in New York City were equipped with alternators which were mounted directly on the crank shafts of large reciprocating engines running at a speed of 75 revolutions per minute. The frequency of the alternating electromotive force which is generated is 25 cycles per second. How many field poles are there in one of the field magnets of one of these alternators.

62. A long bar magnet is placed through a short coil, having 20,000 turns of wire, and the magnet is jerked out of the coil quickly so that the magnetic flux through the coil drops to zero in 0.01 second. What is the average value of the electromotive force induced in the coil during the 0.01 second, the strength of one of the poles of the bar magnet being 1,000 units? Ans. 125.66 volts.

Note. Find the amount of magnetic flux which comes in to the south pole and passes out from the north pole of the magnet. This flux passes through the material of the bar magnet, and this is the amount of magnetic flux which passes through the opening of the coil.

63. Imagine a transformer core made of ideal magnetic material such that an indefinitely small magnetizing current in either coil would produce an indefinitely large amount of magnetic flux through the core, and imagine the resistances of the primary and secondary coils to be zero. The primary coil of 1,000 turns is connected to 100-volt direct-current supply mains. Find the steady rate of increase of flux in the core, and find the character and value of the electromotive force induced in the 10-turn secondary coil.

Note. Since the resistance of the primary coil is supposed to be zero the whole of the 100 volts acting on the coil is balanced by an equal and opposite induced electromotive force in the coil.

64. Suppose 20 amperes of current to be taken from the secondary coil (the 10-turn coil) of the ideal transformer arranged as specified in the previous problem. What current would be delivered to the primary coil by the 100-volt direct-current supply mains? What amount of power would be delivered by the secondary coil? What amount of power would be delivered to the primary coil?

Note. We have assumed that an indefinitely small magnetizing "force" would produce in the transformer core an indefinitely large amount of magnetic flux, and this means that the magnetizing action of any current in the primary coil must necessarily be balanced at each instant by the opposite magnetizing action of current in the secondary coil. This can be shown as follows: (*a*) Suppose that an *excess* of primary current exists for a moment, then the core flux would mount suddenly to an indefinitely large value, rising at an indefinitely rapid rate, and the induced electromotive force in the primary coil (which opposes the flow of current) would suddenly be much larger than the supply voltage; therefore the momentary excess of primary current could not come into existence. (*b*) Suppose that a *deficiency* of primary current exists for a moment, then the core flux would drop suddenly to an indefinitely large negative value because of the greater magnetizing force of the secondary current, the flux would drop at an indefinitely rapid rate, and an indefinitely large electromotive force would be induced in the primary coil in the direction to help the production of current by the supply voltage.

The magnetizing "force" of a coil is measured by the product Zi, where Z is the number of turns of wire and i is the current in the coil.

An actual transformer cannot, of course, be used for the steady transformation of direct-current as indicated in problems 63 and 64, because the iron core quickly reaches what is called magnetic saturation. An actual transformer would, however, behave approximately as indicated in problems 63 and 64 *for a few thousandths of a second*, because only a very small net magnetizing force is required to produce a moderate magnetic flux, and because the resistances of the coils are in fact nearly negligible; then if the supply voltage were to be reversed, the transformer would act as indicated for another period of a few thousandths of a second; another reversal of the supply voltage would permit the action to continue for another short period; and so on. This is exactly what takes place in the transformation of alternating current, and the operating engineer nearly always thinks of the action of a transformer as idealized and simplified by the assumptions which have been made in problem 63.

65. The winding of an electromagnet has a resistance of 22 ohms, and when the winding is connected across 110-volt supply mains the current in the coil rises from zero at the beginning to 5 amperes ultimate value. The current must therefore pass in succession the values 1 ampere, 2 amperes, 3 amperes and 4 amperes. Consider the instant when the current is passing the value of 2 amperes, and calculate the values of the following quantities at this instant: (*a*) The rate at which work is being delivered to the coil, (*b*) The rate at which work is being spent to generate heat in the coil, (*c*) The rate at which work is being spent in overcoming a back electromotive force in the coil due to the increasing magnetism of the rod, and (*d*) The value of the back electromotive force due to the increasing magnetism of the rod. Ans. (*a*) 220 watts; (*b*) 88 watts; (*c*) 132 watts; (*d*) 66 volts.

Note. In this arrangement work is spent in generating heat in the coil in accordance with Joule's law and in magnetizing the rod.

The portion of the electromotive force of 110 volts which is being used to overcome resistance is equal to RI, where R is the resistance of the coil and I is the value of the current at any instant. The remainder of the electromotive force of 110 volts overcomes the back electromotive force and is equal thereto.

66. The coil in the previous problem has 10,000 turns of wire. Find how fast the magnetic flux through the coil is increasing as the current is passing the value of 2 amperes. Ans. 660,000 maxwells per second.

67. A boat is started from rest by a steady force of 110 poundals, and after some time the boat reaches a steady speed of 5 feet per second. The velocity must therefore pass in succession the values 1 foot per second, 2 feet per second and so on. Consider the instant when the velocity is passing through the value of 2 feet per second, and assume that the part of the total force which is being used to overcome friction to be proportional to the velocity of the boat; and calculate the following quantities at this instant: (*a*) The rate at which work is delivered to the boat, (*b*) The rate at which work is being spent in overcoming friction, (*c*) The rate at which work is being spent in increasing the velocity of the boat, and (*d*) The value of the inertia reaction of the boat. Ans. (*a*) 220 foot-poundals per second; (*b*) 88 foot-poundals per second; (*c*) 132 foot-poundals per second; (*d*) 66 poundals.

Note. 110 poundals overcomes friction at 5 feet per second, and, according to our assumption, proportionately less force is used to overcome friction at 2 feet per second. The inertia reaction of an accelerated body is equal and opposite to the force which is producing acceleration.

68. A coil of wire has a resistance of 20 ohms and an inductance of 0.6 henry. When connected to 110-volt supply mains the current rises to a steady value of 5.5 amperes. How fast does the current start to increase in the coil when it is first connected to the 110-volt supply mains and how long would it take for the current to reach its steady value of 5.5 amperes if it continued to increase at this initial rate? Ans. 183.3 amperes per second; 0.0289 second.

Note. When the current is zero no portion of the supply voltage is used to overcome resistance, it is all used to make the current increase. The time which would be required for the current to reach its final steady value at its initial rate of increase is called the ***time constant*** of the circuit.

69. From the definition of the time constant of a circuit as given in the note to the previous problem show that the time constant of any circuit is equal to L/R where L is the inductance of the circuit and R is the resistance of the circuit.

Note. Find an expression for the initial rate of increase of the current in the circuit for any value E of supply voltage, and then find how long it would take at this rate to reach the value E/R.

70. Find the rate of growth of current in the coil of problem 68 at the instant that the current is passing the value of 3 amperes. Ans. $83\frac{1}{3}$ amperes per second.

71a. A current is left to die away in a circuit of which the resistance is 0.6 ohm and the inductance is 0.05 henry. Find the rate of decrease of the current as it passes the value of 10 amperes. Ans. 120 amperes per second.

Note. In this problem no electromotive force is supposed to act on the circuit; the resistance drag, however, is RI volts and it is this drag which causes the current to decrease.

71b. A current of I amperes is left to die away in a circuit of which the inductance is L henrys and the resistance is R ohms. Find an expression for the decreasing current i after t seconds. Ans. $i = I\epsilon^{-(R/L)\cdot t}$.

Note. The resistance-drag Ri is an electromotive force acting to reduce the current so that $Ri = -L\frac{di}{dt}$, and the desired expression is obtained by integrating this differential equation. This problem is fully discussed in Appendix A under the heading "The stopping of a boat." See Franklin, MacNutt and Charles's *Calculus*, pages 172–173 and 179–180.

71c. A circuit of resistance R and inductance L is connected to a battery of electromotive force E. Find an expression for the increasing current i after t seconds. Ans. $i = \frac{E}{R} - \frac{E}{R} \cdot \epsilon^{-(R/L)\cdot t}$.

Note. A portion Ri of the electromotive force of the battery is used to overcome resistance and the remainder $E - Ri$ causes the current to increase. Therefore $E - Ri = L\frac{di}{dt}$. The desired expression is obtained by integrating this differential equation. This problem is fully discussed in Appendix A under the heading "The starting of a boat." See Franklin, MacNutt and Charles's *Calculus*, pages 172–173 and 177–178.

CHAPTER IV.

ELECTRIC CHARGE AND THE CONDENSER.

50. The elimination of the water hammer effect by an air cushion. The elimination of the spark at break by a condenser.—The water hammer effect which is produced when a hydrant is suddenly closed is sometimes sufficiently intense to burst the pipe or injure the valve of the hydrant. In some cases, therefore, it is desirable to protect the hydrant by an air cushion as indicated in Fig. 82. When the hydrant in Fig. 82 is closed (however quickly) the moving water in the pipe is brought to rest gradually as it slowly compresses the air in the chamber *CC*.

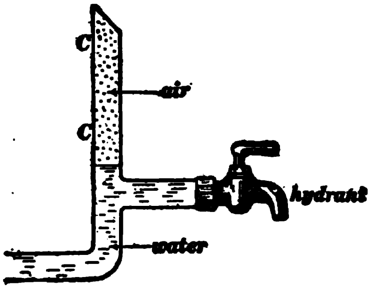

Fig. 82.

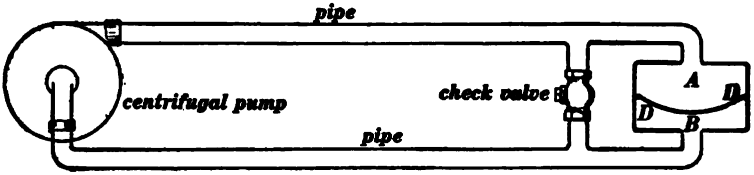

Fig. 83.

Figure 83 shows a centrifugal pump maintaining a stream of water through a circuit of pipe. If the check valve is suddenly closed, the water will continue for a short time to flow through the circuit into the chamber *A* and out of the

Figure 84 shows a battery maintaining a " current of electricity " through a circuit. If the circuit is broken at *p*, the electric current will continue for a short time to flow through the circuit into the metal plate *A* and out of the metal plate chamber *B*. This continued flow of the water into chamber *A* and out of chamber *B* causes a bending (a mechanical stress) of the elastic diaphragm *DD*, and this bending soon stops the flow of water; then the diaphragm unbends and produces a reversed surge of water through the circuit of pipe.

B. This continued flow of the electric current into plate *A* and out of plate *B* causes what we may think of as an "electrical bending" (an electrical stress) of the layer of insulating material *DD*, and this "electrical bending" soon stops the flow of current; then the layer of insulating material "unbends" and produces a reversed surge of electric current through the circuit.

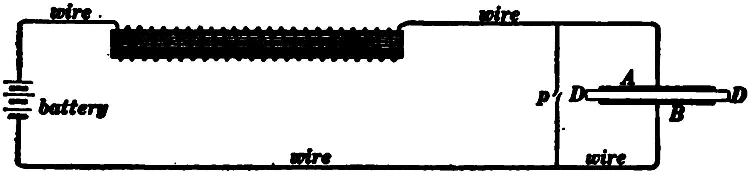

Fig. 84.

The two metal plates *A* and *B*, Fig. 84, together with the layer of insulating material between them constitute what is called a *condenser*. A condenser is usually made of sheets of tin foil separated by sheets of waxed paper. Thus the heavy horizontal lines in Fig. 85 represent sheets of tin foil, and the fine dots represent insulating material. In order that the following effects may actually be observed the condenser in Fig. 84 must be made of a large number of sheets of tin foil and waxed paper.

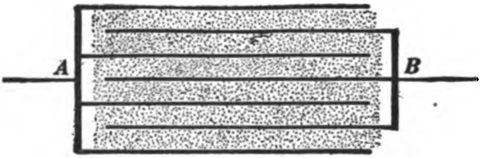

Fig. 85.

The actual flow of current into the metal plate *A* and out of the metal plate *B* when the circuit is broken at *p* in Fig. 84, is shown by the fact that no spark at break is produced when the condenser *AB* is connected, whereas a very perceptible spark at break is produced when the condenser *AB* is not connected.

The reversed surge of current which takes place after the original current has been stopped in Fig. 84, may be shown as follows: Disconnect the condenser *AB*, make and break contact at *p*, hold a magnetic compass near one end of the core of the inductance* coil, and the core will be found to have retained a large portion of its magnetism; in other words, the core will not have become by any means completely demagnetized when the circuit is broken and the current reduced to zero. Then connect the condenser *AB*, make and break the circuit at *p* as before, and again test the core of the inductance coil with a compass. The core will now be found to have lost nearly the whole of its magnetism because of the reversed surge of current.

A reversed surge of current from the condenser in Figs. 64 and 65 is the cause of the very quick demagnetization of the core of an induction coil.

51. The momentary flow of current in an open circuit. Idea of electric charge.—When the metallic contact at *p* in Fig. 84 is broken the electric circuit remains closed as long as the current continues to flow across the break in the form of a spark. The intensely heated air in the path of a spark is a conductor. When the condenser *AB* is connected as shown in Fig. 84, there is no spark at break, and the circuit is actually opened at the moment the contact at *p* is broken. The continued flow of current through the circuit after the contact at *p* in Fig. 84 is broken *is an example of the momentary flow of current in an open circuit.* The current continues for a very short time to flow through the open circuit into plate *A* and out of plate *B* after the circuit is broken at *p*, and *the two plates A and B are said to become electrically charged.* The plate into which the momentary current flows is said to become *positively charged*, and the plate out of which the momentary current flows is said to become *negatively charged.*

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* Use a very long bundle of soft iron wires wound with insulated copper wire.

The flow of current in an open circuit may be shown by connecting a small incandescent lamp and a condenser in series to alternating-current supply mains as shown in Fig. 87. With each reversal of the alternating supply-voltage a momentary current flows through the lamp, and the repeated pulses of current heat the lamp filament to incandescence. When the lamp and condenser are connected to direct-current supply mains, as shown in Fig. 86, a single momentary current flows through the lamp when the connection is first made, but this single momentary pulse of current has no appreciable heating effect on the lamp filament.

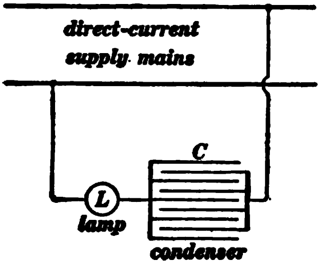

Fig. 86.

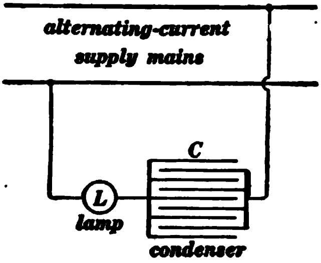

Fig. 87.

52. The function of choke coil and condenser in the lightning arrester.—Figure 89 shows the essential features of the lightning arrester for protecting a dynamo *G* from a lightning stroke. When the lightning strikes the trolley wire, a very large electromotive force acts for a very short time between the trolley wire and ground, the spark gap breaks down, and the lightning discharge flows across the spark gap to earth. Meanwhile the large electromotive force has started an appreciable current through the choke coil, and nearly the whole of this current flows into one plate of the condenser and out of the other plate of the condenser to earth. But the current has been of such extremely short duration that the voltage across the condenser terminals (and across the dynamo terminals) has not risen to any considerable value. Thus the dynamo *A* is protected from the action of a high voltage across its terminals.

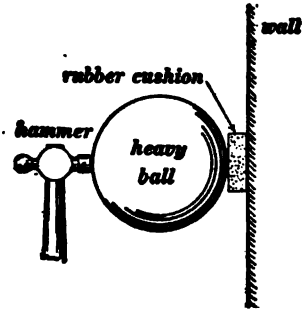

Fig. 88.

Wall well protected from shock.

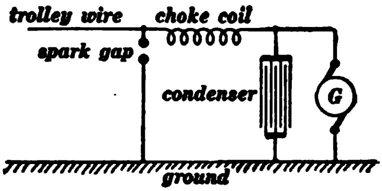

Fig. 89.

Dynamo well protected from lightning stroke.

When the hammer strikes the ball in Fig. 88, it exerts a very large force on the ball for a very short time.	When lightning strikes the trolley wire in Fig. 89, a very large voltage acts from trolley wire to ground for a very short time.
This force sets the ball in motion.	This electromotive force sets up a current through the choke coil.
This motion continues for a relatively long time, slowly squeezing the cushion.	This current continues for a relatively long time, slowly "squeezing" the condenser,* that is, slowly charging the condenser.
Therefore a comparatively small force is exerted against the wall for a comparatively long time.	Therefore a comparatively small electromotive force is exerted across the condenser terminals and across the dy-

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* Let one consider how unsuggestive it is to speak of *charging* a condenser and how extremely suggestive it is to speak of "squeezing" a condenser.

namo terminals for a comparatively long time.

The very large and short-duration force which is exerted on the ball by the hammer is converted by the ball and cushion into a much smaller force of long duration as exerted against the wall.

The very large and short-duration electromotive force from trolley wire to ground (due to the lightning) is converted by the choke coil and condenser into a much smaller electromotive force of long duration as exerted across condenser and dynamo terminals.

53. Electrostatic attraction. The electrostatic voltmeter.—When a momentary current flows into plate A and out of plate B in Fig. 84, the plates are said to become electrically charged, as stated in Art. 51, the plate into which the momentary current flows is said to become positively charged and the plate out of which the momentary current flows is said to become negatively charged. *Two plates which have thus been oppositely charged attract each other when the intervening insulating material is a fluid like air or oil.*

This attraction between two oppositely charged metal plates is utilized in the *electrostatic voltmeter* which consists of a very delicately suspended metal plate and a stationary metal plate, both carefully insulated. The electromotive force to be measured is connected to these plates, a momentary flow of current charges one plate positively and the other plate negatively, the suspended plate is moved by the electrostatic attraction between the plates, and a pointer attached to the movable plate plays over a divided scale. Figure 90 shows the essential features of an electrostatic voltmeter. The moving element of the instrument consists of two very light metal vanes VV (seen edgewise in the figure)

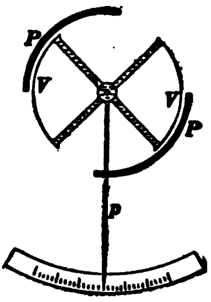

Fig. 90.

Westinghouse type electrostatic voltmeter.

mounted on a pivot and carrying a pointer p, and the stationary element consists of two metal plates PP (also seen edgewise in the figure). The plates PP are connected together and to one terminal of the voltage to be measured, and the other terminal of the voltage is connected to the moving element VV by means of the controlling hair spring.

Electrostatic attraction is familiar to every one. A hard-rubber comb, for example, is charged with electricity when it is passed through dry hair, at the same time the hair is oppositely charged, and each hair is attracted by the comb.

54. Definition of the coulomb.—The electric current in a wire may be looked upon as the transfer of "electricity" along the wire, and the quantity Q of "electricity" which flows past a point on the wire during t seconds may be defined as the product of the strength of the current and the time, that is we may write:

$$Q = It \tag{20}$$

in which I is the strength of the current in amperes, and Q is the quantity of electricity which flows past a point on the wire during t seconds. It is evident from equation (20) that the product of amperes and seconds gives quantity of electricity, and therefore the unit of quantity of electricity is most conveniently taken as one *ampere-second*, meaning the amount of electricity which during one second flows past a point on a wire in which a current of one ampere is flowing. The ampere-second is usually called the *coulomb*. One *ampere-hour* is the quantity of electricity carried in one hour by a current of one ampere.

The c.g.s. unit of charge in the " electromagnetic system " is the amount of charge carried in one second by a current of one abampere, and it is called the *abcoulomb*.

The c.g.s. unit of charge in the "electrostatic system" is that amount of charge which if concentrated on a very small body would repel a similar charge with a force of one dyne at a distance of one centimeter. The "electrostatic system" system of units is not used in this text.

55. The capacity of a condenser.—A ballistic galvanometer BG, a condenser and a number of dry cells are connected as shown in Fig. 91. One terminal of the condenser is connected to a flexible wire which is fixed to the end of a glass handle. By touching the wire W to the point b, the electromotive force E of one dry cell acts upon the condenser, and the momentary flow of current which charges the condenser produces a throw of the ballistic galvanometer. The condenser can then be discharged by touching the wire W to the point a. By touching the wire W to the point c, the electromotive force $2E$ of two dry cells acts upon the condenser, and the momentary flow of current which charges the condenser produces a throw of the ballistic galvanometer. The condenser may then be discharged as before. By touching the wire W to the point d, the electromotive force $3E$ of three dry cells acts upon the condenser, and the momentary flow of current which charges the condenser causes a throw of the ballistic galvanometer; and so on. In this way the throws of the ballistic galvanometer may be observed when the condenser is charged by an increasing series of voltages E, $2E$, $3E$, $4E$, and so forth, and it is found that the throw of the ballistic galvanometer becomes larger and larger in proportion to the voltage. But the throw of the ballistic galvanometer is proportional to the charge which is drawn out of one plate and forced into the other plate of the condenser. *Therefore the amount of charge which is drawn out of one plate and forced into the other plate of a condenser is proportional to the electromotive force which*

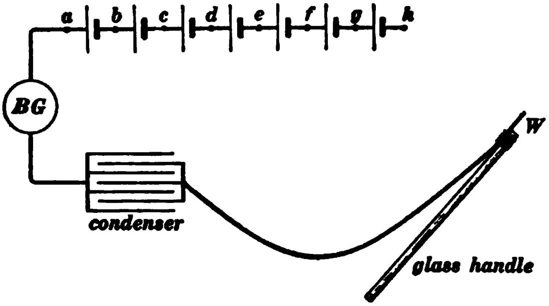

Fig. 91.

acts upon the condenser. Therefore we may write:

$$Q = CE \tag{21}$$

where Q is the quantity of charge which is drawn out of one plate and forced into the other plate of a condenser when an electromotive force of E volts is connected so as to act upon the condenser, and C is a constant for a given condenser. The factor C is adopted as a measure of what is called the *capacity* of the condenser.

It is evident from the above equation that C, the capacity of a condenser, is expressed in *coulombs-per-volt.* One coulomb-per-volt is called a *farad*, that is to say *a condenser has a capacity of one farad when an electromotive force of one volt will draw one coulomb out of one plate of the condenser and force one coulomb into the other plate of the condenser.*

Condenser capacities as usually encountered in practice are very small fractions of a farad. Thus the capacity of a condenser made by coating with tin foil the inside and outside of an ordinary one-gallon glass jar would be about one five-hundred-millionth of a farad, or 0.002 of a microfarad. A *microfarad* is a millionth of a farad, and in practice capacities of condensers are usually expressed in microfarads.

The approximate dimensions of a one-microfarad condenser are as follows: 501 sheets of tin foil separated by sheets of paraffined paper 0.05 centimeter in thickness, the overlapping portions of the tin foil sheets being 25 centimeters × 25 centimeters, as shown in Fig. 92.

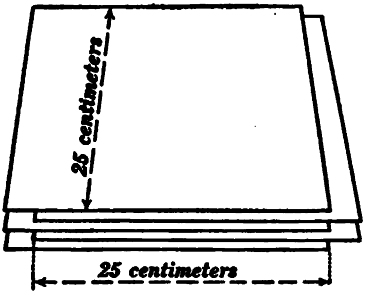

Fig. 92.

Two pieces of metal of any shape separated by insulating material constitute a condenser; the only reason for using sheets of metal with thin layers of insulating material between is to obtain a large capacity.

56. Inductivity of a dielectric.—The insulating material between the plates of a condenser is called a *dielectric.* Indeed, the insulating material between any two oppositely charged bodies is called a dielectric. The capacity of a condenser depends upon the size of the plates, upon the thickness of the dielectric and upon the nature of the dielectric. The dependence of the capacity of the condenser upon the nature of the dielectric is a matter which must be determined purely by experiment. Thus Fig. 93 represents two metal plates with air between them, and Fig. 94 represents the same plates immersed in oil. The distance between the plates is understood to be the same in Figs. 93 and 94. Let C be the capacity of the condenser in Fig. 93 with air as the dielectric, and let C' be the capacity of the condenser in Fig. 94 with a given kind of oil as the dielectric. The

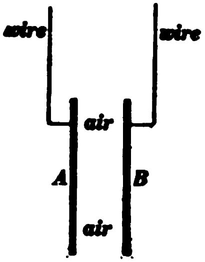

Fig. 93.

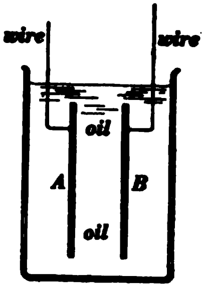

Fig. 94.

TABLE.

Inductivities of Various Substances.

Crown glass (according to composition)	3.2 to 6.9
Flint glass (according to composition)	6.6 to 9.9
Hard rubber	2.08 to 3.01
Sulphur (amorphous)	3.04 to 3.84
Paraffin	2.00 to 2.32
Shellac	2.74 to 3.67
Ordinary rosin	2.48 to 3.67
Mica (according to composition)	5.66 to 10
Petroleum	about 2.04
Water	about 90.

ratio C'/C is called the *inductivity** of the oil. Thus the inductivity of kerosene is about 2.04, that is, the capacity of a given condenser is 2.04 times as great with kerosene between the plates as with air between the plates. The accompanying table gives the inductivities of a few dielectrics.

57. Dependence of capacity of a condenser upon size and distance apart of plates.—When the dielectric of a condenser is of uniform thickness and when the metal plates are large as compared with their distance apart (thickness of dielectric), then the capacity C of the condenser is proportional† to a/x for a given dielectric, where x is the thickness of the dielectric and a is the area of the sheet of dielectric between the plates. Therefore, if we choose a given dielectric, we may write

$$C = B \cdot \frac{a}{x} \tag{i}$$

in which B is a constant. When x is expressed in centimeters, and a in square centimeters; when air is chosen as the dielectric; and when C is expressed in farads; then the value of B as found by experiment is $884 \div 10^{16}$. Therefore we have:

$$C_{\text{in farads}} = 884\frac{a}{x} \div 10^{16} \tag{22}$$

When a dielectric of inductivity k is used instead of air, the capacity of the condenser is k times as great, or:

$$C_{\text{in farads}} = 884\frac{ka}{x} \div 10^{16} \tag{23}$$

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* What is here called the *inductivity* of a dielectric is sometimes called *dielectric constant*, or *specific capacity* of a dielectric, or *specific inductive capacity* of a dielectric.

† It can be shown from almost purely geometrical considerations that C is proportional to a/x, but it is sufficient to accept this proportional relation as the result of experiment.

Suppose one were to take the density of a fluid as unity, arbitrarily, and express the velocity of sound in terms of a unit of velocity entirely consistent with the unit of density so chosen. Then the compressibility of the fluid is related to the velocity of sound in the fluid in exactly the same way that the value of the factor B is related to the velocity of light. Many students having in mind this relation think that the factor B need not be determined by experiment; but it must be so determined directly or indirectly.

in which C is the capacity in farads of a condenser of which the plates are separated by a layer of dielectric x centimeters thick and a square centimeters in area (between the plates), and k is the inductivity of the dielectric. The meaning of a may be understood with the help of Fig. 92. If there are 501 sheets of tin foil there will be 500 sheets of dielectric, and a will be equal to 500 × 25 centimeters × 25 centimeters, which is 312,500 square centimeters.

58. **The work done by an electromotive force E in pushing a given amount of charge, Q, through a circuit.**—When Q coulombs of electric charge flow through a battery of which the electromotive force is E, the amount of work W done by the battery is EQ joules. That is:

$$W = EQ \tag{24}$$

This is evident from the following considerations. Imagine a current I flowing through the battery; then EI watts is the rate at which the battery does work, and EIt joules is the amount of work done in t seconds. But the product It is the amount of charge Q (in coulombs) which has been pushed through the circuit. Therefore the work done, namely EIt joules, is expressible as EQ joules.

59. **The potential energy of a charged condenser.**—A charged condenser represents a store of potential energy in much the same way that a stretched spring or the distorted diaphragm DD in Fig. 83 represents a store of potential energy, and before con-

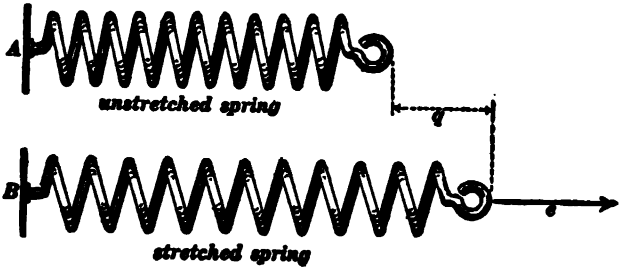

Fig. 95

sidering the amount of potential energy in a charged condenser it is helpful to consider the amount of potential energy in a stretched spring.

Let q represent the elongation of a spring due to a stretching force e as shown in Fig. 95. As is well known, q is proportional to e; therefore if we plot corresponding values of q and e as abscissas and ordinates respectively, we will get a straight line cc as shown in Fig. 96.

Consider the total amount of work W which is done while the spring is being stretched from $q = 0$ to $q = Q$, and while the stretching force is increasing from $e = 0$ to $e = E$. The average value of the stretching force is $\frac{1}{2}E$, as may be understood from Fig. 96, and the work done is equal to the product of the total stretch Q and the average stretching force $\frac{1}{2}E$. That is:

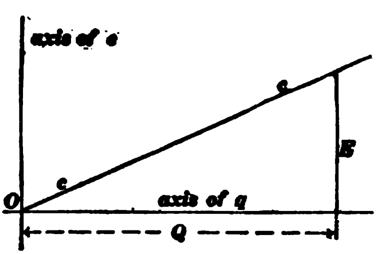

Fig. 96.

$$W = \tfrac{1}{2}EQ$$

and this work W is stored in the stretched spring as potential energy. Thus a stretch of 3 centimeters $(= Q)$ is produced in a large spring while the stretching force rises from zero to 2,000,000 dynes $(= E)$. The average value of the stretching force is 1,000,000 dynes $(= \frac{1}{2}E)$, and the work done is 3,000,000 ergs $(= \frac{1}{2}EQ)$. This work is stored in the stretched spring as potential energy.

A condenser is charged by applying it to an electromotive force which begins at zero and rises to E volts, and the amount of work W which is done in charging the condenser is equal to $\frac{1}{2}EQ$, where $\frac{1}{2}E$ is the average value of the charging electromotive force, and Q is the total charge which is drawn out of one plate of the condenser and pushed into the other plate. This statement is in accordance with equation (24) of Art. 58.

Therefore:

$$W = \tfrac{1}{2}EQ \tag{25}$$

where W is the potential energy of a charged condenser, E is the voltage acting on the charged condenser, and Q is the charge which has been drawn out of one plate of the condenser and pushed into the other plate; W is expressed in joules when E is in volts and Q in coulombs.

We may substitute CE for Q in equation (25), according to equation (21) of Art. 55, and we get:

$$W = \tfrac{1}{2}CE^2 \tag{26}$$

or we may substitute Q/C for E in equation (25), according to equation (21) of Art. 55, and we get:

$$W = \tfrac{1}{2}\frac{Q^2}{C} \tag{27}$$

Following is a rigorous derivation of equation (27) as applied to a stretched spring. Let q be the elongation of the spring when the stretching force is e. Then q and e are proportional, so that:

$$q = Ce \tag{i}$$

where C is a constant for the given spring. Let Δq be the added elongation due to an increment Δe of the stretching force, and let ΔW be the work done on the spring to produce the added elongation. Then:

$$\Delta W \text{ is greater than } e \cdot \Delta q$$

and

$$\Delta W \text{ is less than } (e + \Delta e) \cdot \Delta q$$

or

$$\frac{\Delta W}{\Delta q} \text{ is greater than } e \text{ and less than } (e + \Delta e).$$

Therefore $\Delta W/\Delta q$ approaches e as a limit when Δe and Δq both approach zero or, using differential notation, we have $dW/dq = e$; or, using the value of e from equation (i), we have:

$$\frac{dW}{dq} = \frac{q}{C} \tag{ii}$$

Now the potential energy W of the spring when its elongation is Q, is the amount of work done in stretching the spring from $q = 0$ to $q = Q$, and this is found by integrating equation (ii) from $q = 0$ to $q = Q$, which gives:

$$W = \frac{1}{2}\frac{Q^2}{C} \tag{iii}$$

60. Disruptive discharge. Dielectric strength.—When the electromotive force which charges a condenser is increased more and more, the dielectric of the condenser is eventually broken down; this break-down occurs in the form of an electric spark, it discharges the condenser, and it is called a *disruptive discharge.* By a condenser is here meant two metal bodies of any shape separated by insulating material. The electromotive force required to break down a dielectric depends upon three things, namely, (*a*) the shape of the metal bodies, (*b*) the minimum distance* between the metal bodies, and (*c*) the nature of the dielectric. The dependence upon the shape of the metal bodies is illustrated by the fact that a given electromotive force will produce a much longer spark between points than between flat metal surfaces. *In the whole of the following discussion the dielectric is assumed to be between flat metal plates.*

When the dielectric is perfectly homogeneous like air or oil, the voltage required to break it down is very nearly proportional to its thickness, and the voltage required to break down such a dielectric divided by the thickness of the dielectric gives what is called the *specific strength* of the dielectric. Thus the specific dielectric strength of air is about 35,000 volts per centimeter. When the dielectric is non-homogeneous, the voltage required to break it down is not even approximately proportional to its thickness. The most familiar example of a non-homogeneous dielectric is the material which is used for insulating the windings of dynamos and transformers. This material is made up of layers of cloth and varnish and mica with occasional layers of air.

If a tank is made with one wall of porous material like unglazed earthenware, the pressure of the fluid in the tank has three important effects upon the wall, namely, (*a*) fluid soaks through the wall at a certain rate, (*b*) the wall is slightly elastic and it yields a little to the fluid pressure, and (*c*) the wall has a certain ultimate strength and it will burst if the pressure exceeds a certain value. Similarly the electromotive force which acts on a condenser has three important effects upon the dielectric of the condenser, namely, (*a*) a certain amount of electric current " soaks " through the dielectric as it were, because the dielectric is an electrical conductor although a very poor one, (*b*) the dielectric has a certain amount of electrical " elasticity " (inductivity as it is properly called), and it " yields " a little to the electromotive force and allows a certain amount of charge to be drawn out of one plate and forced into the other plate of the condenser, and (*c*) the dielectric has a certain ultimate strength and it will be ruptured if the electromotive force exceeds a certain value.

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* This is not true when the distance is very small or when the bodies are in a very good vacuum. See Art. 79.

61. The spark-gauge.—The electromotive force required to produce a spark between polished metal spheres in air depends upon the length of the air gap between the spheres and upon the diameter of the spheres; and the accompanying table gives the observed sparking voltages corresponding to different lengths of air gap and different diameters of spheres.

The *spark-gauge* is an arrangement for measuring an electromotive force by observing the length of spark it will produce. As an example consider the following test of the break-down voltage of the rubber insulation on a wire. The arrangement is shown in Fig. 97. The two spheres *BB* of the spark-gauge are connected to a high-voltage electric machine,* one of the spheres is connected to the metal core of the wire to be tested, and a wire from the other sphere is wrapped around the outside

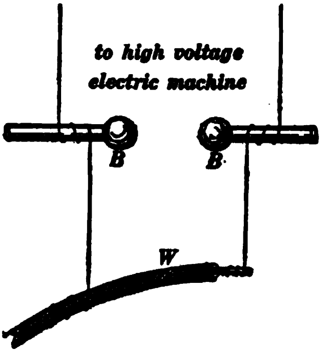

Fig. 97.

Spark-gauge diagram.

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* The familiar Toepler-Holtz machine. This refers to a very simple laboratory experiment. Very full specifications for voltage tests are given in Sections 480–541 of the 1916 edition of The Standardization Rules of the American Institute of Electrical Engineers.

of the insulation to be tested, as shown. The spheres *BB* are near together at the start, and they are slowly separated until the spark breaks through the insulation on the wire and ceases to jump between the spheres. For example the air gap between the spheres was increased to 0.6 centimeter before the insulation on the wire broke down, and the spheres were each 2 centimeters in diameter; therefore the break-down voltage, as taken from the table, was 20,400 volts. Break-down tests are nearly always made in practice by using alternating voltage from a step-up transformer, and the spark-gauge usually has needle points instead of polished metal spheres.

TABLE.[*]

Sparking Voltages in Air at 18° C. and 745 mm. Pressure.

Spark gap in centimeters.	Between polished metal spheres 0.5 centimeter diameter. Volts.	Between polished metal spheres 1.0 centimeter diameter. Volts.	Between polished metal spheres 2 centimeters diameter. Volts.	Between polished metal spheres 5 centimeters diameter. Volts.
0.1	4,830	4,800	4,710	.....
0.2	8,370	8,370	8,100	.....
0.3	11,370	11,370	11,370	.....
0.4	13,800	14,400	14,400	.....
0.5	15,600	17,400	17,400	18,300
0.6	17,100	19,800	20,400	21,600
0.7	18,300	21,900	23,100	24,600
0.8	18,900	24,000	26,100	27,300
0.9	19,500	25,500	28,800	30,000
1.0	20,100	27,000	31,200	32,700
1.1	20,700	.....	33,300	35,700
1.2	21,000	.....	35,400	38,400
1.3	21,600	.....	37,200	41,100
1.4	21,900	.....	38,700	43,800
1.5	22,200	.....	40,200	46,200
1.6	.....	.....	41,400	48,600

62. Mechanical oscillations and electrical oscillations.—In the following discussion the ball in Fig. 98 is assumed to move without friction, and the circuit *fLg* in Fig. 99 is assumed to have zero resistance for the sake of simplicity of statement.

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[*] A table giving the sparking voltage between needle points is given in Section 537 of the 1916 edition of The Standardization Rules of the American Institute of Electrical Engineers. See Sections 538–541 for more complete information concerning the spark between metal spheres.

The ball is pulled to one side by the thread until the thread breaks; then the ball is free to be set in motion by the push of the spring.

The voltage between wires *a* and *b* is raised more and more (thus charging the condenser more and more) until a spark forms across the spark gap; then the "push" of the charged condenser begins to set up current through the circuit *fLg*.

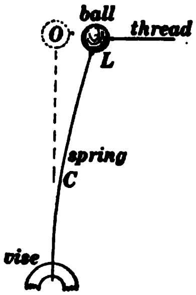

Fig. 98.

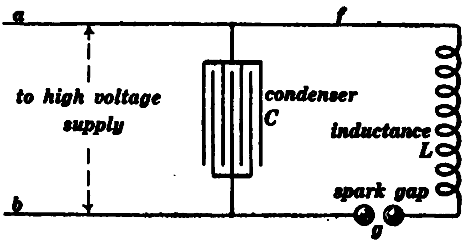

Fig. 99.

The velocity i of the ball increases until the spring is unbent.

The momentum of the ball carries it onwards, however, thus bending the spring in the reverse direction, until the ball, or $i = 0$.

The reversed bend then sets up a reversed motion of the ball, and so on, over and over again.

The current i increases until the condenser is completely discharged.

The "momentum" of the current keeps up the flow for a time, however, thus charging the condenser in a reversed direction, until $i = 0$.

The reversed charge on the condenser then sets up a reversed current in the circuit *fLg*, and so on, over and over again.

A condenser and an inductance arranged as shown in Fig. 99 constitute what is called an *electric oscillator*.

In a mechanical oscillator like Fig. 98 the energy is all in the spring as potential energy when the bend is a maximum and the velocity of the ball is momentarily zero, and the energy is all in the ball as kinetic energy when the bend of the spring is momentarily zero and the velocity of the ball a maximum.

In an electrical oscillator like Fig. 99 the energy is all in the condenser as electric energy [$= \frac{1}{2}CE^2$ according to equation (26)] when the charge of the condenser is a maximum and the current is momentarily zero, and the energy is all in the inductance as magnetic energy [$= \frac{1}{2}Li^2$ according to equation (19)] when the charge of the condenser is momentarily zero and the current at its maximum value.

The complete analogy between Figs. 98 and 99 is best shown by comparing the differential equations of motion, on the assumption that the ball moves to and fro in a viscous fluid so that the force of friction may be at each instant equal to Ri, where i is the velocity of the ball and R is a constant.

63. Differential equation of motion of the ball in Fig. 98.— Imagine the ball to be oscillating to and fro, and consider the instant when the ball is at a distance $+q$ from O, and moving at velocity $+i\left(=+\frac{dq}{dt}\right)$. The plus sign means to the right and the negative sign means to the left. The forces exerted on the ball at the given instant are (a) The pull of the spring, which, being proportional to q and towards the left, can be written $-\frac{1}{C}\cdot q$, where $1/C$ is a constant, and (b) The force of friction, which, being proportional to i and opposed to i, can be written $-Ri$ or $-R\frac{dq}{dt}$. The total force exerted on the ball is therefore $-R\frac{dq}{dt}-\frac{1}{C}\cdot q$; and this force must be equal to $L\frac{d^2q}{dt^2}$, where L is the mass of the ball and $\frac{d^2q}{dt^2}$ is its acceleration. Therefore we get

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}\cdot q = 0 \tag{28a}$$

Differential equation of "motion" of the circuit $CfLg$ in Fig. 99.—Imagine electric charge to be flowing back and forth through fLg, and, reckoning from the instant when the condenser has no charge, consider the instant when an amount of charge q has passed through the circuit in a chosen positive direction. Then the accumulated charge q on the condenser causes the condenser to push *backwards* on the circuit with an electromotive force equal to q/C; and the positive current $\frac{dq}{dt}$ is *opposed* by the resistance drag of Ri or $R\frac{dq}{dt}$. Therefore the total electromotive force acting on the circuit is $-R\frac{dq}{dt} - \frac{1}{C}\cdot q$, and this must be equal to $L\frac{di}{dt}$ or $L\frac{d^2q}{dt^2}$ according to equation (18). Therefore we get

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}\cdot q = 0 \tag{28b}$$

The identity of equations (28*a*) and (28*b*) shows that the mathematical theory of mechanical oscillations is the same as the mathematical theory of electrical oscillations. A familiar proposition in elementary dynamics is the following: If it takes q/C dynes to pull the ball in Fig. 98 a distance of q centimeters to one side and hold it there, then, if the ball is released it will perform n complete oscillations per second where

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \tag{29*}$$

L being the mass of the ball in grams.

This identical equation expresses the number of complete electrical oscillations per second in Fig. 99, L being the value in henrys of the inductance, and C being the capacity in farads of the condenser.

Remark concerning Fig. 99.—One would think that the break-down of the spark gap would constitute a serious short-circuit of the high-voltage supply. However, the to and fro surging of current which follows each break-down lasts but an extremely short time, and any tendency for the formation of a steady arc across the spark gap can be overcome by several devices the simplest of which is a powerful blast of air blown through the gap, or a strong blow-out magnet arranged as in Fig. 25. The high-voltage supply used in Fig. 99 is usually the high-voltage terminals (10 to 20 thousand volts) of a small step-up transformer.

64. The closed oscillator and the open oscillator.—Two metal rods AB, Fig. 100, have a spark gap G between them. The rods are connected as indicated to the high-voltage terminals of an induction coil, and at each interruption of the primary circuit of the induction coil the rods A and B are charged sufficiently to produce a spark across the gap G. This spark is followed by a back and forth surging of current across the gap and along the rods A and B.

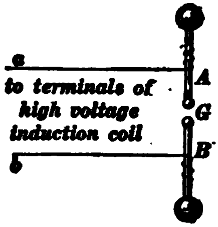

Fig. 100.

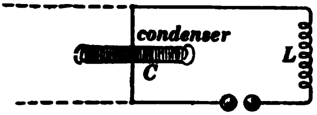

Fig. 101.
Closed oscillator.

The arrangement in Fig. 99 is called a *closed oscillator* because the energy when it is all electric is all located in the narrow region between the condenser plates as indicated by the fine lines of force (lines of force of the electric field) in Fig. 101; and the arrangement in Fig. 100 is called an *open oscillator* because the energy when it is all electric is spread out in the whole surrounding region as indicated by the fine lines of force (lines of force of the electric field) in Fig. 102.

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* See Franklin and MacNutt's *General Physics*, pages 53–56. The McGraw-Hill Book Co., New York, 1916. See also Appendix A.

A closed oscillator gives out but little energy in the form of electric waves, whereas an open oscillator gives out very energetic electric waves.

The difference between the closed oscillator and the open oscillator is a difference of degree only, and the usual form of wireless antenna is intermediate between Figs. 101 and 102 as may be seen from Fig. 103 which shows the lines of force of the electric field.

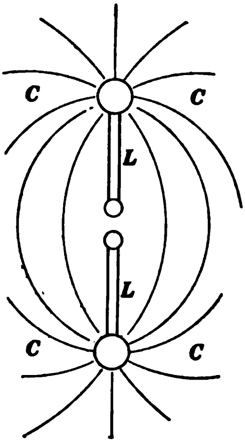

Fig. 102.

Open oscillator.

65. Wireless telegraphy.—The essential features of a wireless installation are shown in Fig. 104. The arrangement *abCfLg* is the oscillator of Fig. 99, the two coils *L* and *S* (without iron) constitute a transformer, the surging of alternating current back and forth through *L* induces a high-frequency electromotive force in *S*, this high-frequency electromotive force causes current to surge up and down in the sending antenna, and the antenna gives out electric waves. These waves cause a very weak alternating current to surge up and down in the receiving antenna and through the primary coil *P* of the transformer *PS′* (without iron). If the secondary *S′* were connected directly to

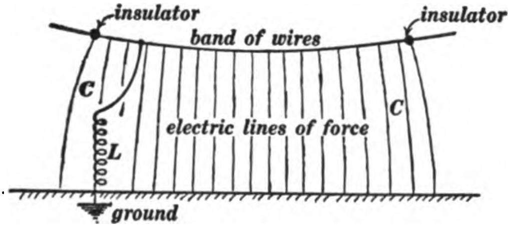

Fig. 103.

the telephone no audible effect would be produced because of the extremely high frequency, but by connecting a crystal rectifier cm in the telephone circuit every alternate half-wave of the secondary current i flows through the telephone.

As long as the high-voltage supply ab is connected, a torrent of sparks is maintained across the air gap g, each spark is followed by a few hundred surges of high-frequency alternating current up and down the sending antenna, and up and down the receiving antenna; and when delivered by S', these few hundred cycles of high-frequency current blend as one pulse of direct current producing a sharp click in the telephone. Thus the torrent of sparks across g is heard as a continuous buzz in the telephone, and by repeatedly interrupting the high-voltage supply ab long and short periods of buzzing sound can be produced in the telephone as dots and dashes of the telegraph code. The arrangements for " tuning " are not shown in Fig. 104.

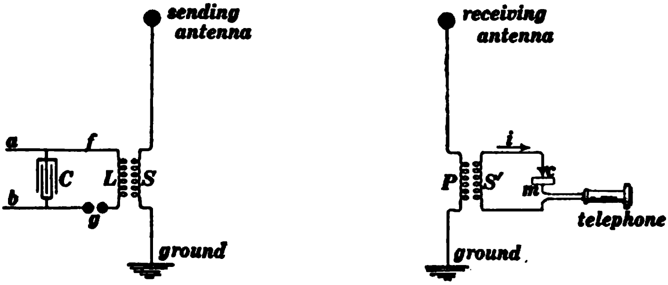

Fig. 104.

66. Coupled circuits.—The circuit $CfLg$ in Fig. 104 is an oscillating circuit (closed type), the coil S together with the sending antenna is an oscillating circuit (open type), the two coils L and S are related to each other as the primary and secondary coils of a transformer (without iron), and the two oscillating circuits are said to be *coupled.** Figure 105 shows the same arrangement except that both oscillating circuits are of the closed type.

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* Another type of coupling is called *electrostatic coupling*. Figures 104 and 105 show *transformer coupling*.

An iron core cannot be magnetized and demagnetized at an extremely high frequency, therefore *oscillation transformers*, like LS Fig. 104 and L_1L_2 Fig. 105, are always made without iron, and much of the magnetic flux which is produced by the primary (or secondary) current does not pass through or link with the secondary (or primary) coil. If only a small fraction of the flux due to a current in L_1 links with L_2 the coupling of the two circuits in Fig. 105 is said to be *loose*; if a large fraction of the flux due to a current in L_1 links with L_2 the coupling is said to be *close*. The effect of loose coupling may be described in general as follows: Suppose the circuit C_1fL_1g in Fig. 105 is suddenly set oscillating by the formation of a spark across the gap g; then if the coupling between L_1 and L_2 is loose each surge of current through L_1 will induce in L_2 a small electromotive force,[*] and many repetitions of this small electromotive force will be required to bring the circuit L_2C_2 into violent oscillation.[†]

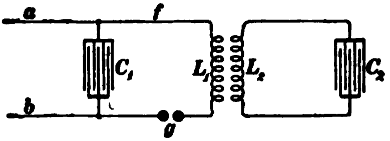

Fig. 105.

67. The mechanical analog of the transformer and the mechanical analog of coupled circuits.—Every student of electrical engineering has no doubt wondered what might possibly be the relation between the familiar devices for multiplying force in mechanics and the devices for multiplying electromotive force. A steady force of 110 pounds is easily "transformed" into a steady force of 1100 or 11,000 pounds by means of a lever; but a steady electromotive force of 110 volts cannot be converted into a steady electromotive force of 1100 or 11,000 volts by a transformer. Therefore it would seem that the lever and the transformer are wholly unrelated. However, a lever with *a heavy weight* instead of *a fixed fulcrum* is entirely analogous to a transformer; and the equations of motion of such a lever can be reduced to forms identical to the equations of a transformer including such complications as magnetizing current and magnetic leakage.

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[*] This statement applies to the simplest case, namely, when the coils L_1 and L_2 have the same number of turns of wire; and by a small electromotive force is meant an electromotive force which is small in comparison with the electromotive force acting at the same instant across the terminals of coil L_1.

[†] A very clear discussion of the mathematical theory of coupled circuits is given by Professor F. R. Lyle in the *Philosophical Magazine*, Vol. 25, pages 567–592.

An ideal lever consisting of a bar without weight (mass) and having an indefinitely large weight (mass) fixed to its middle is the exact equivalent of an ideal transformer in which *all* the flux produced by current in the primary (or secondary) coil links with the secondary (or primary) coil. The to and fro *velocities* of the ends of the lever correspond to primary and secondary *currents*, the *force* (alternating) applied to one end of the lever corresponds to the *voltage* applied to the primary of a transformer, and the *force* (alternating) exerted by the other end of the lever corresponds to the secondary terminal *voltage* of a transformer. The velocity values are directly as the lengths of the two arms of the lever (this corresponds to the fact that the primary and secondary currents of an ideal transformer are inversely as the respective numbers of turns of wire), and the two forces are inversely as the lengths of the two arms of the lever (this corresponds to the fact that the primary and secondary voltages of an ideal transformer are directly as the respective numbers of turns of wire).

When the working end of the lever is rigidly fixed, we have a condition corresponding to open-circuited secondary. In this case the hand-end of the lever will not move perceptibly if the fulcrum-mass is very great; but if the fulcrum-mass is moderate, the hand-end of the lever will move as the fulcrum-mass is accelerated and decelerated. This motion of the hand-end of the lever corresponds exactly to the magnetizing current of a transformer.

If the beam of the lever were without mass the forces at the ends of the lever would be in exact inverse proportion to the lengths of the arms of the lever; but if the mass of the lever beam is considerable or if there are weights attached to the ends of the lever then the lever acts like a transformer with magnetic leakage.* Part of the force exerted by the hand is used to accelerate and decelerate the weight at the hand end, the *remainder* of the force exerted by the hand is transmitted to the other end of the lever (being multiplied in exact inverse proportion to the lengths of arms of lever); a portion of the force so developed at the working end of the lever is used to accelerate and decelerate the weight at the working end, and the *remainder* is exerted on the receiving device.

In all of the above the lever is supposed to be frictionless and the transformer coils are assumed to have zero resistance. There is no need to trace out the analogies between mechanical friction and electrical resistance, because, to make the analogy complete, one has to assume that the force of friction is proportional to the velocity of the moving body.

Figure 106 shows a mechanism which is exactly analogous to the coupled (transformer coupled) circuits in Fig. 105. The lower car *P* together with its controlling springs *CC* corresponds to the primary circuit $C_1 f L_1 g$ in Fig. 105, the upper car *S* together with its controlling springs $C_1 C_1$ corresponds to the secondary circuit $L_2 C_2$ in Fig. 105, and the lever *ll'* with its attached weight *W* corresponds to the transformer coupling in Fig. 105. If the mass *W* is large as compared with the mass of either car we have *close coupling*, if the mass

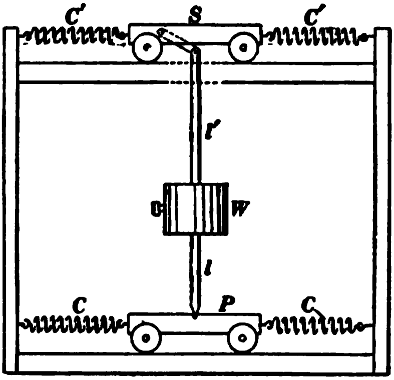

Fig. 106.

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* Only a fraction of the flux produced by a current in the primary (or secondary) coil links with the secondary (or primary) coil.

W is small as compared to the mass of either car we have *loose coupling.*

Constrained motion of Fig. 106.—There are two distinct frequencies of oscillation of the system in Fig. 106 when one car or the other is held fast (equivalent to having one circuit or the other in Fig. 105 open so that no current can flow in it), namely, (*a*) The oscillation of car P at frequency a when car S is held fast. (*b*) The oscillation of car S at frequency b when car P is held fast.

Free motion of Fig. 106.—There are two simple modes of oscillation of the system in Fig. 106 when the cars are free (and, of course, two simple modes of oscillation of the system in Fig. 105 when both circuits are closed so that current can flow), *and the system usually performs both of these simple modes of oscillation simultaneously.* These two simple modes of oscillation are:

(*c*) The perfectly steady oscillation of the entire system at a higher frequency c, with velocities of cars opposite at each instant (primary and secondary currents opposite at each instant in coils L_1 and L_2 in Fig. 105); and

(*d*) The perfectly steady oscillation of the entire system at a lower frequency d, with velocities of cars in the same direction at each instant (primary and secondary currents in the same direction at each instant in coils L_1 and L_2 in Fig. 105).

The simultaneous performance by each car of two steady simple oscillations of slightly different frequencies means that the resultant motion of each car would be as represented in Fig. 107. At the instant A car P has a maximum amplitude of motion and car S has a minimum amplitude of motion, and at instant B car P has a minimum amplitude of motion and car S has a maximum amplitude of motion. The energy of the system is evidently handed from P to S and back again repeatedly.

The mathematical theory of coupled circuits (or the mathematical theory of the mechanism of Fig. 106) shows that this back and forth transfer of energy is complete when the two

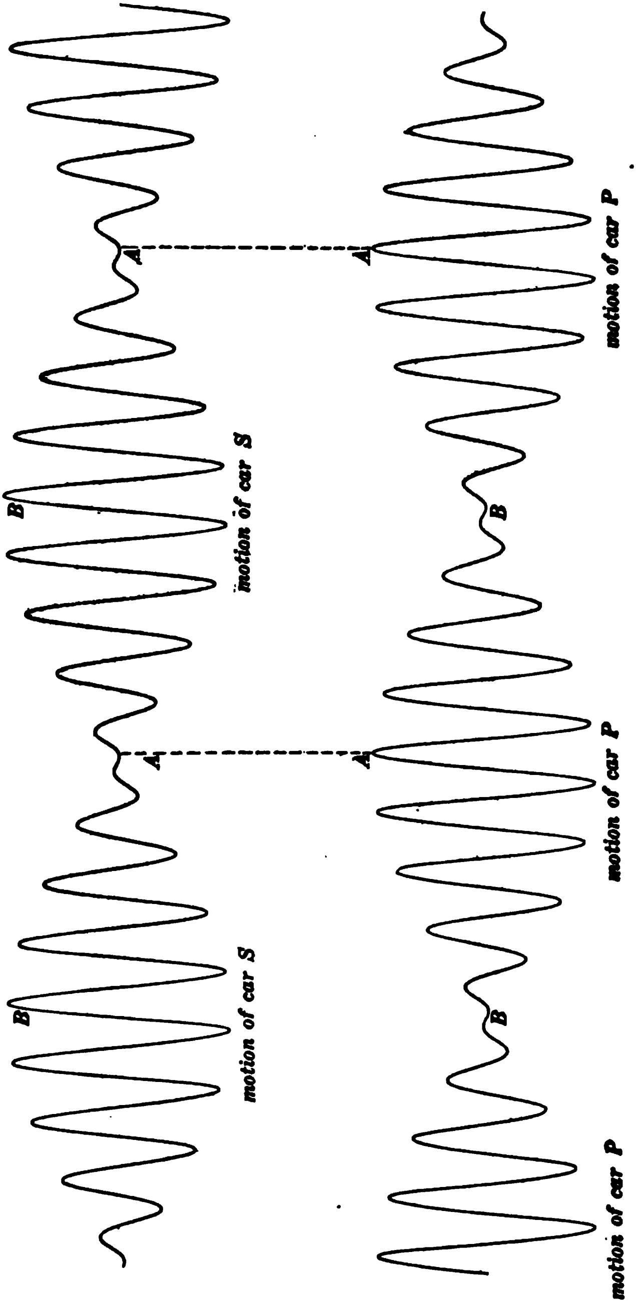
A
A
B
B
A
A
B
B
motion of car S
motion of car S
motion of car P
motion of car P
motion of car P

Fig. 107.

constrained frequencies a and b are equal; now, it is desirable, in Fig. 104, that all of the energy which resides in the oscillating circuit $CfLg$ immediately after the formation of a spark in the gap g *be handed over to the antenna circuit.* Therefore the system should be adjusted by changing the value of L or C to bring the two constrained frequencies a and b to equality. But when the energy has been all handed over to the secondary circuit, it is handed back again to the primary if the primary circuit is closed; therefore it is important to provide for the opening of the primary circuit by what is called the *quenched spark* the moment the primary circuit loses all its energy.

The sending antenna in Fig. 104 sends out three distinct wave-lengths of electric waves when a quenched spark is used, namely, wave-lengths corresponding to the component oscillations c and d while the primary circuit is closed, and wave-length corresponding to frequency b after the primary circuit is opened by the quenched spark.

68. Expression for total kinetic energy of the mechanism of Fig. 106. Definition of mutual inductance.—Let i_1 be the velocity of car P, and P its mass; then $\frac{1}{2}Pi_1^2$ is its kinetic energy. Let i_2 be the velocity of car S, and S its mass; then $\frac{1}{2}Si_2^2$ is its kinetic energy. Let us consider the simplest case in which the weight W is at the middle of the lever; then the velocity of W is $\frac{1}{2}(i_1 + i_2)$; let W be the mass of the weight W, then its kinetic energy is $\frac{1}{2}W\left(\frac{i_1 + i_2}{2}\right)^2$ or $\frac{1}{8}Wi_1^2 + \frac{1}{4}Wi_1i_{11} + \frac{1}{8}Wi_2^2$. Therefore the total kinetic energy of the system is

$$T = \frac{1}{2}(P + \frac{1}{4}W)i_1^2 + \frac{1}{4}Wi_1i_2 + \frac{1}{2}(S + \frac{1}{4}W)i_2^2$$

or, let us write

$$L_1 = P + \frac{1}{4}W \qquad \text{(i)}$$

$$L_2 = S + \frac{1}{4}W \qquad \text{(ii)}$$

$$M = \frac{1}{4}W \qquad \text{(iii)}$$

then

$$T = \tfrac{1}{2}L_1i_1^2 + Mi_1i_2 + \tfrac{1}{2}L_2i_2^2 \qquad (30)$$

this equation also expresses the total kinetic energy of two coupled circuits; L_1 is the ***inductance*** or ***self-inductance*** of one circuit, the primary circuit let us say, L_2 is the ***self-inductance*** of the secondary circuit, and M is called the ***mutual inductance*** of the two circuits (see Art. 48).

Suppose both cars in Fig. 106 are moving towards the right, car P at velocity i_1 and car S at velocity i_2, then if one increases i_1 by pushing to the right on car P the lever will exert a force on car S towards the left, a negative force; and therefore if one should wish to keep the velocity i_2 of car S from decreasing while the velocity i_1 of car P is being increased an outside positive force (to the right) would have to be exerted on car S to balance the backward kick of the lever.

Exactly the same conditions exist in Fig. 105 or in Fig. 108 if the secondary current i_2 is to be kept constant while the primary current i_1 is increased; and the expression for the "backward kick" in the secondary circuit may be derived as follows. We are here concerned only with transformer action and therefore we may consider two coupled circuits without condensers as indicated in Fig. 108. The total energy (kinetic) of these two circuits is given by equation (30). Let us assume* that both circuits have zero resistance, and let us determine precisely how work must be done to keep the current i_2 constant and make current i_1 increase at a specified rate $\frac{di_1}{dt}$.

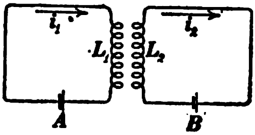

Fig. 108.

Differentiating equation (30) we get

$$\frac{dT}{dt} = Li_1\frac{di_1}{dt} + Mi_2\frac{di_1}{dt}$$

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* This assumption is not necessary but it avoids tedious qualifications in the following argument.

or, rearranging terms, we get

$$\frac{dT}{dt} = \left(L_1 \frac{di_1}{dt}\right) i_1 + \left(M \frac{di_1}{dt}\right) i_2$$

but the rate of increase of the energy T of the system is necessarily the rate of doing work on the system, and rate of doing work on an electrical circuit is always equal to ei where e is an electromotive force acting on the circuit and i is the current flowing in that circuit. Therefore

(1) The term $\left(L_1 \frac{di_1}{dt}\right) i_1$ is power expended on the primary circuit, and $L_1 \frac{di_1}{dt}$ is the outside electromotive force, due to battery A for example, which must act on the primary circuit to make the primary current increase [see equation (18)].

(2) The term $\left(M \frac{di_1}{dt}\right) i_2$ is power expended on the secondary circuit, and $M \frac{di_1}{dt}$ is the outside electromotive force, due to battery B for example, required to overcome the backward kick in the secondary circuit due to the increasing primary current.

Therefore the electromotive force e_2 (the backward kick) induced in the secondary circuit due to the increasing primary current is

$$e_2 = - M \frac{di_1}{dt} \tag{31}$$

This backward kick in the secondary is induced therein by the changing value of the flux Φ_2 through the secondary coil due to the primary current. Therefore, according to equation (17), we have $e_2 = - N_2 \frac{d\Phi_2}{dt}$, so that

$$M \frac{di_1}{dt} = N_2 \frac{d\Phi_2}{dt}$$

where N_2 is the number of turns of wire in the secondary coil, whence, integrating* from $\Phi_2 = 0$ and $i_1 = 0$ we get

$$N_2\Phi_2 = M i_1 \tag{i}$$

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* The equation $N_2 \frac{d\Phi_2}{dt} = M \frac{di_1}{dt}$ means that Φ_2 increases M/N_2 times as

A similar equation, namely,

$$N_1\Phi_1 = Mi_2 \tag{ii}$$

expresses the value of $N_1\Phi_1$ where Φ_1 is the flux through the primary coil due to a secondary current i_2 and N_1 is the number of turns of wire in the primary coil.

Relation between L_1, L_2 and M when there is no magnetic leakage. From equation (iii) of Art. 48 we have $\Phi N_1 = L_1i_1$ where Φ is the magnetic flux through the primary coil due to the primary current i_1. But if there is no magnetic leakage $\Phi = \Phi_2$ where Φ_2 is the magnetic flux through the secondary coil due to the primary current i_1. Therefore

$$L_1i_1 = N_1\Phi_2 \tag{iii}$$

and, similarly, we have

$$L_2i_2 = N_2\Phi_1 \tag{iv}$$

Multiplying equations (i) and (ii) member by member, we get

$$N_1N_2\Phi_1\Phi_2 = M^2i_1i_2 \tag{v}$$

and multiplying equations (iii) and (iv) member by member, we get

$$N_1N_2\Phi_1\Phi_2 = L_1L_2i_1i_2 \tag{vi}$$

whence, from equations (v) and (vi), we get

$$M = \sqrt{L_1L_2}$$

PROBLEMS.

72. During 0.03 second a charge of 15 coulombs passes through a circuit. What is the average value of the current during this time?

73. Suppose the strength of a current in a circuit to increase at a uniform rate from zero to 50 amperes in 3 seconds. Find the number of coulombs of charge carried through the circuit by the current during the 3 seconds.

Note. The average value of the current during the 3 seconds is half the sum of the initial and final values, because the current changes at a constant rate.

74. A condenser consists of two square flat metal plates each 2 meters by 2 meters, and the plates are 1 centimeter apart. What is the capacity of the condenser in farads? Ans. 0.003536 microfarad.

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fast as i_1 so that Φ_2 must always be M/N_2 times as large as i_1 if Φ_2 and i_2 start from zero together.

75. What would be the capacity of the condenser specified in the previous problem if the whole were submerged in light petroleum (kerosene)? Ans. 0.0072 microfarad.

76. A condenser is to be built up of sheets of tin foil 12 centimeters by 15 centimeters. The overlapping portions of the sheets are to be 12 centimeters by 12 centimeters. The sheets are to be separated by leaves of mica 0.05 centimeter thick. How many mica leaves and how many tin foil sheets are required for a one-microfarad condenser? Assume the inductivity of the mica to be equal to 6. Ans. Mica leaves, 655; tin foil sheets, 656.

77. The two metal plates arranged as in problem 74 are charged by connecting them to 110-volt direct-current supply mains. Find the amount of charge on each plate, and find the energy of the charged condenser. Ans. 0.389 microcoulomb; 21.4 microjoules or 214 ergs.

78. The charged plates which are specified in the previous problem are insulated so that the charge on each plate must remain constant, and one of the plates is then moved until the plates are 2 centimeters apart. What is the voltage between the plates after the movement? What is the energy of the charged condenser after the movement? Ans. 220 volts; 428 ergs.

79. The increase of energy from 214 ergs to 428 ergs due to the movement of the plate as specified in problem 78, is equal to the work done in pulling the plates apart against their mutual force of attraction F. Find the value of F. Ans. 214 dynes.

Note. Consider that the work done against the force of F dynes is equal to F multiplied by the movement in centimeters.

The force of attraction F is independent of the distance apart of the plates *for the given amount of charge* q, provided the distance between the plates is small in comparison with the size of the plates.

The following problems can be solved by the method illustrated in this problem and the preceding one.

80. Find the force of attraction of the plates referred to in problem 79 when the charge on each plate is such as would be produced by connecting the plates to 110-volt supply mains when the plates are 2 centimeters apart. Ans. 53.5 dynes.

81. Find the force of attraction of the plates referred to in problem 79 when the plates are connected to 1,100-volt supply mains when the distance between the plates is 1 centimeter. Ans. 21,400 dynes.

82. The two plates referred to in problem 79 are submerged in kerosene (inductivity 2.04). Find the force with which they attract each other when they are at a distance of 1 centimeter apart and connected to 1,100-volt supply mains. Ans. 43,660 dynes.

83. Find the frequency of oscillation of the circuit *CfLg* in Fig. 99 when the capacity of the condenser is 0.003536 microfarad (see problem 74), and the inductance *L* is 0.00016 henry (the inductance of a coil of 50 turns of wire wound in one layer on a wooden cylinder 20 centimeters in diameter and 50 centimeters long). Ans. 21,100 cycles per second.

CHAPTER V.

THE ELECTRIC FIELD.

69. The electric field.—When a momentary electric current flows through an open circuit, certain important effects are produced in the gap which breaks the circuit. In order that these effects may be easily observed a very high voltage must be used. The most convenient device for generating a high voltage is the familiar Toepler-Holtz influence electric machine. Figure 109 shows two brass balls which have been charged by a momentary electric current drawn out of one ball and pushed into the other by an influence machine. When an ordinary wooden tooth-pick suspended by a fine thread is placed in the region between the balls, the tooth-pick points in a definite direction at each point very much as a magnetic needle points in a definite direction at each point when it is placed between magnet poles. The short black lines in Figs. 109 and 110 represent the various positions of the tooth-pick.

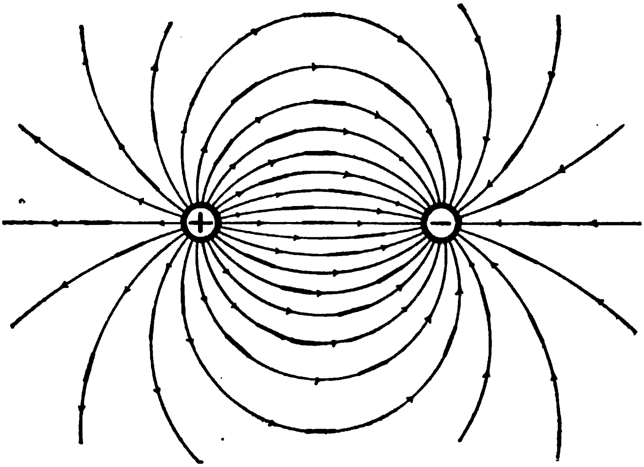

Fig. 109.

The behavior of the tooth-pick shows that the whole region surrounding the charged metal balls in Fig. 109 is in a peculiar condition, and this region is called an *electric field*. The direction of the electric field at each point is indicated by the direction of the tooth-pick when it is placed at that point, and lines drawn through the electric field so as to be, at each point, in the direction of the field at that point are called the *lines of force* of the electric field. Figure 110 shows the lines of force of the electric field between two charged flat metal plates. The lines of force in the region between the plates are straight lines, and the electric field is said to be *uniform*.

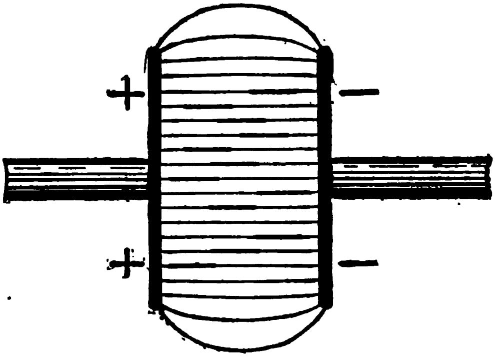

Fig. 110.

70. Intensity of electric field.—It would be permissible to adopt arbitrarily the ratio E/x as a measure of the intensity of the uniform electric field between the flat metal plates in Fig. 110, E being the electromotive force between the plates and x being the distance between the plates. Thus the intensity of the electric field would be expressed in *volts per centimeter* or *volts per inch*. It is desirable, however, to base the definition of electric field intensity upon some observable effect as in the following discussion.

Two metal plates A and B, Fig. 111, are connected to an electric machine giving a high electromotive force E. The electric machine is represented in Fig. 111, as a battery for the sake of clearness. A small metal ball b is suspended between A and B by a silk thread. If this ball is started it continues to vibrate back and forth from plate to plate.

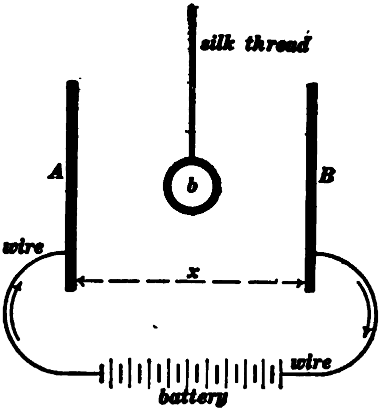

Fig. 111.

The ball b oscillates to and fro.

Regarding the behavior of the vibrating ball the following statements may be made:

(*a*) Work evidently is done to keep the ball b oscillating back and forth, and this work is evidently done by the battery.

(*b*) The only way the battery can do work is by continuing to draw charge out of one plate and push it into the other plate. It is evident therefore that the ball carries charge back and forth between the plates.

(*c*) The successive movements of the ball are similar, and therefore if the ball carries charge at all it must carry a definite amount each time it moves across. Let this definite amount of charge be represented by q; this charge is positive when the ball moves from A to B, and negative when it moves from B to A. At each movement of the ball the battery supplies the amount of charge q, drawing it out of plate B and pushing it into plate A. Therefore at each movement of the ball the battery does an amount of work Eq according to equation (24) of Art. 58.

(*d*) Let F be the average mechanical force acting on the ball b while it is being pulled across from plate to plate. Assuming the ball b to be very small in diameter, it moves the distance x in traveling from plate to plate. Then Fx is the amount of work done on the ball while it moves from plate to plate.

(*e*) The work Eq done by the battery during one movement of the ball is equal to the mechanical work Fx done on the ball, therefore we have $Fx = Eq$, or

$$F = \frac{E}{x} \cdot q \qquad \text{(i)}$$

Any region in which a charged body is acted upon by a force* is called an electric field. Thus the region between A and B in Fig. 111 is an electric field because the charged ball b is acted upon by the force F.

The force F with which an electric field pulls on a charged body placed at a given point in the field is proportional to the charge q on the body so that we may write:

$$F = fq \qquad (32)$$

in which f is the proportionality factor, and it is called the *intensity of the electric field at the point.*

From equations (i) and (32) it is evident that the intensity of the electric field between the plates A and B in Fig. 111 is:

$$f = \frac{E}{x} \qquad (33)$$

that is, the intensity of the electric field between the plates is equal to the electromotive force between the plates divided by the distance between the plates. In the above discussion F is spoken of as the average force acting on the ball b in Fig. 111 while the ball is moving from plate A to plate B. This force is constant if the ball is small and if the plates are large.

Direction of electric field at a point.—The direction of an electric field at a point is the direction in which the field would pull on a *positively charged body* placed at that point.

Tension of the lines of force in an electric field.—Two oppositely charged metal plates attract each other as stated in Art. 53. Thus the oppositely charged plates in Fig. 110 attract each other. This attraction may be thought of as due to a

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* A force which depends upon the charge on the body and which does not exist when the body is not charged.

tension of the lines of force; that is, the lines of force may be thought of as if they were rubber-like filaments stretching from plate to plate and pulling the plates towards each other.

If the lines of force in an electric field are like stretched filaments of rubber one would expect the lines of force to pull outwards on every part of the surface of a charged body. In fact each part of the surface of a charged body is pulled outwards by the surrounding electric field. This outward pull may be beautifully shown by pouring melted rosin in a thin stream from a metal ladle which is supported by an insulated handle and connected to one terminal of an electric machine. The lines of force which emanate from the lip of the metal ladle pull the melted rosin into extremely fine jets which shoot straight outwards from the lip. These jets congeal in the form of excessively fine fibers which float about in the air.

71. The idea of electric charge as the ending of electric lines of force.—Figure 112 represents two metal bodies A and B to which a battery is connected as shown. The battery draws a certain amount of charge out of one body B and forces it into the other body A, and the entire surrounding region becomes an electric field, the lines of force of which are shown in the figure. *The positive charge on body* A *may be thought of as the beginning of the lines of force, and the negative charge on body* B *may be thought of as the ending of the lines of force*, the directions of the lines of force being indicated by the arrow heads in the figure.

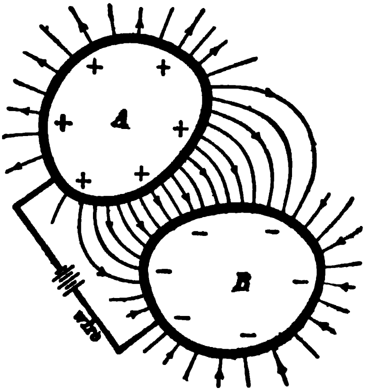

Fig. 112.

72. Electric charge resides wholly on the surface of a metal body.—Experiment shows that to whatever degree a hollow metal shell may be charged, no effect of the charge can be observed inside of the shell, however thin the shell may be; that is to say, the lines of force of the outside electric field do not penetrate into the metal but terminate at its surface. Therefore, the electric charge on a metal body may be thought of as residing on the surface of the body. Figure 113 shows a hollow metal ball *C* placed between two charged bodies *A* and *B*. The presence of the ball *C* modifies the trend of the lines of force as may be seen by comparing Fig. 113 with Fig. 112, but the lines of force do not penetrate to the interior of the ball *C*. *The interior of a metal shell is entirely screened from outside electric field.** This is an experimental fact. An electric field may be detected by its action upon a very light body like a suspended tooth-pick or a suspended pith ball. No evidence of electrical field can be detected inside of the ball *C* by such a device.

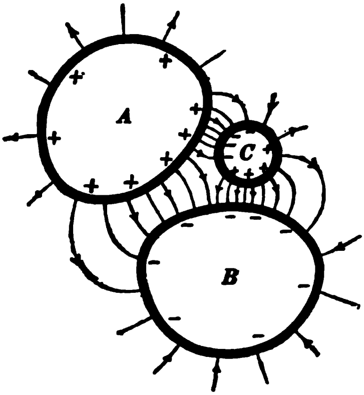

Fig. 113.

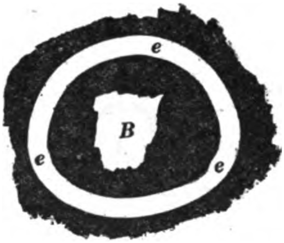

Fig. 114.

Mechanical analogue of electrical screening.—The narrow space *eee* in Fig. 114 represents an empty space between two masses of steel (steel represented by black patches). Stress and strain in the surrounding steel cannot be transmitted across the empty space *eee*, and therefore the empty space *eee* protects or screens the inner mass of steel. This empty space, in its behavior towards mechanical stress is analogous to a metal shell in its behavior towards electrical stress (electric field).

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* The screening is not complete while an electric field is changing rapidly.

73. **A charged conductor shares its charge with another conductor with which it is brought into contact.**—A brass ball with a glass handle may be charged by touching it to one terminal of an influence machine, and if the brass ball is brought near to a suspended charged pith ball, the charge on the brass ball will be indicated by the behavior of the pith ball.

A brass ball A is charged by touching it to a terminal of an influence machine, the ball A is then touched to another brass ball B (*A* and *B* both have glass handles), *then both A and B are found to be charged.* The charged ball *A* has shared its charge with ball *B*. The original charge on ball *A* is represented in Fig. 115, which shows the lines of force emanating from ball *A*; and the lines of force in Fig. 116 show how the charge originally on *A* has spread over *A* and *B*. See Art. 71.

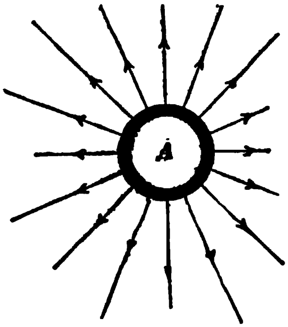

Fig. 115.

Charge on single ball.

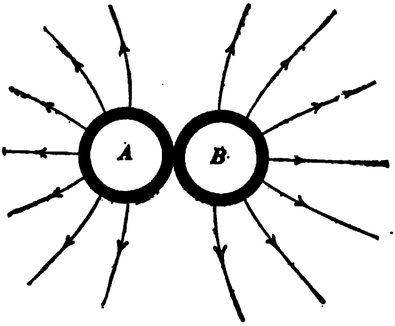

Fig. 116.

Charge shared by two balls.

74. **Giving up of entire charge by one body to another.**—Figure 117 shows a charged ball and an insulated metal can. If the charged ball is placed inside of the can, brought into contact with the inner wall of the can, and then removed, it will give up its entire charge to the can. *This is true whatever charge the can may have to begin with.* This giving up of the entire charge

on one body to another, as described, is an essential feature in the action of the electric doubler as described later.

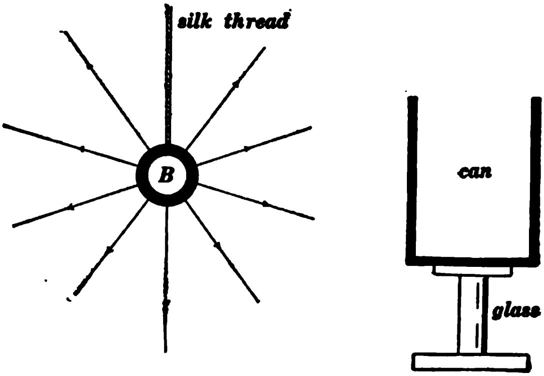

Fig. 117.

The charged ball *B* is lowered into the can, and the can is closed by the metal lid as indicated in Figs. 118 and 119. As the charged ball is lowered into the can, each line of force that emanates from the ball is cut in two, as it were, by the wall of the can; so that when the ball is entirely enclosed by the can as many

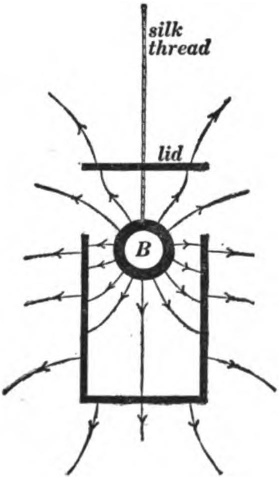

Fig. 118.

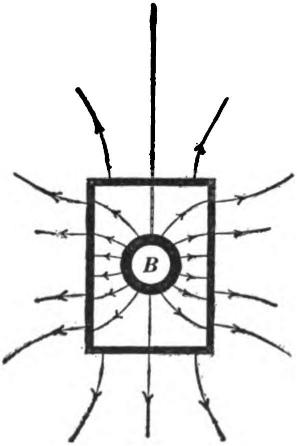

Fig. 119.

lines of force emanate from the external surface of the can as from the ball *B*, as shown in Fig. 119.

After the ball *B* has been completely inclosed by the metal can, the distribution of the electric field outside of the can is entirely independent of what takes place inside of the can, because the walls of the can act as a complete screen as explained in Art. 72.

If the ball *B* is then brought into contact with the inner wall of the can, as shown in Figs. 120 and 121, the lines of force which emanate from the ball disappear, as shown in Fig. 121, and all of the charge originally on *B* is left on the outside surface of the can. The ball may then be removed from the can and it will be found to be without charge, all of its original charge will have been given to the can. This giving up of the entire charge by the ball takes place however great the initial charge on the can may be.

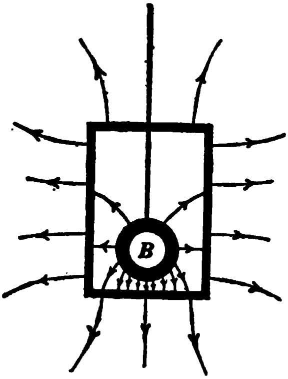

Fig. 120.

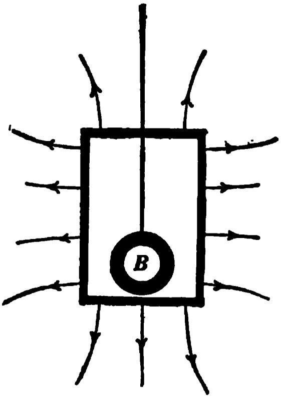

Fig. 121.

75. Charging by influence.—Charging by influence is essentially the cutting of electric lines of force in two by a sheet of metal so that one face of the metal sheet is negatively charged where the lines of force come in upon it, and the other face of the metal sheet is positively charged where the lines of force emanate from it. Thus Fig. 122 shows two metal balls *A* and *B* which have been brought in between two oppositely charged bodies *C* and *D*. The lines of force from *C* converge upon *A*, and spread out from *B* as shown. If *A* and *B* are now separated from each other and withdrawn from the region between *C* and *D*, then *B* will be left positively charged (lines of force emanating from it), and *A* will be negatively charged (lines of force coming in upon it); the charge on *C* and *D* being the same in amount as in the beginning.

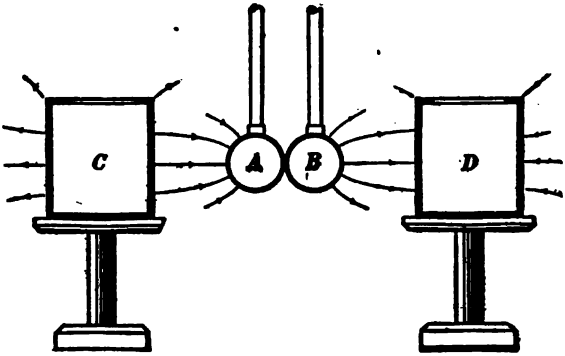

Fig. 122.

76. The electric doubler.—The charged bodies *C* and *D*, Fig. 122, are metal cans supported on insulating stands,* and

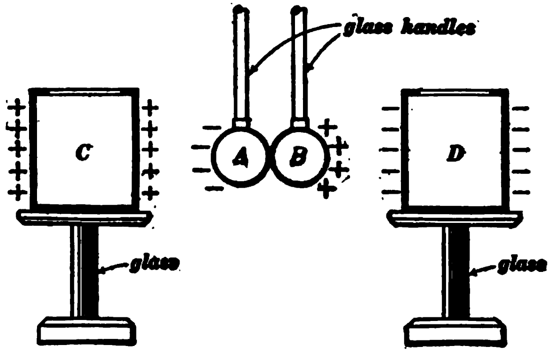

Fig. 123.

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* The cans should stand on freshly scraped blocks of cast sulphur or paraffine, and the handles should be made of quartz tube closed at one end.

Fig. 123 is a duplicate of Fig. 122 but showing the charges on C, D, A and B by positive and negative signs. *The ball A may be made to give up its entire charge to D by being placed inside of D and brought into contact with D; and the ball B may be made to give up its entire charge to C in the same way.* The two balls A and B may then be charged anew by being brought into the positions shown in Figs. 122 and 123; these new charges may then be delivered to the cans C and D as before; and so on.

As the charges on the balls A and B, Figs. 122 and 123, are repeatedly delivered to the respective cans as stated, the amount of charge on each can increases step by step. But at each step the charge on each ball is a certain fraction of the existing charges on C and D. Therefore the increasing series of values of charge on each can is a geometrical progression if the insulation is perfect. Suppose, for example, that the charges on the balls are one quarter of the charges on the cans. Then each repetition of the above described "doubling operation" will multiply the charges on C and D by 1.25, and 100 repetitions would multiply the initial charges on C and D by the hundredth power of 1.25 or approximately 5 thousand million times.

The familiar *Toepler-Holtz influence machine* is essentially an electric doubler.

CHAPTER VI.

THE ELECTRON THEORY.*

77. Electrons and ions.—When a charged body is left standing it soon loses its charge however carefully insulated it may be. This loss of charge has long been known to be due in part to a leakage of the electricity through the surrounding air, although usually it is due mostly to a leakage of electricity along the insulating supports. That is to say, air conducts electricity to some extent, and there are a number of influences which cause the air (or any gas) to become a fairly good electrical conductor. Thus gas which is drawn from the neighborhood of a flame or from the neighborhood of glowing metal or carbon is a fairly good conductor; gas which has been drawn from a region through which an electric discharge has recently passed is a fairly good conductor; a gas becomes a fairly good conductor under the action of Roentgen rays or under the action of the radiation from radioactive substances.

The conductivity which is imparted to a gas by these various agencies may be destroyed by filtering the gas through glass wool or by placing the gas for a few moments between electrically charged metal plates. This effect of filtration seems to show that the conductivity of the gas is due to something which is mixed with the gas, and this something seems to be electrically charged because it is dragged out of the gas by the electric field, just as the charged ball in Fig. 111 is pulled towards plate *A* or plate *B*. From some such considerations as these came the hypothesis that the electrical conductivity of a gas is due to

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* This chapter on the electron theory is necessarily very brief. This subject is developed in close touch with experiment in Sir J. J. Thomson's *Conduction of Electricity Through Gases* (1906) and *Rays of Positive Electricity* (1913), in Sir Ernest Rutherford's *Radioactive Substances and Their Emanations* (1913), and in Professor O. W. Richardson's *Emission of Electricity from Hot Bodies* (1915).

electrically charged particles floating around in the gas. These particles are called *ions*, and the process by which a gas is made into a conductor is called *ionization*. This hypothesis has been used extensively and with remarkable success in the study of electrical discharge through gases and in the study of radioactivity.

The *electron* is a negatively charged particle of which the mass is about 1/800 of the mass of a hydrogen atom.

An *atom* is supposed to be a kind of central nucleus with a number of electrons revolving round it, very much like the solar system which consists of the sun with the planets revolving around it.

A *molecule* is a group of atoms clinging together.

An ordinary atom or molecule is electrically neutral, that is, it has no apparent electrical charge, the negative charges of the revolving electrons being offset by a positive charge in the nucleus. But when the molecules of a gas are broken up by chemical action, by the effect of high temperature or by Roentgen rays, the *pieces* are electrically charged, and these pieces may be *electrons* or *simple ions*.

A *simple ion* is an atom from which a negatively charged electron has been detached thus leaving the remainder of the atom (the simple ion) positively charged; or a simple ion may be a neutral atom to which an electron has become attached thus giving the atom (the simple ion) a negative charge.

The *compound ion* is an aggregate of two or more atoms or molecules with an extra electron attached to it thus giving it a negative charge or from which a negatively charged electron has been detached thus leaving the aggregate positively charged. In highly rarefied gases electrons and simple ions, only, exist; whereas compound ions may exist in gases at moderate or high pressure.

The term *ion* is often used in a general sense to include electrons as well as ions as here defined.

78. Ionization of a gas by collision.—According to the kinetic theory,* a molecule of a gas travels on the average a certain distance between successive collisions with neighboring molecules. This distance is called *the mean free path of a molecule.* The mean free path of an electron is 5 or 6 times as great as the mean free path of a molecule, and the mean free path of an ion is only slightly greater than the mean free path of a molecule.

Consider a body of gas between oppositely charged metal plates as in Fig. 110 or Fig. 111. The electric field between the plates exerts a pull on any electron (or ion) which may be floating around in the gas, and a certain amount of velocity is imparted to the electron (or ion) between successive collisions with the gas molecules. If this velocity exceeds a certain value, the electron (or ion) breaks the molecules of the gas to pieces as it collides with them, and when a molecule of the gas is thus broken to pieces a new electron and a new ion are produced.† This process is called *ionization by collision.*

Experiment shows that in air at normal atmospheric pressure an electric field of about 13,000 volts per centimeter gives to an electron sufficient velocity between collisions to enable the electron to ionize the air molecules with which it collides, and that an electric field of about 32,000 volts per centimeter gives to a simple ion sufficient velocity between collisions to enable the ion to ionize the air molecules with which it collides.

79. The electric spark in a gas.—Let us consider air at normal atmospheric pressure for the sake of definiteness. Figure 124 shows two oppositely charged metal plates AA and BB, and the electric field between them is approximately uniform.

Suppose that the electric field has been increased to about 13,000 volts per centimeter. Then a stray electron e, as it is drawn towards plate AA, will ionize the air along its path p, and each newly formed electron as it travels along with e will ionize the air; *but all of the simple ions thus formed will travel towards* BB *without producing any ionization.* In this case, therefore, ionization takes place only along the path p, and all ionizing action stops when e and the newly formed electrons reach plate AA.

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* The standard general treatise on this subject is O. E. Meyer's *Kinetic Theory of Gases* (translated from the German by R. E. Baynes), Longmans, Green & Co., London, 1899; *Vorlesungen über Gas Theorie*, Ludwig Boltzman, Leipzig, 1896, is a very important work.

† We are not here concerned with the fact that a single collision may produce more than one new electron and more than one new ion.

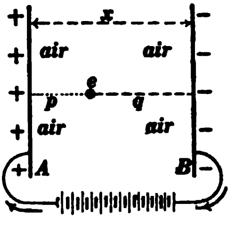

Fig. 124.

Suppose that the electric field has been increased to about 32,000 volts per centimeter. Then any stray electron e, as it is drawn towards plate AA, will ionize the air along its path, and, of course, the newly formed electrons will do likewise. In this case, however, the newly formed simple ions will produce new electrons and simple ions as they travel towards plate BB. It is evident, therefore, that the ionizing action does not come to an end, but increases indefinitely all along the path pq, and the result is the production of a spark along pq. The beginning of the intense ionization along the path pq is here attributed to the stray electron e, but the action may of course be started by a stray simple ion because the field intensity is great enough to enable a simple ion to produce new ions and electrons by collision.

According to the kinetic theory of gases, the mean free paths of the electrons and simple ions are doubled when the pressure (or density) of the gas is halved, and with doubled mean free paths sufficient velocity to ionize by collision is produced by an electric field one half as intense. Therefore the dielectric strength of air should be proportional to its pressure (temperature being constant). This, in fact, is the case. Thus the dielectric strength of air at normal atmospheric pressure is about 32,000 volts per centimeter, at a pressure of 10 atmospheres it is about 320,000 volts per centimeter, and at a pressure of 0.1 atmosphere it is about 3,200 volts per centimeter. The dielectric strength of air reaches a minimum, however, at a pressure of about 2 millimeters of mercury, and increases when the pressure falls below this value.

The idea of specific dielectric strength is based on the assumption that the electromotive force required to produce a discharge is proportional to the length of the spark, so that the quotient, volts divided by spark length, may be a constant for a given material. This is approximately true in gases under moderate or high pressure,* but when the pressure is very low *a greater electromotive force is required to strike across a short gap than is required to strike across a long gap.* This curious behavior of gas at low pressure is illustrated by an experiment due to Hittorf. Two electrodes were sealed into the walls of two glass bulbs and the tips of the electrodes were one millimeter apart, as shown in Fig. 125. The two bulbs were connected together by a spiral tube 375 centimeters long, and, when the pressure of the gas in the bulbs was reduced to a very low value, the discharge took place through the long tube and not across the one millimeter gap space between the points of the electrodes.†

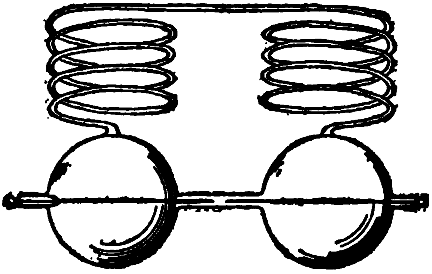

Fig. 125.

80. The corona discharge.—The curved lines in Fig. 109 show, approximately, the mode of distribution of the electric field between two parallel wires; the plane of the paper is at right angles to the wires, and the small circles represent the wires in section. The electric field is much more intense near the wires where the lines of force are crowded together than it is at greater distances from the wires where the lines of force spread more widely apart. If the wires are very fine the electric field may be very intense in a very small region around each wire.

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* And, of course, between flat electrodes. See Art. 60.

† This behavior of a gas at low pressure is fully explained by the atomic theory. See J. J. Thomson's *Conduction of Electricity through Gases*, pages 430–527.

Imagine the electromotive force between a given pair of wires to be increased until the field intensity near the wires is about 13,000 volts per centimeter. Then a stray electron starts an ionizing action which quickly comes to an end either when the original electron and the newly formed electrons reach the surface of the wire (the positively charged wire) or when the original electron and the newly formed electrons pass out from the neighborhood of the negatively charged wire into a region where the field intensity is decidedly less than 13,000 volts per centimeter.

Imagine the electromotive force between the wires to be increased until the field intensity near each wire is about 32,000 volts per centimeter. Then a stray electron or a stray simple ion near either wire will start an ionizing action which *continues* indefinitely. But this ionizing action does not *increase* indefinitely as in the formation of a spark as described in connection with Fig. 124. The reason for this limitation is as follows: Let us consider the corona around the positively charged wire, for example. In the region *rr* near the wire the electric field is sufficiently intense to enable electrons and ions both to produce new electrons and ions by collision. The electrons (negatively charged) travel to the surface of the wire where they are absorbed, but the positively charged ions, having to travel far through the surrounding region *RR*, accumulate in this region in large numbers as an extensive cloud of positive charge surrounding the region *rr* on all sides as indicated by the dots in Fig. 126. This accumulation of a cloud of positive charge (positive ions), as shown by the dots in Fig. 126, reduces the intensity of the electric field near the wire because the lines of force do not all come out from the positively charged wire but

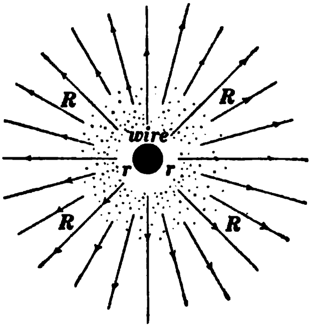

Fig. 126.

many of these lines originate in the cloud of positive charge. Excessive production of ions in the region *rr* in Fig. 126 would thus reduce the field intensity in this region so that no more ions would be formed and therefore excessive ionization in the region *rr* cannot take place.

There is a sharply defined value* of the voltage between parallel wires for which the electric field intensity near the wires is sufficiently intense to maintain steady ionization (by electron collisions and ion collisions). The corona appears when the voltage between the wires reaches or slightly exceeds this value, and for greater values of voltage the corona spreads out farther and farther from the wires.

81. The Cottrell process for the precipitation of dust and smoke particles.—A fine wire is stretched along the axis of a metal tube; a steady (*not* alternating) voltage high enough to produce corona discharge around the fine wire is connected between wire and tube; and the result is that the particles of smoke or dust are deposited on the walls of the tube and to a very slight extent on the wire.

The most satisfactory arrangement for demonstrating the Cottrell process is shown in Fig. 127. A fine wire *AA*, supported by two insulating glass posts, is stretched through a large glass tube, and a piece of strap iron *BB* is laid along the

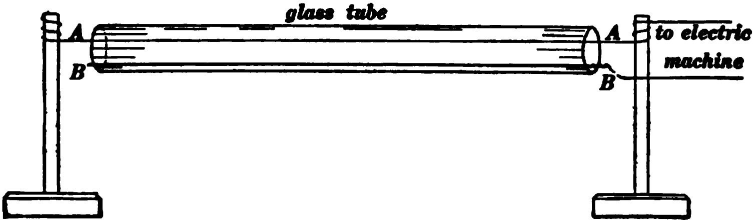

Fig. 127.

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* Depending on size of wires and their distance apart. The mathematical theory of the distribution of electric field between parallel wires is developed in Chapter VIII of Franklin and MacNutt's *Advanced Electricity and Magnetism*, The Macmillan Co., New York, 1915.

bottom of the tube as shown in the figure. The fine wire and iron strap are connected to the terminals of a Toepler-Holtz influence machine, and the smoke to be cleared is blown in at one end of the tube. The effect is most strikingly shown by short-circuiting the influence machine by means of a small metal rod, filling the tube with smoke, and then suddenly removing the short circuit.

The Cottrell separator works almost equally well whether the wire be made positive or negative. If the wire is positive, great numbers of simple ions with their positive charges travel out from the corona, these ions attach themselves to the particles of dust or smoke, these particles thus become positively charged, and the electric field between wire and tube drags the positively charged particles to the negatively charged wall of the tube. If the wire is negative, great numbers of electrons with their negative charges travel out from the corona and attach themselves to the smoke or dust particles, and the particles thus negatively charged are attracted by and travel to the positively charged wall of the tube.

82. The ozonizer.—The ozonizer is a device for converting oxygen into ozone. The essential features of the device are shown in Fig. 128. Two metal plates *A* and *B*, with a glass plate between, are connected to the terminals, *ab*, of the high-voltage coil of a step-up transformer, a blast of air is blown through between the plates, and a portion of the oxygen of the air is converted into ozone.

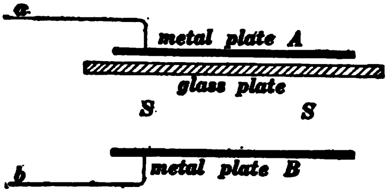

Fig. 128.

Essential features of ozonizer (*a* and *b* connected to high voltage supply of alternating current).

The high electromotive force (alternating, of course) between the metal plates would produce a spark discharge from metal plate to metal plate in the absence of the glass plate, but the glass plate, if it is not punctured, prevents the formation of one single path of indefinitely intense ionizing action (like the path *pq* in Fig. 124 as explained in Art. 79), so that moderately intense ionizing action takes place throughout the entire region *SS*.

The conversion of oxygen into ozone takes place as follows: Ordinary oxygen has two atoms in the molecule. These molecules are split into atoms in the ionizing process, some of these atoms recombine as triplets, and these triplets are ozone molecules.

83. Emission of electrons by hot bodies.*—Electrons are emitted in great numbers by a hot body if the temperature is sufficiently high. Thus, Fig. 129 represents an ordinary tungsten lamp filament *c* which is heated to a high temperature by the battery *A*. The hot filament emits electrons at a rate sufficient to carry a few hundredths of a coulomb of negative charge per second from *c* to *a*, that is to say, at a rate sufficient to permit the battery *B* to produce a few hundredths of an ampere of current in the direction of the arrow; the travel of negative charge from *c* to *a* is equivalent to the travel of positive charge from *a* to *c*, and the direction of a current is thought of as the direction of travel of positive charge.

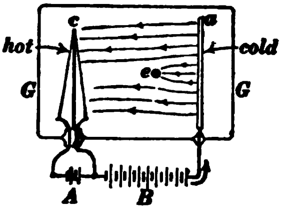

Fig. 129.

If the electromotive force of the battery *B* is large and if the bulb *GG* contains gas, the gas molecules are ionized by collision, and the additional electrons and ions thus produced carry a greatly increased current from *a* to *c*. Indeed, with gas in the bulb an indefinite increase of ionization and indefinite increase of current take place if the voltage of battery *B* is high. *The simplest practical applications of electron emission by a hot body depend upon the absence of gas in the bulb GG and the consequent elimination of ionization of the gas molecules by collision.*

The cold electrode in Fig. 129 is an anode (positively charged) and the lamp filament is a cathode. The fine lines represent the lines of force of the electric field between *a* and *c*. Some of these lines of force terminate on the electrons which are enroute from *c* to *a*. Thus three of the lines of force in Fig. 129 are shown ending on an exaggerated electron *e*. Therefore if there are enough electrons enroute at each instant *all* the lines of force from *a* will end on the moving electrons, and no lines of force will reach the cathode *c*; that is to say, the electric field in the neighborhood of *c* will be zero. Under these conditions the electron emission of the filament *c* might be greatly increased (by raising its temperature) without any increase of current; because the electrons are emitted by *c* at very low velocities and, unless they are drawn away from *c* by an electric field they form a comparatively stagnant cloud around *c*.

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* This subject has been extensively investigated particularly by Professor O. W. Richardson, who has brought most of his researches together in his recent book *The Emission of Electricity from Hot Bodies*, Longmans & Co., London, 1915.

The emission of electrons by a hot metal surface is made use of in three important practical appliances, namely, (*a*) The Coolidge X-ray tube or Roentgen-ray tube, (*b*) The vacuum tube rectifier and (*c*) The telephone repeater or amplifier.

84. The Coolidge tube.—The essential features of the *Coolidge tube* are shown in Fig. 130. A high-voltage supply maintains a very intense electric field between a metal cathode *CC* and a massive tungsten anode *AA*. A fine filament of tungsten wire, *f*, like an incandescent lamp filament, is heated by the insulated

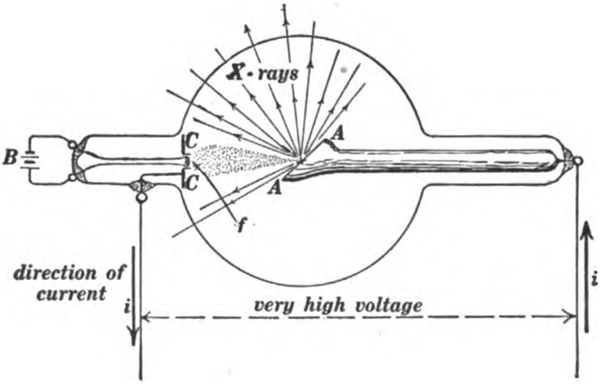

Fig. 130.

battery B, and it is connected to and forms part of the cathode CC. The electrons which are given off by the hot filament f are pulled across* from C to A by the electric field in a slightly converging stream as indicated by the fine dots,† and they impinge on a small spot on the face of the tungsten anode. The stream of electrons constitutes what is called the *cathode rays*, and the fine radiating lines represent the Roentgen rays or X-rays which radiate from the spot where the cathode stream strikes the tungsten plate AA.

With the Coolidge tube the current through the tube can be controlled by changing the temperature of f thus changing the number of electrons per second which are available for carrying negative charge from C to A, and the supply voltage E can be high or low as you please. The higher the voltage E the more penetrating the X-rays, and the greater the current *with a given voltage* the more intense (the greater the energy value of) the X-rays. With the Coolidge tube these two qualities of the X-rays can be adjusted independently of each other.‡

Current is carried through the Coolidge tube by the negative electrons, and negative electrons are supplied only at the plate CC (at the filament f).§ *Therefore current can flow through the Coolidge tube in one direction only*, in the direction corresponding to the travel of negative charge from C to A, as indicated by the arrows *ii*. The Coolidge tube can, indeed, be operated by connecting C and A to the high-voltage coil of a step-up transformer. When the alternating voltage is in one direction a stream of electrons is carried across from C to A and X-rays are produced, but when the alternating voltage is reversed no current can flow.

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* See Art. 70. The charged ball in Fig. 111 is pulled towards plate A or towards plate B, depending upon whether the charge on the ball is positive or negative.

† The convergence of this stream of electrons is due to the breadth of the cathode CC.

‡ With a given voltage the current is limited, and, under ordinary conditions, partly determined by the distribution of negative charge on the electrons which are en route between C and A, and because of this space-charge effect the statement that *intensity* and *penetration* of the X-rays can be independently controlled is not strictly true, although practically it is true.

§ The plate AA may become very hot on account of the bombardment by the particles in the cathode stream, and if AA does thus become hot it gives off electrons.

85. The vacuum-tube current-valve or rectifier.—When the Coolidge tube is operated by connecting cathode and anode, C and A in Fig. 130, to high-voltage alternating supply mains, current can flow in one direction only, as above explained. That is to say, the Coolidge tube acts like a valve. When this valve action alone is desired the cathode is made of a plane zig-zag or grid of fine tungsten wires which are kept at a high temperature by a small auxiliary battery connected as in Fig. 130, and the cold anode is made of two flat metal plates, one on each side of the cathode. *Current can flow only in the direction corresponding to the movement of negatively charged electrons from hot cathode to cold anode,* as indicated by the arrows in Figs. 130, 131 and 132.

Figure 131 is a diagram showing a single current valve ranged to allow current from an alternating supply to flow in one direction only through a receiving circuit, and Fig. 132

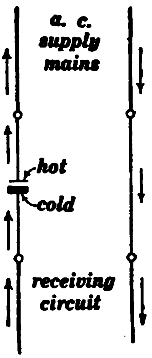

Fig. 131.

When the supply voltage is reversed no current can flow.

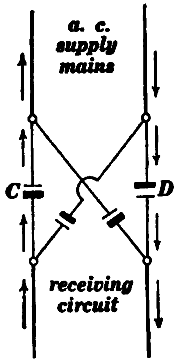

Fig. 132*a*.

One "wave" of alternating current flowing through C and D.

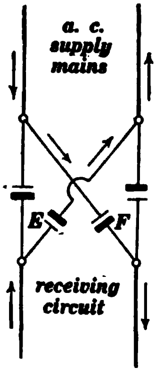

Fig. 132*b*.

Reverse "wave" of alternating current flowing through E and F.

shows an arrangement of four current valves for rectifying both waves of alternating current so that they flow in the same direction in the receiving circuit.

86. The use of the vacuum-tube current valve as a receiver or detector in wireless telegraphy.—The essential features of a wireless receiving station using a vacuum-tube current valve are shown in Fig. 133. The electric waves from the sending antenna (see Fig. 104) produce high-frequency alternating current in the receiving antenna, high-frequency current is induced in the secondary coil *S*, every alternate half-wave of this high-frequency current flows through the current valve, and these half-waves of the alternating current blend into a pulse of direct current in the telephone.

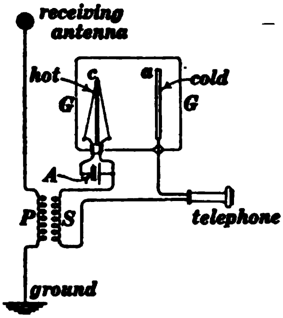

Fig. 133.

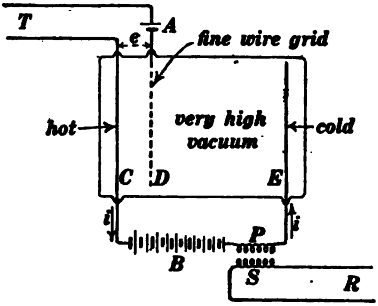

Fig. 134.

87. The amplifier.—The action of the amplifier is most easily described by considering its use as a *telephone repeater*, a device in which a very weak " current wave " from a distant telephone transmitter controls a new supply of energy so as to reproduce the " wave " on a greatly intensified scale so that the " wave" has a new lease of life, as it were, and can travel over another long stretch of telephone line and actuate a distant telephone receiver.

The essential features of the telephone repeater are shown in Fig. 134, in which the two wires *T* come from the distant telephone transmitter, and the two wires *R* lead to the distant telephone receiver. To understand the action of the device we must consider the action of the fine wire grid as follows.

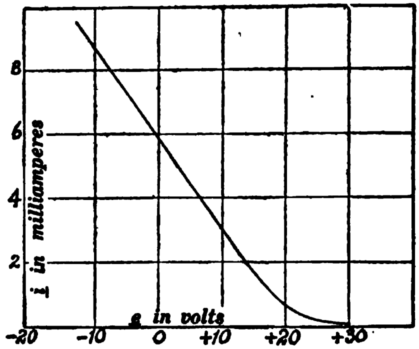

Fig. 135.

Letters *e* and *i* refer to Fig. 134. The positive direction of *e* is understood to be such as to carry positive charge from *C* to *D* or hinder the movement of negative electrons from *C* to *D*.

Imagine a small voltage *e* to act between *C* and *D* *in a direction to charge C positively and D negatively*; this small voltage *e* produces no current between *C* and *D* because current can flow only from *D* to *C*, not from *C* to *D*, as explained in Arts. 84 and 85; *but some of the electrons which shoot out from the hot filament pass through the meshes of D and* **thus carry negative charge from** *C* **to** *E* (or, in other words, permit battery current to flow as indicated by the two arrows *ii*). Increasing *e* decreases the current *i* as indicated by the coordinates of the curve in Fig. 135, and we may consider this curve as approximately a straight line.

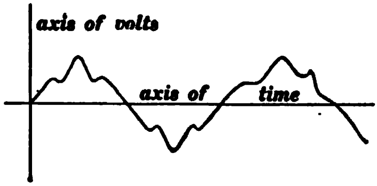

Fig. 136.

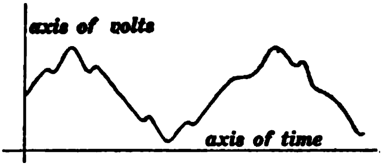

Fig. 137.

The "current wave" coming in over the wires *T* would alone cause the electromotive force *e* to vary as represented by the ordinates of the curve in Fig. 136, *but by connecting the battery A so as to tend to charge D negatively*, this entire curve is raised above the zero axis as shown in Fig. 137. Under these conditions the current *i* varies with *e* as shown by the coordinates of the curve in Fig. 135. But the current i flows through the primary coil P of a transformer or induction coil, and as i varies it induces electromotive forces in the secondary coil S, thus reproducing the original wave greatly intensified.

88. The use of the amplifier for exciting and maintaining electric oscillations.—The essential features of the arrangement for using an amplifier for exciting and maintaining electric oscillations are shown in Fig. 138. Suppose the circuit Lgh to be oscillating, then there is an alternating voltage across the points $a'b'$. By connecting the points $a'b'$ to the points ab as indicated by the dotted lines, the voltage between C and D will vary so as to allow a pulse of current from the high-voltage direct-current supply to flow through P once during each complete oscillation of Lgh, and this pulse of current will occur at the right time to induce a momentary helping electromotive force in the coil L thus maintaining the oscillations of Lgh. The two coils P and L constitute an ironless transformer.

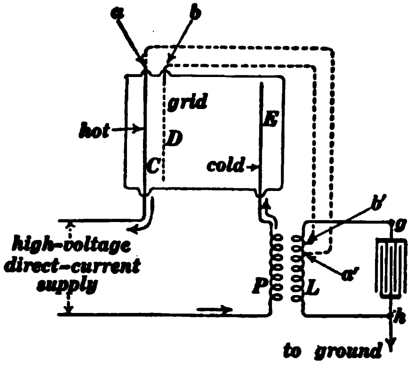

Fig. 138.

A small output of alternating current may be taken from the points g and h, to excite a wireless antenna, for example, g being connected to the antenna and h to ground.

In a *wireless telephone* sending station an arrangement is used to make the very high frequency oscillations of the circuit Lgh in Fig. 138 increase and decrease in violence as the telephone transmitter diaphragm moves to and fro, thus causing the waves emitted by the sending antenna to rise and fall in intensity, thus producing increase and decrease of the high frequency alternating current in the receiving antenna of Fig. 133, thus causing increase and decrease of the rectified telephone current, and thus eventually reproducing the original sound. There are several methods for controlling the intensity or violence of the sending antenna oscillations by means of a telephone transmitter as stated, and perhaps the simplest arrangement is shown in Fig. 139.

The electromotive force *e* in Fig. 139 is always in the direction to charge the grid *D* negatively, and the value of *e* rises and falls with the to and fro movements of the transmitter diaphragm as represented by the ordinates of the curve in Fig. 137. Thus more or less current can flow from the point *p* to ground during each alternate half-oscillation of the antenna, thereby causing more or less loss of energy by the oscillating antenna and reducing the intensity of the antenna oscillations more or less as the voltage *e* rises and falls in value.*

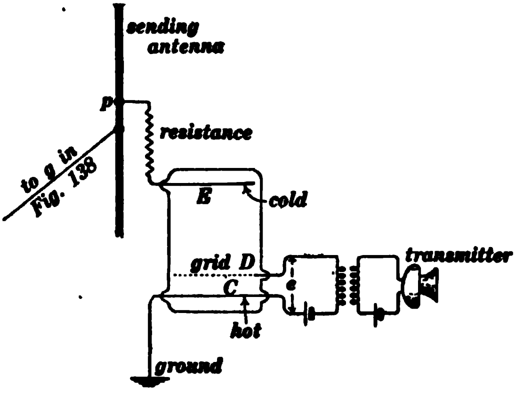

Fig. 139.

89. The use of the electric arc for the maintenance of electric oscillations.—An electric arc is maintained between a carbon rod *A* and a water cooled copper vessel *B*, and the arc is shunted by a circuit containing an inductance *L* and a condenser *C* as shown in Fig. 140. Under these conditions a steady back and forth surging of current through the circuit *LC*, is maintained, the frequency being $\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ cycles per second. The existence of these electric oscillations can be shown by bringing near to the coil L a small secondary coil which is connected to a small glow lamp. The alternating current in L induces an alternating current in the secondary coil and lights the lamp. Also, if the frequency is less than about 10,000 cycles per second, the arc emits a shrill whistling tone due to expansion and contraction of the arc as the current flowing through it increases and decreases.

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* Antenna oscillations are supposed to be maintained by a separate arrangement like Fig. 138 or like Fig. 140.

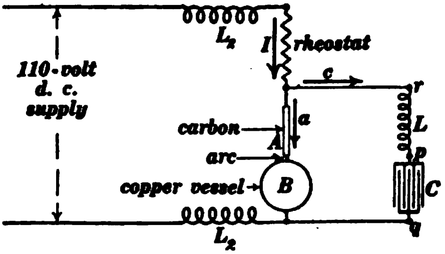

Fig. 140.

The explanation of the steadily maintained oscillations of the circuit LC in Fig. 140 is as follows: A peculiarity of the electric arc is that increase of current through an arc causes a decrease of voltage across the arc. Suppose the supply current I to be constant;* then while the current c is charging the condenser the current a is small, and the voltage across the arc and across the circuit LC is large; and while a reversed current $-c$ is discharging the condenser the current a is large, and the voltage across the arc and across the circuit LC is small. That is to say, the voltage across the circuit LC rises and falls as the current c flows in and out, and thus the oscillations once started are maintained.

The arrangement shown in Fig. 140 is improved in its action by arranging a magnetic blow out to act upon the arc, and by forming the arc in an atmosphere of hydrogen; frequencies ranging from about 4 or 5 thousand cycles per second to a million or more cycles per second can be obtained; and it is possible to take from the points p and q (or p and r) a few watts of power in the form of very high frequency alternating current, such as would be used, for example, to excite the sending antenna of a wireless telegraph installation.

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* It is indeed desirable to put a large inductances L_1L_2 in circuit with the rheostat so as to keep I steady.

The characteristic of the electric arc which is utilized in the above device is shown by the curves in Fig. 141. The ordinates

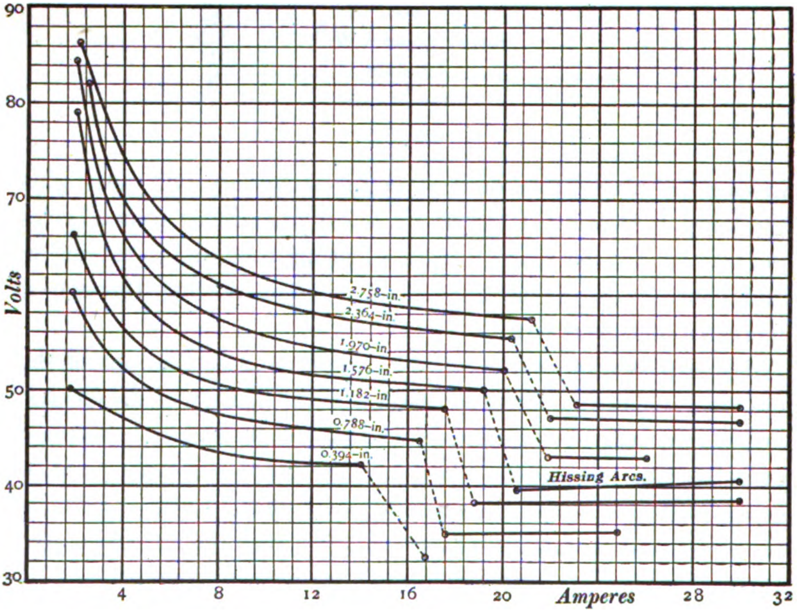

Fig. 141.

of these curves represent *volts across arc*, the abscissas represent *amperes flowing through arc*, and the different curves refer to arcs of different lengths as indicated. The dotted portions of the curves represent extremely unsteady conditions where a very small increase of current is accompanied by a big decrease of voltage across the arc, and *vice versa*.

CHAPTER VII.*

ELECTROMAGNETS. MAGNETISM OF IRON.

90. **Bunched windings and distributed windings.**—The electromagnet, such as is used for the field magnet of a dynamo, consists of a rod of iron surrounded by a winding of insulated copper wire, through which an electric current flows. The iron rod is called the *core* of the electromagnet, and the electric current which flows through the winding of wire is called the *exciting current.* The exciting current produces a magnetic field in the region occupied by the iron core, and the effect of this *magnetizing field*, as it is called, is to magnetize the core and cause a certain amount of magnetic flux to " flow " through it. The iron rod or core usually forms a complete or nearly complete circuit, called a *magnetic circuit,* through which the magnetic flux " flows."

Two distinct cases occur in the arrangement of the windings of wire upon the iron core as follows:

(a) **Uniformly distributed winding.**—The winding of wire may be distributed uniformly along the entire length of the iron rod or core. In this case the magnetic field which the winding produces in the region occupied by the iron core has sensibly the same value at every point in the rod and is sensibly parallel to the rod at each point. *The magnetizing action of such a field upon an iron rod depends simply upon the intensity of the field.* Electromagnets having uniformly distributed windings are seldom used in practice except in the magnetic testing of iron, for which purpose the iron to be tested is usually made into a test ring,† which is wound uniformly with wire.

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* This text is devoted primarily to operating engineering, and therefore this chapter might better, perhaps, have been omitted. It touches problems which were seriously faced by electrical engineers thirty years ago. In recent years improved varieties of silicon-steel have been developed for use in transforming cores, and remarkable developments in the atomic theory of magnetism have been made.

† This applies to the ballistic method of testing iron, which is due to Rowland. In Ewing's method the test piece of iron is in the form of a long slim rod.

(b) **Bunched winding.**—The winding of wire may be bunched at one or more places along the iron core. In this case the magnetic field which the winding produces in the region occupied by the iron core varies greatly in intensity from point to point in the core, and the direction of this field is not parallel to the rod at each point. The magnetizing action of such a field upon an iron rod is discussed in the next article.

Electromagnets usually have bunched windings.

91. The magnetizing action of a bunched winding. Definition of magnetomotive force.—The magnetizing action upon an iron rod of a non-uniform magnetic field, such as the magnetic field produced by a bunched winding, depends upon the *average value (along the rod) of the component** *of the magnetizing field parallel to the rod.* **The product of this average value and the length of the rod is called the magnetomotive force along the rod.** That is:

$$F = lA \tag{34}$$

in which F is the magnetomotive force along a rod in c.g.s. units, l is the length of the rod in centimeters, and A is the average value, along the rod, of the component of the magnetizing field parallel to the rod. The rod may be straight or curved.

A rod passing through a magnetic field determines a certain line or path through the field, and we speak of the magnetomotive force along this path, whether the rod is there or not.

92. Proposition.—*The magnetomotive force along a path in a magnetic field is equal to the work per unit pole done by the magnetic field upon a magnet pole while the pole is made to travel along the path*; that is, the magnetomotive force along a path is equal to W/m, where W is the work done by the field upon a pole of strength m while the pole is made to travel along the path. That is:

$$F = \frac{W}{m} \tag{35}$$

in which F is the magnetomotive force along the path,

---

* At each point.

Proof.—The product mA is the average value along the path of the component parallel to the path of the force $f(=mH)$ with which the field acts on the pole, the meaning of A being explained in Art. 91; therefore lmA is the work done on the pole by the field as the pole travels from one end of the path to the other. That is:

$$W = lmA$$

Dividing both members of this equation by m, and remembering that $lA = F$, according to equation (34), we have equation (35).

The above proposition is usually taken as the *definition* of magnetomotive force along a path, and from this definition it may be shown that the magnetomotive force along a path is equal to what is called the line integral* of the magnetic field along the path, as follows: Consider an element Δl of a path pp', Fig. 142. Let H represent the intensity of the magnetic field at this element and ϵ the angle between H and Δl. Then the component of H parallel to Δl is $H \cdot \cos \epsilon$.

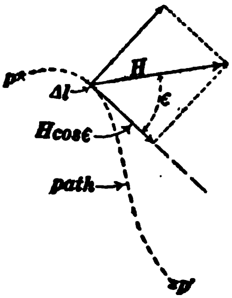

Fig. 142.

Let a magnet test pole of strength m be moved along Δl. The force with whieh the magnetic field acts upon this pole is mH, and the component of this force parallel to Δl is $mH \cos \epsilon$, so that $mH \cos \epsilon \times \Delta l$ is the work ΔW done by the field on the pole as the pole moves along Δl. That is:

$$\Delta W = mH \cos \epsilon \cdot \Delta l$$

from which, by integration, we have:

$$\frac{W}{m} = F = \int H \cos \epsilon \cdot dl$$

93. **Magnetomotive force of a coil.**—The most important case of magnetization of an iron rod or core by a bunched winding is the case in which the core is in the form of an endless rod which passes through, or *links with*, the coil as shown in Fig. 143. *The magnetomotive force along the rod in this case is entirely inde-*

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* A very simple and complete discussion of line and surface integrals such as are used in electromagnetic theory is given in Chapter IX of Franklin, MacNutt and Charles's *Calculus*, South Bethlehem, Pa., 1913.

pendent of the shape and length of the rod and of the shape and size of the coil. **It depends only upon the number of turns of wire in the coil and upon the strength of the current in the wire.**

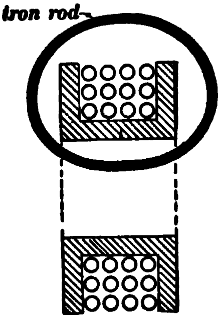

Fig. 143.

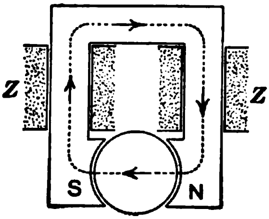

Fig. 144.

This definite magnetomotive force along an endless rod which links with a coil of wire in which an electric current is flowing, is called the **magnetomotive force of the coil.**

Examples.—Fig. 144 shows a two-pole dynamo with two field coils ZZ. The field-magnet iron of this dynamo and the iron of the armature together constitute a nearly endless rod or core of iron which links with both field coils, and which constitutes what is called the *magnetic circuit* of the dynamo.

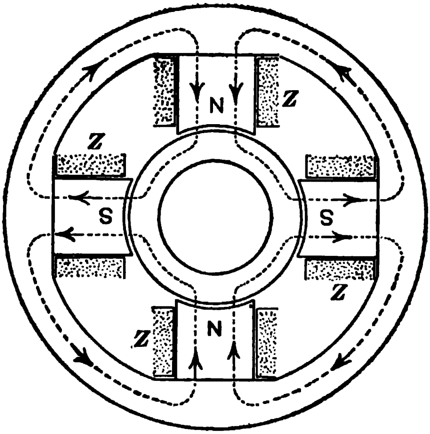

Fig. 145.

Figure 145 is a sketch of a four-pole dynamo with four field coils $ZZZZ$. The iron of the field magnet and armature presents in this case four distinct magnetic circuits, as indicated by the dotted lines. Through each of these magnetic circuits, flux is forced by the combined magnetomotive forces of the two field coils through which the magnetic circuit passes, that is, with which it links.

The total magnetomotive force acting on any magnetic circuit is equal to the sum of the magnetomotive forces of the coils with which the circuit links.

94. **Proposition.**—The magnetomotive force of a coil is given by the equation:

$$F = 4\pi Zi \tag{36}$$

in which Z is the number of turns of wire in the coil, and i is the strength of the current in the wire in abamperes. When the current is expressed in amperes we have:

$$F = \frac{4\pi}{10} Zi \tag{37}$$

Proof of equation (36).—Before proceeding to the derivation of equation (36), it is necessary to find an expression for the total work W done in keeping the current i in a coil constant while the magnetic flux through the opening of the coil is increased by a specified amount Φ; W being expressed in ergs, i in abamperes, and Φ in maxwells or lines. Now, while the flux is increasing, a back electromotive force equal to $Z\frac{d\Phi}{dt}$ abvolts is induced in the coil. and therefore (assuming the coil to have zero resistance for the sake of simplicity of statement) an electromotive force $e = Z\frac{d\Phi}{dt}$ will have to act on the coil to keep the back electromotive force from decreasing the current. Consequently work will have to be done on the coil at the rate ei, or $Z\frac{d\Phi}{dt} \times i$, to keep the current from decreasing. That is:

$$\frac{dW}{dt} = Zi \cdot \frac{d\Phi}{dt} \tag{38}$$

But Zi is a constant, because i is being kept from changing.

Therefore by integrating* (38) from $W = 0$ and $\Phi = 0$ we get:

$$W = Zi\Phi \tag{39}$$

in which W is the work in ergs done to keep a current of i abamperes constant in a coil (of zero resistance) while the magnetic flux through the opening of the coil increases by the amount of Φ maxwells, and Z is the number of turns of wire in the coil.

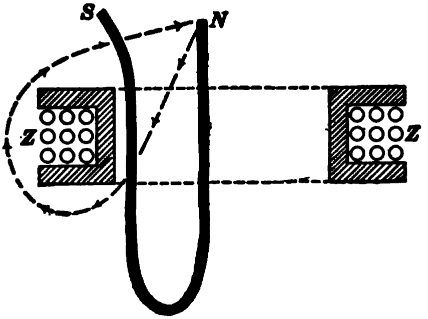

Fig. 146.

We shall now proceed to the proof of equation (36). Let ZZ, Fig. 146, represent a coil of Z turns of wire. Imagine NS to be a *flexible* magnet. Let the north pole of this flexible magnet be carried through the coil and around to its initial position along the dotted path. The flexible magnet will then link with the coil of wire as shown in Fig. 147. Let F be the magnetomotive force along the dotted path, and let m be the strength of the pole N which has been carried around the path. Then, according to Art. 92, Fm is the work done on the pole by the magnetic field of the coil. *This work done on the moving pole by the magnetic field of the coil is the work which is spent in keeping the current constant in spite of the electromotive force induced in the coil by the moving pole.* This is evident when we consider that work done on the pole by the field must be made good, that is, energy must be supplied from somewhere: and when we con-

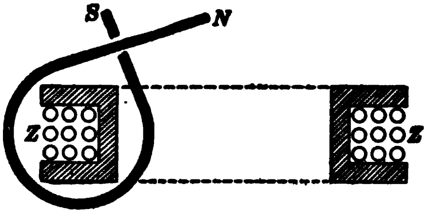

Fig. 147.

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* According to equation (38), the quantity W increases Zi times as fast as Φ; therefore, while Φ is increasing from zero to any assigned value Φ, the increase of the quantity W must be Zi times as great.

sider that the energy stored in the system is the same in Figs. 146 and 147. Therefore, the only possible source of the energy for doing work on the moving pole is the work that is done to keep the current in the coil constant.

Now the two poles of the flexible magnet are in the same positions before and after the movement; therefore, of the total number of lines of force which *radiate* from these poles, the same number pass through the coil before and after the movement. On the other hand, the flux $4\pi m$ (see Art. 11), which passes along the magnet from pole to pole, passes through the coil after the movement, so that the flux through the opening of the coil is increased by the amount $4\pi m$ by the movement of the pole. Therefore, according to equation (39), $Zi \times 4\pi m$ is the work spent in keeping the current constant during the movement of the pole and, since this is equal to the work Fm done upon the pole by the magnetic field of the coil, we have

$$Fm = 4\pi i Zm$$

or

$$F = 4\pi Zi$$

95. Units of magnetomotive force.—The product lH (length of path in centimeters multiplied by intensity of magnetic field in gausses) gives the magnetomotive force along the path (when l and H are parallel, of course) in c.g.s. units. The name *gilbert* has been adopted by the American Institute of Electrical Engineers for the c.g.s. unit of magnetomotive force.

The magnetomotive force along a path which links with one turn of wire carrying one ampere of current is called *one ampere-turn*. The magnetomotive force of any coil, in ampere-turns, is equal to the product of the current flowing in the coil in amperes and the number of turns of wire in the coil. In magnetic calculations it is usually convenient to reduce ampere-turns of magnetomotive force to c.g.s. units (gilberts), which is done by multiplying ampere-turns by $4\pi/10$, according to equation (37).

The product, field intensity multiplied by length of path, gives magnetomotive force, so that the quotient obtained by dividing magnetomotive force by length of path is field intensity. When a magnetomotive force which is expressed in ampere-turns is divided by length of path, we have a magnetic field intensity expressed in *ampere-turns-per-centimeter*, or in *ampere-turns-per-inch*, as the case may be.

96. Intensity of magnetic field inside of a coil wound uniformly on a long tube.—Figure 148 shows a sectional view of a coil AB.

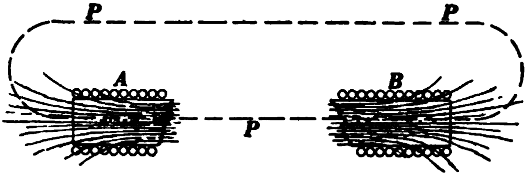

Fig. 148.

The magnetomotive force along the closed path PP is equal to $4\pi Zi$, where Z is the number of turns of wire in the coil and i is the current in abamperes. Let us consider *a coil which is very long in comparison with its diameter;* then inside the coil is an intense magnetic field as indicated by the fine lines (lines of force) in Fig. 148, but this magnetic field spreads out at the ends of the coil and becomes of negligible intensity in the region outside of the coil. Therefore the magnetomotive force along the path PP is nearly all contributed by the magnetic field inside the coil, and, according to equation (34), it is equal to lH, where l is the length of the coil and H is the intensity of the field inside the coil. This is evident when we consider that the field inside the coil must be sensibly uniform and parallel to the axis of the coil. Therefore we have

$$4\pi Zi = lH$$

whence

$$H = 4\pi \frac{Z}{l} \cdot i \qquad (40)^*$$

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* This equation is derived in a more straightforward manner in Franklin and MacNutt's *Advanced Electricity and Magnetism*, pages 23–25, The Macmillan Co., New York, 1915.

97. Inductance of a long coil without iron core.—It is interesting to make use of equation (40) to get an expression for the inductance of a long coil. Multiplying the area of opening of coil (πr^2 where r is the radius of the coil) by H we get the flux through the opening of the coil, namely, $4\pi^2 r^2 Zi/l$, using equation (40), and this is the quantity which is represented by bi in Art. 48. Therefore the value of b is $4\pi^2 r^2 Z/l$. But the inductance L of the coil is equal to bZ, according to Art. 48. Therefore we have

$$L = 4\pi^2 r^2 Z^2/l \tag{41}$$

where the inductance L of the coil is expressed in abhenrys, or

$$L_{\text{in henrys}} = \frac{4\pi^2 r^2 Z^2}{10^9 l} \tag{42}$$

r and l being expressed in centimeters.

98. Magnetizing force in iron.—Consider an iron rod in a coil of wire as indicated in Fig. 149. The rod is, of course,

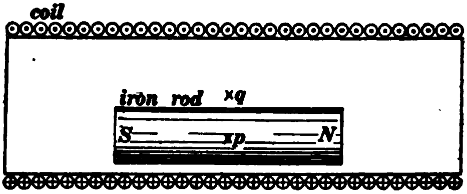

Fig. 149.

magnetized, and two distinct causes act to produce magnetic field at any point q, namely, (a) The current in the coil would alone produce a certain field intensity at q, and (b) The free magnetic poles N and S of the rod would alone produce a certain field intensity at q. The same may be said of any point p in the iron, *and the " magnetizing force " at p is the net resultant field intensity at p due to the combined action of coil and of free magnet poles N and S.*

Example 1.—When an iron rod in the form of a closed ring is magnetized by a winding of wire there are no free magnet poles,* and the magnetic field due to the winding is itself the net magnetizing field or " magnetizing force."

Example 2.—A very long slim iron rod is placed in a long coil and magnetized. In this case the free magnet poles are weak (because the rod is slim) and they are far distant from the middle portions of the rod because the rod is long. Therefore the magnetic field produced near the middle portions of the rod by the free poles of the rod is negligibly weak, and the field due to the coil $(4\pi Zi/l)$ is itself the net magnetizing field or " magnetizing force."

Example 3.—Suppose the intensity of the field due to the coil in Fig. 149 is 60 gausses, and suppose the iron rod to be 20 centimeters long and magnetized so as to give each pole a strength of 2700 units. In this case the middle point p is approximately 10 centimeters from each pole, and the actual field near the middle of the rod is made up of three parts, namely, (*a*) The field due to the coil alone, intensity 60 gausses directed towards the right, (*b*) The field due to the N-pole, intensity $2700/10^2$ directed towards the left, and (*c*) The field due to the S-pole, intensity $2700/10^2$ directed towards the left. [See equation (3) Art 9.] Therefore the net magnetizing field at p is 60 gausses $-$ 27 gausses $-$ 27 gausses, which is 6 gausses to the right.

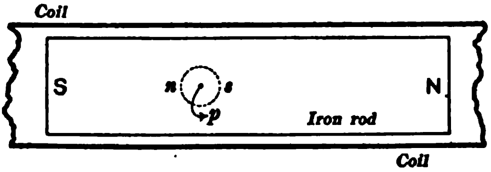

Fig. 150.

Explanation.—The fundamental definition of the intensity of a magnetic field at a point, as given in Art. 8, is the force in dynes which would act on a unit test-pole placed at the point, and this definition has no meaning unless it is possible to place a test-pole at the point, that is to say, the point must be in air. But if a cavity is made round the point p in Fig. 149, as shown by the dotted circle in Fig. 150, the walls of this cavity will be free magnet poles as indicated by the letters n and s, and the field intensity in this cavity will *not* be that which is due merely to the combined action of the coil and poles N and S at the ends of the rod. If, however, the cavity around p is very long and narrow, as shown in Fig. 151, then the poles n and s at the ends of this cavity are weak and far away from p, so that these poles n and s do not produce at p any perceptible field, and therefore the field intensity in the narrow longitudinal cavity shown in Fig. 151 is that which is produced by the combined action of the coil and the free poles N and S at the ends of the rod. That is to say, the actual field intensity in the narrow longitudinal cavity in Fig. 151 is the "magnetizing force" at the point p.

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* The armature ring in Fig. 28, as magnetized by the flow of current which is indicated in the figure, has free magnet poles at *a* and *b*. The above statement refers to the case in which the ring is magnetized in the same direction all the way round.

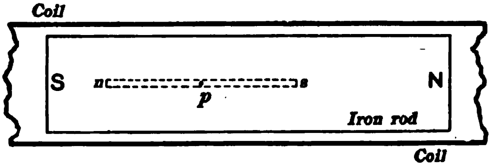

Fig. 151.

99. Intensity of magnetization.—Let m be the strength of each of the magnetic poles at the ends of a magnetized iron rod of which the sectional area is s square centimeters. The **pole-strength-per-unit-sectional-area**, namely m/s, is called the *intensity of magnetization*, I, of the rod. That is

$$I = \frac{m}{s} \tag{43}$$

100. Magnetic flux through a magnetized rod.—Consider a very long slim iron rod, s square centimeters in sectional area, placed in and parallel to a magnetic field, which, but for the presence of the rod, would be a uniform magnetic field of intensity H. Then the actual intensity of the magnetizing force along the middle portions of the rod is H, as explained in Art. 98. The magnetic field in the neighborhood of each end of the rod is a composite field which is the resultant of two distinct parts, a and b; and the magnetic flux which comes up to one end of the rod, flows through the rod, and passes out from the other end of the rod, consists of two distinct parts which correspond to the respective parts, a and b, of the field near the ends of the rod.

(a) The part a of the field is the original uniform field H. The lines of force of this field are shown by the parallel straight lines in Fig. 152. The amount of magnetic flux which comes in to one end of the rod and passes out from the other end of the rod because of this uniform field H, is Hs lines, where s is the sectional area of the rod.

(b) The part b of the field is the magnetic field due to the pole m of the rod. The lines of force of this field are shown by the radiating straight lines in Fig. 152. The amount of magnetic flux which comes in to one end of the rod and passes out from the other end of the rod, because of this field, is $4\pi m$ lines, as explained in Art. 11.

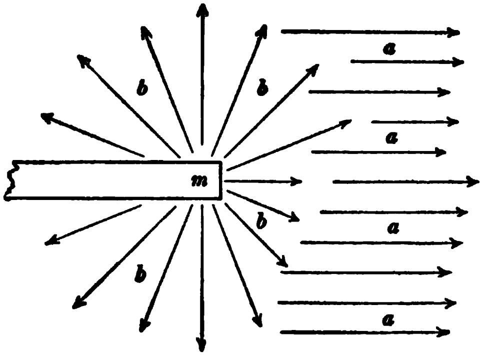

Fig. 152.

The total magnetic flux Φ coming in to one end of the rod and passing out from the other end is equal to the sum of the two parts, a and b, so that we have:

$$\Phi = 4\pi m + Hs \tag{44}$$

and this flux passes through the rod. If H is zero we have $\Phi = 4\pi m$.

Now $m = Is$, from equation (43), and therefore equation (44) may be written:

$$\Phi = 4\pi Is + Hs \tag{45}$$

The part $4\pi m$ or $4\pi Is$ of the flux through an iron rod is due to the magnetized condition of the rod, and the part Hs is due to the magnetizing force directly. This part, Hs, is the amount of flux which would pass through the region occupied by the rod if the rod were removed.

101. Flux density in iron.*—The magnetic flux per unit sectional area of an iron rod, namely Φ/s, is called the *flux density* in the rod, and it is represented by the letter B. That is:

$$B = \frac{\Phi}{s} \tag{46}$$

Substituting Bs for Φ in equation (45), we have:

$$B = 4\pi I + H \tag{47}$$

The flux density in an iron rod is equal to the intensity of the magnetic field in a thin crevasse cut across the rod. This is evident when we consider in the first place that *flux density in air* is identically the same thing as *field intensity* according to Art. 10, and in the second place, that a very thin crevasse does not sensibly disarrange the flux through a rod, so that the flux density in the crevasse is the same as in the rod. If the crevasse is not very narrow, a portion of the flux passes out of the rod, around the crevasse, and back into the rod again. In this case the flux density, or field intensity in the crevasse, is less than the flux density in the part of the rod which is remote from the crevasse.

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* Frequently called *magnetic induction* or simply *induction*.

102. Residual magnetism. Permanent magnets.—An iron rod retains a portion of its magnetism when it is removed from the magnetic field in which it has been magnetized, or, in case of an electromagnet, when the magnetizing current is reduced to zero. The magnetism thus left in a bar of iron (in the core of an electromagnet) is called *residual magnetism.* The poles of a short thick bar produce a strong demagnetizing field along the bar, and therefore a short thick bar does not retain as much residual magnetism as a long slim bar, or as a bar which forms a closed or nearly closed magnetic circuit.

A long slim bar or a closed ring of soft annealed wrought iron or mild steel may retain as much as 90 per cent. of its initial magnetism. But a very weak demagnetizing field or a slight mechanical shock is sufficient to cause the soft iron or steel to lose nearly all of its residual magnetism; cast iron and hard-drawn iron wire retain a smaller portion of their magnetism, but with greater persistence; and hardened steel retains a portion of its magnetism very persistently, even when roughly handled. Magnetized bars of hardened steel are called *permanent magnets.*

The more persistently a sample of iron or steel retains its residual magnetism, the greater the intensity of the magnetizing force required to magnetize it. Thus hardened bars of steel are magnetized by placing them between the poles of a strong electromagnet or by placing them in a coil of wire through which a strong electric current is caused to flow.

103. Magnetization curves.—When an iron rod is acted upon by a given magnetizing force, the intensity of magnetization of the rod (and also the flux density in the rod) may have any value whatever (between certain limits), on account of the tendency of the iron to retain its previous magnetic state. Thus the intensity of magnetization of an iron rod is much greater for a given value of H, if the given value of H is reached by a decrease from a higher value, than if the given value of H is reached by an increase from a lower value.

If, however, an iron rod is subjected to mechanical shocks, or if the magnetizing field pulsates slightly before it settles to a steady value, then the rod tends to settle to a state which depends only on the value of *H*, that is, to a state which is independent of the previous magnetic condition of the rod. Under these conditions, therefore, definite values of *B* and *I* are produced by a given value of *H*. These values are called the ***normal values*** of *H* and *I* corresponding to the given value of *H*. For example, a dynamo having a given exciting current in its field coils tends to settle to a state in which the magnetization of its field magnet is the normal value corresponding to the given value of its field current, because of the mechanical vibrations of the machine and because of slight variations of magnetizing action due to slight changes of field current and of armature current.

The normal *I* and *H* curve.—The curve of which the abscissas represent values of magnetizing force *H* and of which the ordinates represent the corresponding normal values of *I* for a given sample of iron or steel is called ***the normal I and H curve*** of the sample of iron or steel. Figure 153 shows the

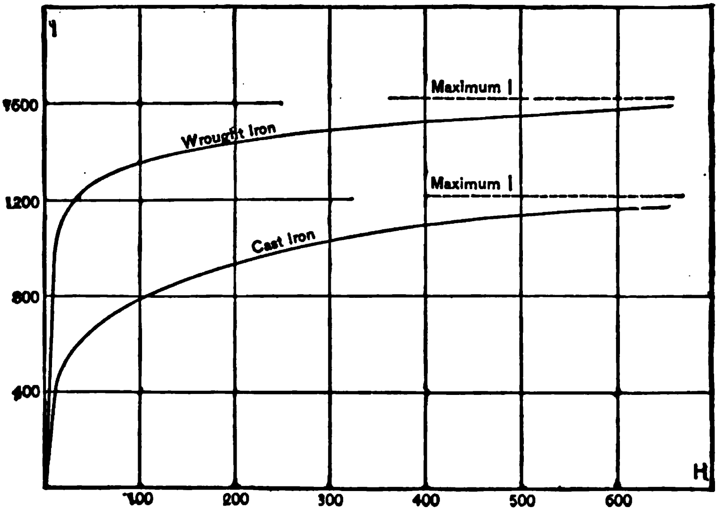

Fig. 153.

I and *H* curves for wrought iron and cast iron.

normal I and H curves for annealed wrought iron and for ordinary cast iron.

The intensity of magnetization I approaches a definite limiting value for a given kind of iron when H is increased indefinitely. This limiting or saturation value of I is about 1,730 units pole per square centimeter for annealed wrought iron, and about 1,250 units pole per square centimeter for ordinary cast iron.

The value of I for a good permanent magnet made of a straight bar of hardened tungsten steel, the length of the bar being about 15 times its diameter, is about 700 or 800 units pole per square centimeter.

The normal B and H curve. The curve of which the abscissas represent values of magnetizing force H and of which the ordinates represent the corresponding normal values of B for a given kind of iron or steel is called the ***normal* B *and* H *curve*** of the iron or steel. Figure 154 shows the normal B and

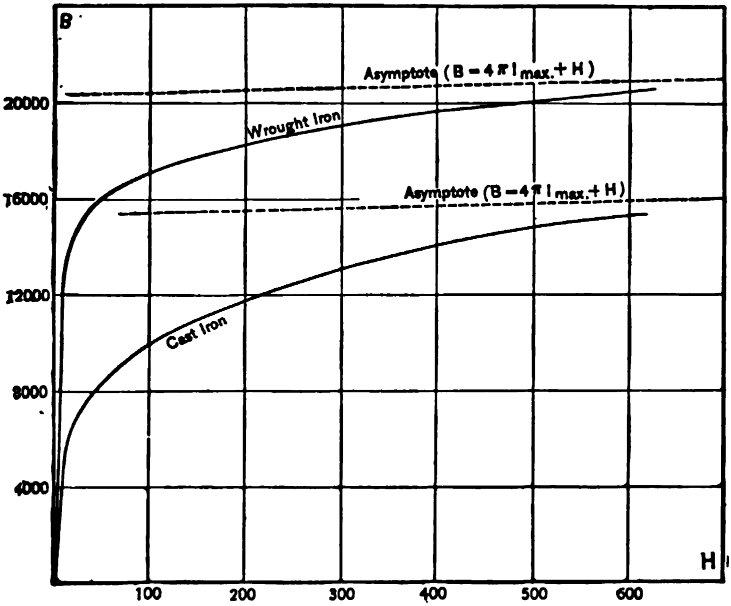

Fig. 154.

B and H curves for wrought iron and cast iron.

H curves for annealed wrought iron and for ordinary cast iron. The value of B increases indefinitely in a given sample of iron when H is increased indefinitely.

The sharp bend in the B and H curve is called the "knee" of the curve. The iron is approximately saturated when H has been increased beyond the knee of the curve, and any further increase of H produces only slight increase of B. Thus a magnetizing force of 10 units produces a flux density of 12,400 lines per square centimeter in a certain sample of annealed wrought iron, and a magnetizing force of double this intensity produces only 14,330 lines per square centimeter; that is, doubling the value of H (from 10 to 20 units) produces only about 15 per cent increase of B. Wrought iron has been subjected to a magnetizing field of about 20,000 units by Ewing who found the corresponding value of B to be about 40,000 lines per square centimeter, and the corresponding value of I to be 1,730 units pole per square centimeter.

The accompanying table gives the corresponding values of B and H for wrought iron, for cast iron and for soft cast steel.

TABLE.

Magnetic Properties of Iron and Steel.

Wrought Iron (Hopkinson).			Cast Iron (M. E. Thompson).*			Soft Cast Steel.		
H	B	μ	H	B	μ	H	B	μ
10	12,400	1,240	10	5,000	500	10	9,700	970
20	14,330	716	20	6,600	330	20	13,380	669
30	15,100	503	30	7,290	246	30	14,500	483
40	15,550	389	40	7,850	195	40	15,250	381
50	15,950	319	50	9,360	169	50	15,840	317
60	16,280	271	60	8,800	146	60	16,300	272
70	16,500	235	70	9,200	131	70	16,750	239

104. Magnetic permeability.—A long coil of wire produces a magnetic field of intensity H in its (air-filled) interior, and this field intensity is of course equal to the magnetic flux per unit sectional area (the flux density) in the air-filled interior of the coil. If the region inside of the coil is filled with iron, the flux density is many times as great as H, as is evident from an inspection of the B and H curves of Fig. 154. From this fact there arises the idea of *the multiplying effect of an iron core, in so far as the production of magnetic flux is concerned.* In conformity with this idea we may write:

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* Thompson, Knight, and Bacon, *Transactions American Institute of Electrical Engineers*, Vol. IX, 1892.

$$B = \mu H \tag{48}$$

This quantity, μ, is called the *permeability* of the iron. It is of course *not* a constant for a given kind of iron (if it were constant the B and H curve would be a straight line), but its value varies with the flux density B, as is clearly shown in the tabulated values of H, B, and μ in Art. 103.

The ordinates of the curves in Fig. 155 show the values of the permeability μ of wrought iron and of cast iron for various values of the flux density B.

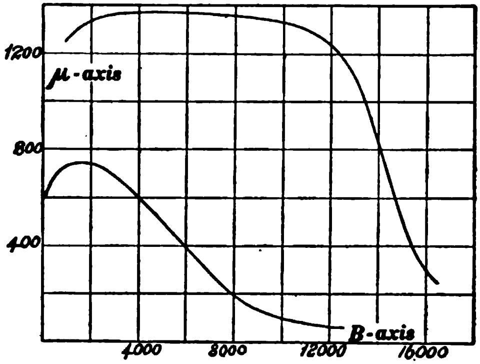

Fig. 155.

μ and B curves for wrought iron and cast iron.

The permeability of air and of all non-magnetic materials such as copper, cotton, wood, etc., is unity.

105. Magnetic circuit calculations.—When an iron rod of uniform size and quality forms a closed magnetic circuit and is subjected to a uniform magnetizing force H along its entire length, the value of the flux density B in the rod may be found from the B and H curve, and the total flux through the rod may be found by multiplying this flux density by the sectional area of the rod. Or, the uniform value of H required to give a specified total flux through the rod may be taken from the B and H curve, after the value of B has been found, by dividing the specified flux by the sectional area of the rod.

When, however, the iron rod varies in size from point to point, or when different parts of the rod are of different kinds of iron, or when there is an air gap in the magnetic circuit, then some kind of an averaging process must be used to calculate the magnetomotive force required to produce a specified flux through the rod, or, conversely, to calculate the flux produced by a specified magnetomotive force.

This calculation is carried out as follows:

(a) To find the magnetomotive force required to produce a specified flux.—Divide the prescribed flux by the sectional area in square centimeters of each part of the circuit, wrought iron, cast iron, or air, as the case may be. This gives the flux density B in each part of the magnetic circuit.* Knowing B for each part of the circuit, take from the B and H tables or curves for the different kinds of iron, the value of H required to produce the known value of B in each part of the circuit, and multiply the value of H for each part of the circuit by the length in centimeters of that part. This gives the magnetomotive force required for each part; and the sum of these magnetomotive forces is the total magnetomotive force required. This total magnetomotive force may then be reduced to ampere-turns by multiplying it by $10/4\pi$.

In this kind of a calculation it is to be remembered that flux density in air is H, so that the flux density in the air gap is to be multiplied by the length in centimeters across the gap to give the magnetomotive force required for the gap.

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* It is here assumed that the whole of the magnetic flux passes through each portion of the iron circuit. This is not always the case, for a portion of the magnetic flux may stray through the air. This portion is called the *leakage flux*. Magnetic leakage is discussed in treatises on dynamo design.

(b) To find the flux produced by a specified magnetomotive force.—In the case of a rod of uniform size and quality this problem (*b*) is simply the reverse of problem (*a*). When, however, the parts of a magnetic circuit are of different sizes or of different materials, the flux produced by a specified magnetomotive force is best determined as follows: Calculate, as explained under (*a*), the magnetomotive force required to produce a series of arbitrarily assumed values of flux. Arrange these results in tabular form, plot them as a curve, and from this plot find the flux corresponding to the prescribed magnetomotive force.

106. Analogy between the magnetic circuit and the electric circuit. Definition of magnetic reluctance.—The statements given in Art. 105 (*a*) and (*b*) are complete statements of the fundamental principles and methods of calculation of the magnetic circuit. A slight modification of the fundamental methods outlined in Art. 105 is, however, extensively used. This modified method is based upon an analogy between the magnetic circuit and the electric circuit, but it contains no physical or mathematical principles in addition to those involved in the fundamental method outlined in Art. 105, and its only advantage is that the fundamental equation is rearranged so as to correspond exactly in form to the familiar equation for Ohm's law.

The fundamental equation

$$B = \mu H \qquad \text{(i)}$$

as applied to an iron rod which forms a magnetic circuit may be transformed as follows: The magnetic flux Φ through the rod is equal to Bs, whence equation (i) may be written:

$$\Phi = \mu s H \qquad \text{(ii)}$$

Now the magnetomotive force F along the rod is equal to Hl where l is the length of the rod. Substituting, therefore, $\frac{F}{l}$ for H in equation (ii) we have:

$$\Phi = \frac{\mu s F}{l} \qquad \text{(iii)}$$

This equation may be rewritten thus:

$$\Phi = \frac{F}{R} \qquad \text{(iv)}$$

in which R is written for $\frac{1}{\mu} \cdot \frac{l}{s}$. That is,

$$R = \frac{1}{\mu} \cdot \frac{l}{s} \qquad \text{(v)}$$

The quantity R is called the *reluctance* of the magnetic circuit, and the reciprocal of the permeability of a sample of iron, $1/\mu$, is sometimes called its *specific reluctance* or *reluctivity*.

A portion of a magnetic circuit one centimeter in length $(l = 1)$, and one square centimeter in sectional area $(s = 1)$, made of a material having a permeability of unity ($\mu = 1$, which is the value of μ for air), has unit reluctance. The name *oersted* has been adopted for this unit of reluctance by the American Institute of Electrical Engineers.

Equation (iv) is exactly similar in form to the equation expressing Ohm's law, namely, $I = E/R$; and equation (v) is similar in form to the equation for calculating the resistance of a wire, having given its length and section and the specific resistance or resistivity of its material.*

To find the magnetomotive force required to produce a specified magnetic flux, using equations (iv) and (v), proceed as follows: Divide the total flux by the sectional area of each portion of the magnetic circuit thus finding the flux density for each portion. Knowing the flux density B for each portion of the circuit, take the corresponding values of μ from the table in Art. 103. Knowing the value of μ, the length l, and the sectional area s of each portion of the circuit, calculate the magnetic reluctance R of each portion. Add these separate reluctances together to get the total reluctance of the entire circuit, and multiply the total flux Φ by this total reluctance to get the required magnetomotive force F, according to equation (iv).

107. Work required to magnetize iron.—When an iron rod is magnetized by sending an electric current through a coil of wire surrounding the rod, an opposing electromotive force is induced in the coil by the growing magnetism of the rod, and *the work done in forcing the current against this opposing electromotive force is the work expended in magnetizing the rod.*

The work W, in ergs, which is done in magnetizing V cubic centimeters of iron from a given initial flux density B' to a given final flux density B'', is given by the equation:

$$W = \frac{V}{4\pi}\int_{B'}^{B''} H \cdot dB \qquad (49)$$

Proof.—In order to avoid the complications which arise on account of the perceptible demagnetizing action of the poles of a short iron rod, let us consider a very long slim rod l centimeters in length and s square centimeters in sectional area. Suppose this rod to be placed in a long coil having z turns of wire per centimeter of length or lz total turns. When the coil of wire is first connected to the battery or other source of current, the current in the coil (beginning at zero) rises in value during the time that the rod is being magnetized, and during this time the magnetic flux through the rod is growing in value. Let $d\Phi/dt$ be the rate at which the flux is increasing at a given instant, and let i be the value of the current at this instant. Then $lz \times d\Phi/dt$ is the induced electromotive force in the coil which at the given instant is opposing the current i, so that $lz \times d\Phi/dt \times i$ is the rate, dW/dt, at which work is being done at the given instant in magnetizing the rod. That is:

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* This analogy between the magnetic circuit and the electric circuit is incomplete, and it is misleading if carried too far.

$$\frac{dW}{dt} = lzi \cdot \frac{d\Phi}{dt}$$

so that:

$$dW = lzi \cdot d\Phi \tag{50}$$

in which dW is the amount of work done during the time that the flux has increased by the amount $d\Phi$ and while the current has the mean value i.

Now $\Phi = Bs$ or $d\Phi = s \cdot dB$ from equation (46), and $zi = H/4\pi$ from equation (40), so that the equation (50) becomes:

$$dW = \frac{ls}{4\pi} H \cdot dB$$

or, since $ls = V$, we have:

$$dW = \frac{V}{4\pi} H \cdot dB$$

or

$$W = \frac{V}{4\pi} \int_{B'}^{B''} H \cdot dB$$

In magnetizing a short iron rod, more work is done than is accounted for by equation (49). The additional work goes to establish the magnetic field in the neighborhood of the magnetic poles of the rod. Equation (49) expresses the work which is spent within the iron.

108. Graphical representation of work done in magnetizing iron.—Let the curve opp', Fig. 156, be drawn so that the coordinates represent corresponding values of B and H for a given sample of iron. The branch op represents the values of B and H when the iron is being magnetized for the first time, and the branch pp' represents the values of B and H when, after the iron has been magnetized up to the point p, the value of H is slowly reduced to zero. *The curve of* B *and* H *for decreasing values of* H *does not coincide with the curve for increasing values of* H. Now, as explained later, the total area opa represents the *work done upon the iron* in magnetizing it up to the point p, and the area $pp'a$ represents the *work which is regained from the iron* when the magnetizing field drops slowly to zero. *The work regained is less than the work required to magnetize the iron.* The work which is lost is represented by the shaded area in Fig. 156.

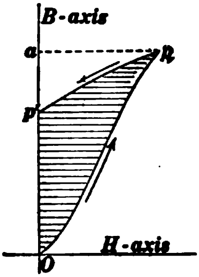

Fig. 156.

That area in Fig. 156 represents work may be shown as follows: Abscissas represent values of H to scale, so that we may write:

$$H = ax \tag{i}$$

Ordinates represent values of B to scale, so that we may write:

$$B = by \tag{ii}$$

or

$$dB = b \cdot dy$$

Substituting these values of H and dB in equation (49), we have

$$W = \frac{abV}{4\pi} \int x \cdot dy \tag{51}$$

in which a is the number of units of H represented by one unit of abscissa, and b is the number of units of B represented by one unit of ordinate in Fig. 156. Now $\int x \cdot dy$ is the area between any portion of the B and H curve and the y-axis. Therefore, $abV/4\pi$ is the number of ergs of work represented by each unit of area between the B and H curve and the y-axis.

109. Magnetic hysteresis. The magnetic cycle.—The divergence of the B and H curve for increasing values of H from the B and H curve for decreasing values of H is called *magnetic hysteresis*; or, rather, the tendency of iron to retain a previous magnetic state which is the cause of this divergence is called magnetic hysteresis. One effect of magnetic hysteresis is that the work regained when iron is demagnetized is less than the work which must be spent to magnetize the iron, as pointed out in Art. 107.

The magnetization of a given portion of the iron core of a transformer is reversed with each reversal of the alternating current which flows through the primary coil, the iron is thus repeatedly carried from a certain degree of magnetization in one direction (a certain positive value of B) to the same degree of magnetization in the opposite direction (the same negative value of B) and back to the original degree of magnetization. Such a magnetic double-reversal is called a *magnetic cycle*. At the end of a cycle the iron comes to precisely the same condition that it had at the beginning of the cycle. The importance of the magnetic cycle in the determination of the amount of heat produced by magnetization will be appreciated from the following two statements:

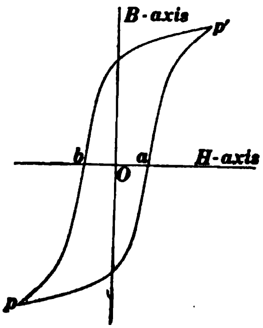

Fig. 157.

(*a*) When a mass of iron is magnetized along the B and H curve op of Fig. 156 and then partially demagnetized along the curve pp', a portion of the work done upon the iron during the first stage op is regained during the stage pp', a portion is lost as heat, and a portion remains in the iron as energy of magnetization; and no experimental method has been devised for determining the second and third portions of work separately.

(*b*) When, however, a mass of iron is carried through a magnetic cycle, the algebraic sum of the work spent upon the iron during the cycle is lost as heat, inasmuch as the magnetic energy in the iron is exactly the same at the beginning and at the end of the cycle.

Figure 157 shows the relation between B and H during a complete magnetic cycle. The total work spent on the iron is given by the value of the integral

$$\frac{abV}{4\pi}\int x \cdot dy \qquad \text{(i)}$$

extended over the whole cycle; but the value of $\int x \cdot dy$ extended over the whole cycle is the area enclosed by the B and H curve. Therefore, the total energy, in ergs, lost in V cubic centimeters of iron per magnetic cycle, is equal to $abV/4\pi \times$ area enclosed by the B and H curve. This energy loss is called the *hysteresis loss*, and it is all converted into heat. The meanings of a and b are explained in Art. 108.

The hysteresis loss per cycle increases with the range of flux density, and it may be expressed with sufficient accuracy for most practical purposes by the empirical equation:

$$W = \eta V B^{1.6} \qquad (52)$$

which is due to Steinmetz. In this equation W is the loss of energy in ergs per cycle, V is the volume of the iron in cubic centimeters, $\pm B$ is the range of flux density during the cycle, and η is a coefficient which is nearly constant for a given kind of iron or steel. The following table gives the approximate values of η for different kinds of iron and steel.

TABLE.

VALUES OF HYSTERETIC COEFFICIENT η.

Best quality of sheet iron for transformer cores, annealed	0.0015
Sheet iron for armature cores, annealed	0.003
Ordinary sheet iron, annealed	0.004
Soft annealed cast iron	0.008
Soft machine steel	0.0095
Cast steel	0.12
Hardened steel	0.25

PROBLEMS.

84. The intensity of the magnetic field in the air gap between the pole face and the armature core of a dynamo is 3,500 gausses and the field is at right angles to pole face and armature surface. The distance across the air gap is 3/8 inch. Find the magnetomotive force across the air gap in gilberts and in ampere-turns. Ans. 3,333.5 gilberts or 2,650 ampere-turns.

85. Find the magnetomotive force in gilberts and in ampere-turns along a verticle line 10 meters long at a place where the intensity of the earth's magnetic field is 0.56 gauss and its dip is 72°. Ans. 533 gilberts or 424 ampere-turns.

86. A slim rod 25 centimeters long is made into a link which passes through a coil of 50 turns of wire in which a current of 15 amperes is flowing. Find the average value along the rod of the component parallel to the rod of the magnetic field due to the coil. Express the result in gausses. Ans. 37.7 gausses.

87. An iron ring is wound with 1,000 turns of wire, and the wire has a current of 2 amperes flowing in it. The ring has an air gap in it and the distance across the gap space is 0.75 centimeter. Find the magnetic field intensity in the gap space assuming that the magnetomotive force of the winding is all concentrated on the gap space. Ans. 3,350 gausses.

88. A fiber tube 3 inches in diameter and 5 feet long has 1,500 turns of insulated wire wound evenly upon it in one layer. Find the intensity of the magnetic field inside of the coil when a current of 15 amperes flows through the coil.

89. How much flux passes through the opening of the coil which is specified in the previous problem when a current of 15 amperes flows through the coil?

90. Find the inductance in henrys of the coil specified in problem 88.

91. The choke coil of a lightning arrester (see Fig. 89) is an open coil (having no iron) 3 inches in diameter and a foot long, and it has 24 turns of wire. Find the approximate inductance of the coil in henrys, using the formula for an indefinitely long coil. How fast does the current grow in this coil when a lightning stroke causes an electromotive force of 125,000 volts to act on the coil? Ans. 11,550 million amperes per second.

92. An iron rod 2 centimeters square and 20 centimeters long is magnetized to an intensity of 1,000 units pole per square centimeter section when it is placed in a region which, but for the action of the poles of the rod, would be a uniform field parallel to the rod and of an intensity of 102 gausses. Assuming the poles of the rod to be concentrated at its ends, calculate the net magnetizing field at the center of the rod. Ans. 22 gausses.

93. Find the total magnetic flux through the middle part of the rod specified in problem 92. Ans. 50,353.6 maxwells or lines.

94. A bar magnet of hard steel is 2 centimeters square and 20 centimeters long, and the strength of each pole of the magnet is 2,000 units. The magnet is placed in a region which, but for the presence of the magnet, would not be a magnetic field. Find the total magnetic flux through the middle part of the magnet. Ans. 24,972.8 maxwells.

Note. In this case the net magnetizing force, H, at the middle of the bar is a demagnetizing force, and the flux due to H is subtracted from $4\pi m$.

95. A coil precisely like the one specified in problem 88 is wound on an iron rod 3 inches in diameter. Let us assume that the permeability of the iron is constant and equal to 1150. On this assumption the flux through the opening of the coil will be proportional to the current in the coil so that the coil will have a definite inductance as explained in Art. 48. Calculate the value of this inductance in henrys.

96. A transformer has a sheet iron core of which the uniform sectional area is 120 square centimeters. The mean length of the magnetic circuit formed by the core is 100 centimeters. How much current must be passed through a winding of 500 turns of wire to produce a magnetic flux of 1,767,000 lines through the core? Ans. 3.88 amperes.

97. How much flux will be produced through the wrought iron core specified in problem 96 by a current of 8.2 amperes through the 500 turns of wire? Ans. 1,920,000 lines.

98. The magnetic circuit of a dynamo consists of wrought iron (in armature core and in field cores), cast iron (in field yoke and pole pieces), and air gap. The wrought iron portion is 50 centimeters long and 120 square centimeters in sectional area, the cast iron portion is 40 centimeters long and 220 square centimeters in sectional area, and the air portion is 2.5 centimeters long and 300 square centimeters in sectional area. How many ampere-turns are required to force 1,600,000 lines of flux through this circuit, ignoring magnetic leakage? Ans. 12,100 ampere-turns.

Note. On account of magnetic leakage the magnetic flux through field yoke, field cores and pole pieces is larger, sometimes very much larger, than the flux which flows across the air gaps and through the armature core. Therefore a slightly greater magnetomotive force is required to overcome the magnetic reluctance of field yoke, field cores and pole pieces than would be required if the magnetic leakage were zero. Magnetic leakage is to be ignored, however, in the solution of the problem.

99. How much magnetic flux would be forced through the magnetic circuit specified in problem 98 by a magnetomotive force of 13,500 ampere-turns? Ans. 1,723,000 lines.

Note. This problem is to be solved by calculating a series of values of ampere-turns for various assigned values of flux. These results are to be plotted. The flux corresponding to 13,500 ampere-turns may then be taken from this curve.

100. (*a*) Find the work in ergs spent in magnetizing a wrought iron bar 3 inches square and 20 inches long from a neutral condition to $B = 16{,}000$ lines per square centimeter, using the tabular values of B and H given in Art. 103. (*b*) Find the work in ergs required to magnetize a cast-iron rod of the same size from a neutral condition to $B = 9{,}000$ lines per square centimeter. Ans. (*a*) 32.6×10^6 ergs. (*b*) 33.9×10^6 ergs.

Note. Plot the B and H curves from the tabular values given in Art. 103. Divide the areas between the curves and the B axis into a number of narrow strips, and calculate the area of each strip. Add these areas to get the total area.

101. A transformer core contains 96 cubic inches of the best quality of transformer iron. The core is carried through 60 magnetic cycles per second between the limits $B = \pm 3{,}500$ gausses, by means of an alternating current. Find the hysteresis loss in the core in watts. Ans. 6.63 watts.

CHAPTER VIII.

THE DIRECT-CURRENT DYNAMO AS A GENERATOR.

110. Bipolar and multipolar direct-current dynamos.—The simple dynamo which is described in Art. 18 has a bipolar field magnet and a ring-wound armature, whereas most direct-current dynamos, especially large ones, have multipolar field magnets and drum-wound armatures.

Figure 28 shows a two-pole dynamo. To drive this dynamo as a motor, current from an outside source is led into the armature winding at one point (at brush *a*) and out of the winding at one point (at brush *b*), as explained in Art. 18. Figure 158 shows current entering a ring winding at two points (at the two brushes *aa*), and leaving the winding at two points (at the two brushes *bb*). The current entering at each brush *a* divides so that there are *four paths* through the armature winding and *one quarter* of the total current *T* flows in each path. The flow of current in the outside armature conductors is indicated by the dots and crosses in Fig. 159; and, if this ring-wound armature with its four brushes is arranged as shown in Fig. 160,

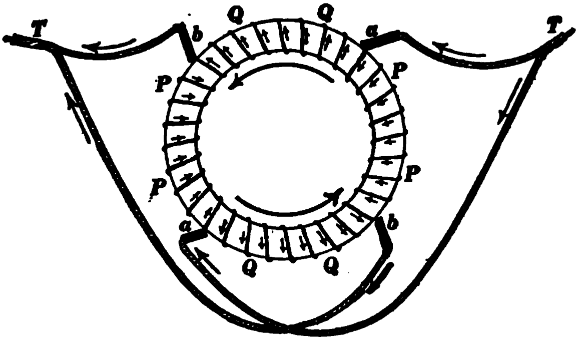

Fig. 158.

the side pushes of the magnetic field on all the armature wires will work together to turn the armature in the direction of the curved arrows. This arrangement is called a four-pole direct-current dynamo. If the armature in Fig. 160 is driven by a steam engine or water wheel, the machine becomes a generator; current flows out of the armature winding at the two brushes *aa*, and back into the winding at the two brushes *bb*. In this case the total current delivered by the machine is four times the current in any particular armature wire.

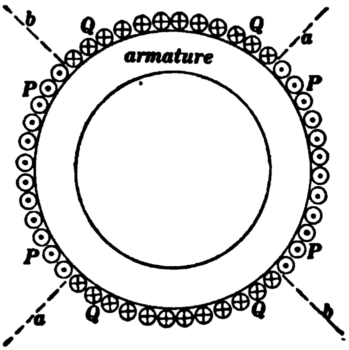

Fig. 159.

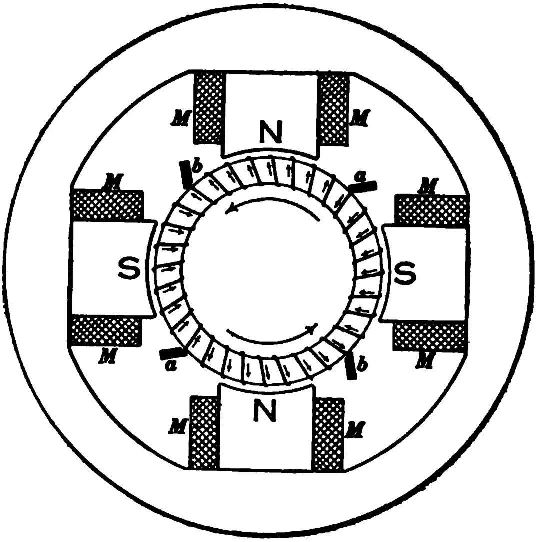

Fig. 160.

111. The ring-wound armature and the drum-wound armature.—The wire on a ring-wound armature passes from one end of the armature to the other on the *outside* of the ring and returns through the *inside* of the ring. In the drum-wound armature, however, the wire crosses over to the opposite* side of the armature and returns on the outside. The relation between the ring and drum windings may be understood from Fig. 161. Each wire *a* on the inside of the ring may be thought of as shifted over to the opposite side of the ring at *b* as shown. In this case it is evident that the wire at *b* must not make electrical contact with the lower brush otherwise the cross-over wires *c* and *d* would be short-circuiting connections from brush to brush.

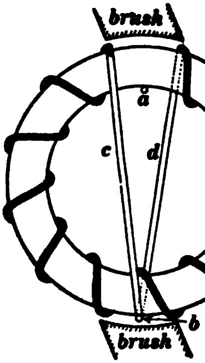

Fig. 161.

Showing relation between ring and drum armature windings.

Every wire on the outside of a ring armature may be a commutator bar or be connected to a commutator bar; whereas every second conductor on a drum armature may be a commutator bar or be connected to a commutator bar. The maximum number of commutator bars is therefore Z for the ring winding and $Z/2$ for the drum winding, Z being the number of conductors on the outside of the armature. In any case the number of commutator bars may be reduced to 1/2, 1/3, 1/4, etc., of the maximum possible number, and the portion of the armature winding between the points of attachment of two adjacent* commutator bars is called a *section* of the armature winding. The sections of an armature winding are usually wound separately on a form, bound with a winding of tape, and impregnated with insulating compound; and these sections or coils are laid in the armature slots and fastened usually by wooden wedges; the ends of each coil being soldered to two adjacent† commutator bars.

112. The shunt dynamo.—In one type of direct-current dynamo the field winding consists of many turns of comparatively fine wire, the winding has a comparatively high resistance, the terminals of the winding are connected directly to the brushes of the machine, and from 2 to 10 per cent. of the permissible‡ current output of the generator flows through the winding and excites the field magnet, the remainder of the permissible output being available for use in the external circuit. In this case the field winding and the outside circuit (the receiving circuit) are in parallel with each other between the brushes so that the field winding is in the relation of a shunt to the receiving circuit. A direct-current dynamo with its field windings arranged in this way is called a *shunt dynamo.* Figure 162 shows the arrangement of a shunt dynamo, and Fig. 163 is a simple diagram showing the connections.

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* This refers to a drum winding which is to be used with a two-pole field magnet. A slight modification adapts the drum winding to a multipolar field magnet.

† Referring to the simplex lap winding. See Appendix C.

‡ When a dynamo electric generator delivers an excessive current the machine becomes dangerously hot. The largest permissible current is that for which the rise of temperature is not sufficiently great to endanger the insulation of the ma-

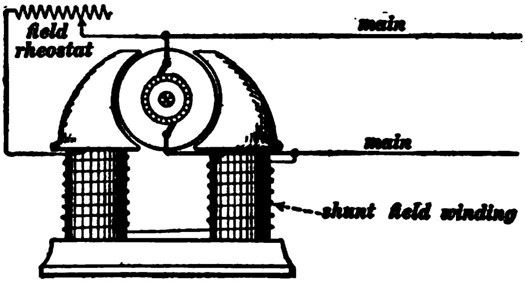

Fig. 162.
Shunt dynamo.

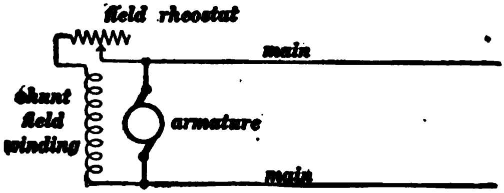

Fig. 163.
Shunt dynamo diagram.

The field rheostat, as shown in Figs. 162 and 163, is an adjustable resistance in circuit with the field windings. By adjusting the field rheostat, more or less current can be made to flow through the field windings, thus strengthening or weakening the

chine. In some cases sparking at the commutator limits the output of a dynamo electric generator.

field magnet and increasing or decreasing the electromotive force of the machine.

113. The series dynamo.—In the series dynamo the field winding consists of a few turns of coarse wire, the winding has a low resistance, the winding is connected in series with the receiving circuit so that the whole current output of the machine flows through it, and from 2 to 10 per cent. of the electromotive force developed by the machine is used to overcome the resistance of the field winding, the remainder being available for forcing the current through the external circuit (receiving circuit). A direct-current dynamo with its field windings arranged in this way is called a *series dynamo*. Figure 164 shows the arrangement of a series dynamo, and Fig. 165 is a simple diagram showing the connections.

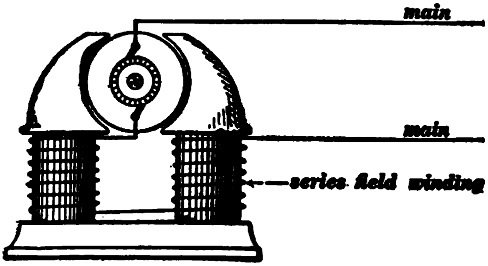

Fig. 164.
Series dynamo.

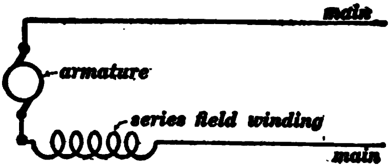

Fig. 165.
Series dynamo diagram.

114. The compound dynamo.—Most direct-current generators as used in practice have two distinct field windings, namely (*a*) a shunt winding of fine wire which is connected between or across the terminals of the machine, and (*b*) a series winding of coarse wire through which the entire current output of the machine flows. A direct-current dynamo with its field windings connected in this way is called a *compound dynamo*. The shunt winding usually supplies the greater part of the field excitation and *the series winding gives an increase of field excitation with increase of current output.* Figure 166 shows the arrangement of a compound generator, and Figs. 167*a* and 167*b* are simple diagrams showing the connections.

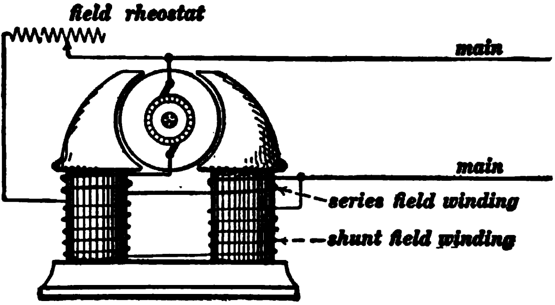

Fig. 166.

Compound dynamo.

Multipolar field windings.—Figures 162, 164 and 166 show bipolar dynamos for the sake of simplicity. The various coils or spools of a multipolar field winding are connected as indicated in Fig. 168. Whenever there is question as to the proper connection of the field coils the magnetic polarity of each field magnet pole may be determined by means of a compass needle.

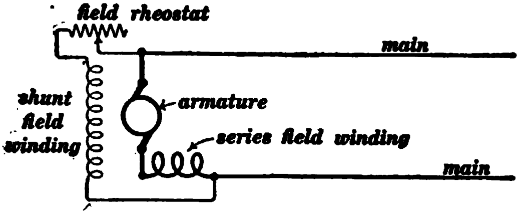

Fig. 167*a*.

Compound dynamo diagram. Long-shunt connection.

115. The fundamental equation of the direct-current generator.—Equations (13) and (14) of Chapter III give the electromotive force which is induced in the armature winding of a two-pole direct-current dynamo as explained in Art. 38. This induced electromotive force opposes the flow of current through the armature of a motor, but in the case of a generator this induced electromotive force enables the machine to deliver current to a receiving circuit. Following is a derivation of equations (13) and (14) differing from the derivation given in Art. 38.

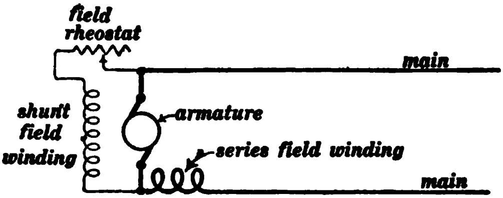

Fig. 167*b*.

Compound dynamo diagram. Short-shunt connection.

Consider a single armature wire while it moves from a to b in Fig. 27. The time occupied by this movement is $1/2n$ of a second, where n is the speed of the armature in revolutions per second; and the amount of magnetic flux cut during the movement is the flux Φ which enters the armature core from the N-pole of the field magnet. Therefore the average rate of cutting lines of force (flux) by the single armature wire is $\Phi \div (1/2n)$ or $2n\Phi$ lines per second, and this is the average value of the electromotive force induced in the single wire while it is moving from a to b. *Now it is evident that the average value of the induced electromotive force in the single wire while it is moving from a to b is equal to the average value of the induced electromotive forces in all the separate individual wires between a and b* (on left side of armature) *at any given instant.* Therefore the

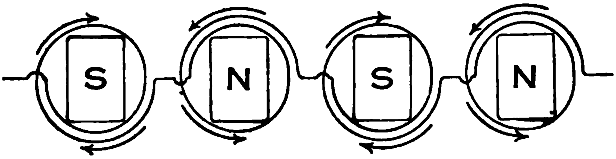

Fig. 168.

average value of the induced electromotive force in the individual wires between a and b (on left side of armature) is equal to $2n\Phi$. But the wires between a and b (on left side of armature) are all in series with each other as may be seen from Fig. 28 and the number of these wires is $Z/2$, where Z is the number of wires on the outside of the armature. Therefore the total electromotive force induced in the left side of the armature winding in Fig. 28 is $2n\Phi \times Z/2$ or ΦZn abvolts. A similar argument shows that ΦZn abvolts is induced in the right side of the armature winding in Fig. 28.

Equation (13) is here derived for a two-pole dynamo with a ring-wound armature; but the general form of the equation, applying to any direct-current dynamo, is

$$E_{\text{in abvolts}} = \frac{p}{p'} \cdot \Phi Zn \tag{53}$$

or

$$E_{\text{in volts}} = \frac{p}{p'} \cdot \Phi Zn \div 10^8 \tag{54}$$

where p is the number of field magnet poles, p' is the number of paths in the armature winding through which current flows from one brush or brush set to the other brush or brush set, Φ is the amount of magnetic flux entering the armature core from one north pole of the field magnet, Z is the number of wires laid on the outside of the armature, and n is the armature speed in revolutions per second. The operating engineer is seldom concerned with the separate values of p' and Z, and therefore for most practical purposes the factor $\frac{p}{p'} \cdot Z \div 10^8$ may be represented by the single letter Z' so that equation (54) becomes

$$E_{\text{in volts}} = \Phi Z'n \tag{55}$$

116. Terminal voltage of a direct-current generator.—Let $\Phi Z'n$ be the electromotive force induced in the armature, I_a

the armature current, R_a the armature resistance and E_t the voltage across the brushes (*terminal voltage*). Then

$$E_t = \Phi Z'n - R_a I_a \tag{56}$$

This is evident when we consider that $R_a I_a$ is the electromotive force required to overcome the armature resistance according to Ohm's law.

The ordinates of the curves in Fig. 169 show how the terminal voltages of the various types of direct-current generators change with increase of current delivered by the respective generators to a receiving circuit, the generator being in each case driven at constant speed, field rheostats when such exist being untouched.

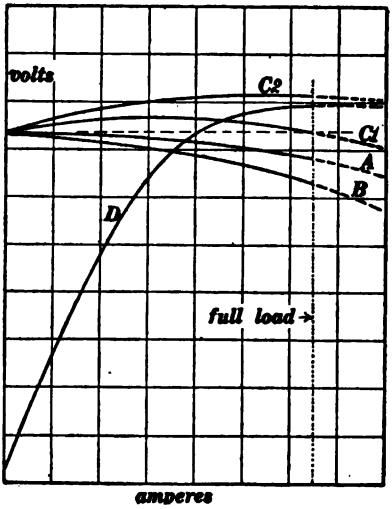

Fig. 169.

Dotted portions of curves correspond to over-load. A Separately excited generator; B Shunt generator; C_1 Flat compound generator; C_2 Over-compound generator; D Series generator.

A. The separately excited generator.—The field windings of a generator are supplied with constant current from an outside source. In this case the field excitation is invariable, but the armature flux decreases slightly with increase of current output because of the demagnetizing action of the current in the armature windings. Therefore the terminal voltage of a separately excited generator falls off for two reasons as the current output increases, namely, (*a*) Because of the slight decrease of Φ which decreases $\Phi Z'n$, and (*b*) Because of the increase of $R_a I_a$. The ordinates of the curve A, Fig. 169, show the decreasing terminal voltage of a separately excited generator with increase of current output.

B. The shunt generator.—The decrease of E_t as explained under A leads to a decrease of field excitation of a shunt generator, and therefore the terminal voltage of a shunt generator falls off more rapidly with increase of current output than does the terminal voltage of a separately excited generator; see curve B, Fig. 169.

C. The flat-compounded generator.—When the current output of a compound generator increases, a very considerable increase of field excitation is produced by the series field coils. When this increase of field excitation is just enough at full load to bring the terminal voltage up to its zero-load value, we have what is called *flat compounding* as shown by the curve $C1$ in Fig. 169. When this increase of field excitation raises the full-load terminal voltage above its zero-load value, we have what is called *over-compounding*, as shown by the curve $C2$ in Fig. 169.

D. The series generator.—When the current output of a series generator is zero the residual magnetism of the field produces a very small terminal voltage, and as the current output increases the field excitation increases so that the terminal voltage rises rapidly as shown by the curve D in Fig. 169.

117. Constant-voltage and constant-current supply.—It is important at this point to consider the two conditions under which a number of distinct receiving units (lamps or motors) may be operated *independently* of each other, so that any lamp or any motor may be put into or taken out of service at will without affecting the other lamps or motors.

(*a*) When a number of lamps, or motors, or both, are connected in parallel with each other across the supply mains, it is necessary to maintain a constant electromotive force between the mains in order that the various lamps and motors may operate independently of each other. This method of supply is called *constant-voltage distribution.* In this system a given lamp or motor is put out of service by disconnecting it from the mains, that is, by simply opening its circuit. When a lamp or motor is put out of service in this way, each remaining lamp or motor takes the same current as before (voltage between mains being constant), and the generator delivers a total current which is less than before by the amount of current which was taken by the disconnected lamp or motor.

The constant-voltage system is almost universally employed nowadays both for direct-current distribution and for alternating-current distribution, and this system is exemplified by nearly every installation for the long distance transmission of power, by nearly every electric railway system, and by great numbers of plants for supplying current to motors and lamps in our cities.

(*b*) When a number of lamps are connected in series, it is necessary to maintain a constant current in the circuit in order that the various lamps may operate independently of each other. This method of supply is called *constant-current distribution.* In this system a lamp is put out of service, not by breaking its circuit, but by short-circuiting it, that is, by closing a switch which establishes a by-pass for the current. The closing of this switch generally reduces the resistance of the circuit of lamps as a whole, and therefore the maintenance of a constant current in the circuit requires a decrease of the electromotive force of the generator when lamps are put out of service and an increase of the electromotive force of the generator when additional lamps are put into service.

The constant current system is advantageous when a fixed number of widely-distributed lamps or motors are to be operated. The lamps or motors are connected in series and supplied with constant current. This system is exemplified by many street-lighting plants in our cities, and by the Thury* system of power transmission.

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* The Thury system (direct-current) is a constant-current system of distribution. It is exemplified by several large, long-distance power-transmission plants in Europe. A description of the Thury system is given by Wieshofer in the *Zeitschrift für Electrotechnik* (Vienna), Vol. XVI, pages 5–10, 1898. See also a paper by Cuénod and Thury in the *Bulletin de la Sociétté Internationale des Électriciens,* Vol. XVII, pages 9–93, January, 1900. See also an article by B. W. Baum, *General Electric Review,* November 1915, Vol. XVIII, page 206.

The constant-current system of distribution is but little used, and where it is used it is nearly always, in American practice, an auxiliary to a large constant-voltage system of distribution. Special generators for delivery of constant current are seldom or never installed.

118. Voltage regulation of a direct-current generator.—*Voltage control* means the deliberate adjustment of a device for bringing the voltage of a generator to a desired value. *Voltage regulation,* as this term is now used by electrical engineers, refers to the departure of the voltage of a generator from a desired value when the generator is left to itself and its load (current output) changes.

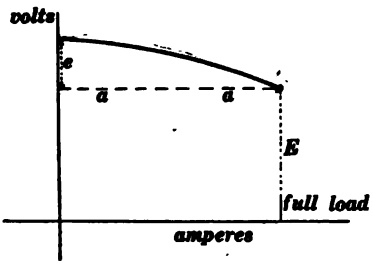

Fig. 170.

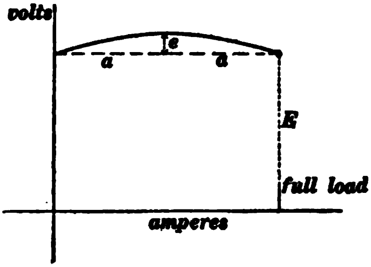

Fig. 171.

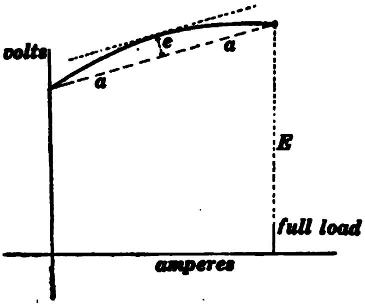

Fig. 172.

Percentage regulation of the shunt generator.—A shunt generator, operating at full load and normal terminal voltage E, has its current output reduced to zero, speed of driving and resistance of field circuit unchanged, and the terminal voltage rises to $E + e$ as indicated in Fig. 170. The percentage voltage regulation is $100e/E$.

Percentage regulation of the flat-compounded generator.—An ideal flat-compounded generator would give an unvarying voltage in changing from full load to zero load. The maximum deviation from this ideally constant voltage is the voltage indicated by e in Fig. 171, and $100e/E$ is called the percentage voltage regulation of the machine.

Percentage regulation of an over-compounded generator.—The terminal voltage of an ideal over-compounded generator would decrease from its full load value E along the dotted line aa in Fig. 172. The maximum deviation from this ideal voltage is represented by e in Fig. 172, and $100e/E$ is called the percentage regulation of the machine.

119. The building up of a shunt or compound generator at starting.*—When a shunt or compound generator is started, the residual magnetism in the field structure induces a small electromotive force in the armature windings, this electromotive force produces a small current in the shunt field winding which *strengthens* the field magnetism thus producing a larger current in the shunt field winding which further strengthens the shunt field current, and so on until the machine is in full action with the iron parts of the field and armature near magnetic saturation. This process is called *building up*, and it usually takes half a minute or more for this building up process to bring a machine to full voltage.

A generator may build up so that either of its brushes or brush sets is positive, *according to the direction of the residual magnetism.* A machine which is repeatedly started and stopped usually builds up in the same direction, but an outside disturbance such as a momentary current from a lightning discharge may reverse the residual magnetism so that at the next starting the machine builds up in a reversed direction.

Failure to build up.—It sometimes happens that the field current produced by the residual magnetism *weakens* the residual magnetism. Under these conditions the machine cannot build up at all; and to reverse the residual magnetism does not overcome the difficulty, because the reversed magnetism produces a reversed field current, and this reversed field current weakens the reversed field magnetism. To make it possible for the machine to build up the field connections must be reversed or the direction of driving must be reversed.

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* The series dynamo is seldom used as a generator in present-day American practice.

120. Rating specifications of direct-current generators.*—The manufacturers always specify the more important conditions under which a generator is to be operated, namely, speed of driving, normal terminal voltage, normal full-load output of current or power, and a certain permissible amount of overload and the time it may be taken from the machine without dangerous heating. Also the kind of field excitation, and, in case of a compound generator, the degree of over-compounding is always specified. Thus a machine might be spoken of as a 500–550 volt over-compounded generator to be driven at a speed of 300 revolutions per minute with a normal full load of 100 kilowatts and an overload capacity of 50 per cent. for one hour.

121. Connections of a shunt or compound generator and its receiving circuit.—Figure 173 is a diagram of connections of a compound generator (short-shunt) with its field rheostat *R*, main fuses at *F* and *F*, main switch *M*, ammeter *A*, voltmeter *V*, and circuit-breaker *BB*. These accessory appliances

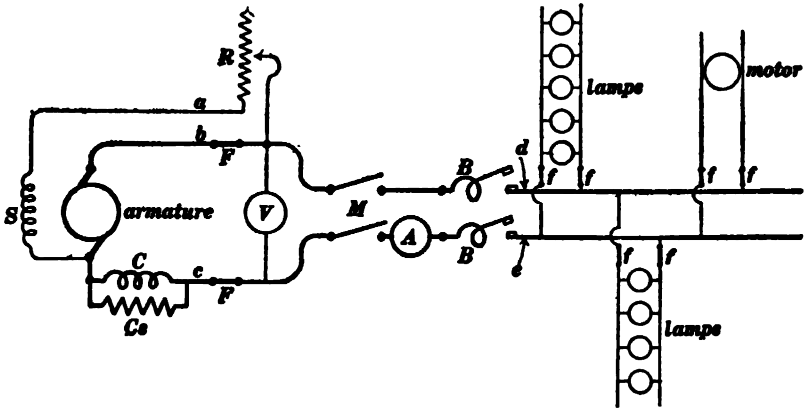

Fig. 173.

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* See Standardization Rules of the American Institute of Electrical Engineers for full information concerning the important topic of rating.

are always mounted in convenient positions on a panel of insulating material, slate or marble, which is called a *switchboard.* The wires *a*, *b* and *c* lead from the generator to the switchboard, and the wires *d* and *e* lead from the switchboard to the street mains. The shunt field winding of the generator is shown at *S*, the series field winding at *C*, and *Cs* is an adjustable shunt by means of which a certain fraction only of the current output of the generator may be made to flow through the series field winding, as explained in the note to problem 116.

The wires leading to the individual groups of lamps and to the motor in Fig. 173 are of course smaller than the mains leading out from the switchboard. Fuse links *ff* should be inserted at every point where a smaller wire attaches to a larger one,* otherwise if a short-circuit should occur in a small group of lamps, enough current might flow to make the smaller copper wires red hot, but not enough current to open the automatic circuit breaker *BB* or to melt the main fuses *FF*, thus giving rise to a series fire risk.

122. The operation of shunt and compound generators in parallel.—A generator operates at its maximum efficiency at or near full load. Therefore when the total output of a station varies from hour to hour, as it always does in electric-lighting and street-railway service, it is desirable to use several generators which may be put into service one after the other as the station output increases, and disconnected one after the other as the station output decreases; the object being to maintain nearly full load at all times upon the generators which are in service. This is especially desirable when each generator is driven by a separate engine, inasmuch as the engines also may then be operated under the most economical conditions. This use of a number of generating units in a station is also advantageous in that a spare unit (engine and generator) may be installed at a moderate cost to serve in case of a breakdown, whereas a single-unit station would have to be completely duplicated in order to provide for such an emergency.

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* This is one of the standard fire insurance rules. The engineer who has to do with the installation of any kind of electrical service should be familiar with the *National Electrical Code* which is published by The National Board of Fire Underwriters, General Agency, 34 Nassau St., New York City.

When several shunt or compound generators are installed in a station as above explained, they are always operated in parallel.*

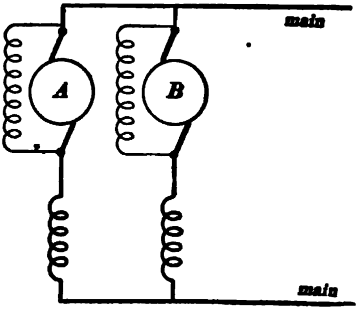

Fig. 174.

Impracticable arrangement.

Two or more shunt generators connected in parallel would operate satisfactorily, that is to say the station load would be shared properly by several shunt generators in parallel, but compound generators (especially over-compound generators) cannot be operated in parallel as indicated in Fig. 174. A slight increase of current through one machine increases its voltage, because of increase of series field excitation, thus causing the machine to deliver more and more current. To obviate this

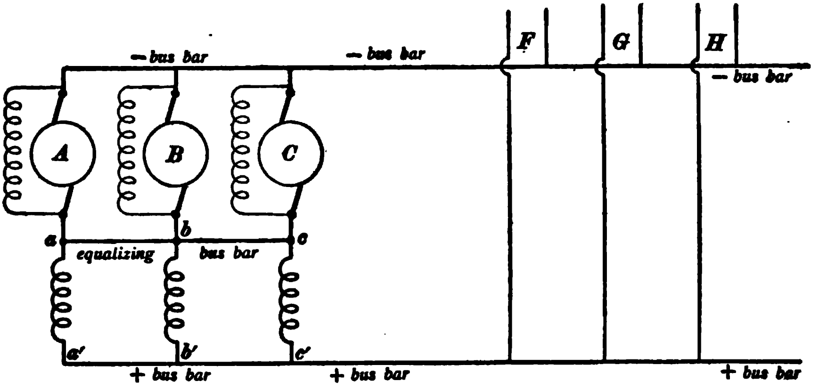

Fig. 175.

Practicable arrangement.

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* When the equipment of a central power station consists of a number of alternating-current generators, the generators are nearly always operated in parallel. See Chapter XIII.

difficulty and ensure a proper division of station load among two or more compound generators the arrangement shown in Fig. 175 must be used. The *equalizing bus bar* makes the three points *a*, *b* and *c* as one, and the current in each series field coil is always a definite fraction of the total current output of the station.

Two distinct conditions must be satisfied by the arrangement shown in Fig. 175 to ensure the proper division of the load among the several generators, namely:

(*a*) Each compound generator must be separately adjusted to give the same degree of compounding.

(*b*) The resistances aa', bb' and cc', Fig. 175, must be inversely proportional to the full-load current outputs of the respective generators. This ensures the proper division of the total current among the several series-field coils.

The use of the equalizing arrangement makes it possible to operate in parallel compound generators which are very different in size and in design.

PROBLEMS.

102. The ring wound armature of a small bipolar direct-current dynamo contains 400 feet of No. 16 Brown and Sharpe copper wire. What is the resistance of the armature from brush to brush, neglecting resistance of brushes and of brush contacts? Ans. 0.4 ohm at 20° C.

103. The above ring wound armature is arranged with six brush sets for use with a six-pole field magnet in a manner similar to Fig. 160. What is the resistance of the armature between brush sets, neglecting resistance of brushes and brush contacts? Ans. 0.0444 ohm at 20° C.

104. The electromotive force between the terminals of a shunt generator is 120 volts, and the resistance of the shunt field winding is 22 ohms. How much current flows through the shunt field winding? If the generator delivers 80 amperes to a group of lamps, how much current is delivered by the armature? Ans. (*a*) 5.45 amperes; (*b*) 85.45 amperes.

Note. See note to problem 106.

105. The resistance of the field winding of a series dynamo is 0.08 ohm, the dynamo, operating as a generator, delivers 80 amperes, and the electromotive force between the brushes is 125 volts. What is the electromotive force between the terminals of the machine, that is, between the points of attachment of the external circuit? Ans. 118.6 volts.

Note. The series dynamo is seldom used as a generator in practice.

106. Find the power expended in field excitation in the two cases specified in problems 104 and 105. Ans. (*a*) 654 watts; (*b*) 512 watts.

Note. After the current in the field winding and the "strength" of the field magnet become steady in value, all of the power delivered to the field winding reappears as heat in the winding in accordance with Joule's law. Therefore Ohm's law applies to the field winding. No power would be required to maintain the magnetism of the field magnet if a field winding of zero resistance could be obtained. When, however, the field magnet *is being magnetized* (during a small fraction of a second at the beginning) then some of the power delivered to the field winding does not reappear as heat in accordance with Joule's law, but is used to establish the magnetism. Thus if E is the voltage across the terminals of a magnet winding, I the current in the winding, and R the resistance of the winding, then EI is the rate at which work is delivered to the winding and RI^2 is the rate at which heat is developed in the winding. Now the ultimate value of I is E/R, and when this ultimate value of I is reached we have $EI = RI^2$, but before I reaches this ultimate value EI is greater than RI^2 and the excess is the power which at each instant is being used to establish the magnetism.

107. The ring wound armature of a bipolar direct-current generator has 300 turns of wire ($Z = 300$). The armature is driven at a speed of 1200 revolutions per minute, and the terminal voltage of the machine with negligible current flowing through the armature is 115 volts as measured by a voltmeter. Calculate the amount of magnetic flux which enters the armature core from the north pole of the field magnet. Ans. 1,917,000 lines.

108. Calculate the average electromotive force induced in a single armature wire of the generator specified in the previous problem while the wire is moving from a to b in Fig. 28. Ans. 0.7668 volt.

109. The armature of problem 107 is used at a speed of 1200 revolutions per minute with a 6-pole field magnet and the armature flux per field pole is 639,000 lines. What is the value of the induced electromotive force between brush sets? Ans. 38.3 volts.

Note. In practice certain standard voltages are almost universally used, and if the armature here specified were to be used with a six-pole field its winding would be modified to give, say, 110 volts.

110. The full-load current of the generator of problem 107 is 150 amperes. What is the full load current of the six-pole generator of problem 109?

111. A direct-current generator has its field separately excited (from an outside invariable source of current), and when the current output of the generator is increased from zero to 70 amperes the terminal voltage drops from 120 to 106 volts. The resistance of the armature including brush contacts and brushes is 0.12 ohm. The armature flux is 1,200,000 at zero load; what is the value of the armature flux at full load? What causes the decrease? Ans. 1,144,000 lines.

112. The generator of the previous problem is arranged as a shunt generator, its field rheostat is adjusted until the terminal voltage at no load is 120 volts, and the terminal voltage is then observed to decrease to 97 volts when the current output of the machine rises to 70 amperes. The armature flux is 1,200,000 lines at zero load; what is its value at full load? What causes the decrease? Ans. 1,054,000 lines.

Note. Strict accuracy of calculation requires a careful consideration of the shunt field current of, say, 3 amperes; because the armature current at zero load is 3 amperes and at full load the armature current is 73 amperes. It is desirable, of course, to adhere to strict arithmetical accuracy, but in this problem it is not worth while. This problem and also problems 113 and 114 are to be solved as if the armature current were zero at zero load, and equal to actual current output at full load.

113. The terminal voltage of a shunt generator at zero load decreases from 115 volts to 98 volts when the speed of the machine is reduced from 1000 to 900 revolutions per minute.

The armature flux at the higher speed is 1,000,000 lines. What is its value at the lower speed? What causes the decrease? Ans. 1,136,000 lines.

114. A shunt generator has a terminal voltage of 110 volts at zero load with a speed of 1200 revolutions per minute, and with a total resistance of 50 ohms in its shunt field circuit. What additional resistance must be inserted in the shunt field circuit in order that the terminal voltage of the machine may be 137.5 volts at zero load when its speed is increased to 1500 revolutions per minute? Ans. 12.5 ohms.

115. A given bipolar shunt generator has 3240 turns of wire in its shunt field winding. The machine gives a terminal voltage of 110 volts at zero load with 2.5 amperes in its field winding, and to get a terminal voltage of 110 volts at full load with the delivery of 50 amperes to a receiving circuit a current of 3.24 amperes is required in its field winding. Find the number of turns required in a series field winding to give flat compounding. Ans. 48 turns.

Note. The connections are supposed to be as shown in Fig. 167*b*, the short-shunt connection.

116. The generator specified in the previous problem requires a shunt field current of 3.905 amperes to bring its full-load terminal voltage up to 110 per cent. of 110 volts or 121 volts. Find the number of turns required in a series field winding (short shunt connection) to give 10 per cent. over-compounding. Ans. 78 turns.

Note. A few more than the required number of series field turns are always provided in compounding a generator, and the series field coil is then shunted with a German silver resistance, *Cs* Fig. 173, which is adjusted until the compounding is exactly correct.

117. The terminal voltage of a shunt generator rises from 110 volts at full load to 134.2 volts at zero load when the receiving circuit is disconnected. What is the percentage regulation of the machine?

118. A flat-compounded generator gives 110 volts between its terminals at full load, and when the load is slowly reduced the terminal voltage rises to a maximum of 114.4 volts and then falls to 110 volts at zero load. What is the percentage regulation of the machine?

119. Two dissimilar over-compounded direct-current generators are to be operated in parallel. The terminal voltage of the smaller machine rises from 500 volts at no load to 550 volts at full-load output of 100 amperes. The terminal voltage of the larger machine rises from 500 volts at no load to 550 volts at full-load output of 150 amperes. The short shunt connection is used as shown in Figs. 167*b* and 175. The resistance of the series field winding of the smaller machine is 0.04 ohm, and the resistance of the series field winding of the larger machine is 0.025 ohm. Additional resistance must be inserted with one or the other series field winding when they are connected as shown in Fig. 175. Which field winding must have this additional resistance and how much must the additional resistance be? Ans. 0.00167 ohm must be added to the series field winding of the larger machine.

Note. In practice this kind of calculation is never made. The two machines are connected as shown in Fig. 175, and resistance is inserted in one or the other field circuits and adjusted until the full station load is properly shared by the two machines.

CHAPTER IX.

THE DIRECT-CURRENT DYNAMO AS A MOTOR.

The alternating-current system of power distribution is coming to be almost universal for general lighting and power service, and therefore alternating-current motors are very widely used. Indeed in most cases direct-current motors cannot be used because direct-current supply is not available.

Direct-current motors are, however, used very extensively in electric railway service and in large shops even when the local system of power distribution uses alternating current; the direct-current being supplied by rotary converters or by independent power generating stations.

123. Connection diagrams of direct-current motors taking current from constant-voltage supply mains.—Figures 176, 177 and 178 are connection diagrams of the three types of direct-current dynamos operating as motors. As explained in Art. 37, it is necessary at starting to have an auxiliary resistance in circuit with the armature, and this auxiliary resistance is cut out (reduced to zero) step by step as the motor speeds up. This proceedure is necessary with series motors and compound motors as well as with shunt motors because the resistance of the series field winding is small, usually less than the armature resistance.

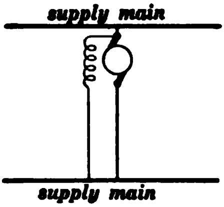

Fig. 176.

Shunt motor connections.

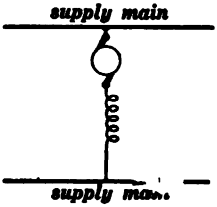

Fig. 177.

Series motor connections.

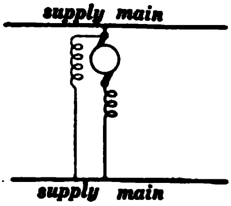

Fig. 178.

Compound motor connections.

The shunt motor.—Once started, the shunt motor runs at very nearly constant speed however its load may vary, unless its load becomes excessive. The alternating-current induction motor (polyphase and single phase) has the same constant-speed characteristic, and direct-current shunt motors *or alternating-current induction motors* are nearly always used where approximately constant speed is desired. The speed characteristics of the shunt motor are fully discussed in Art. 126.

The series motor must be permanently connected to its load because if it is unloaded its speed rises excessively and endangers the motor.

The important peculiarity of the series motor is that the starting current through the armature (which is usually about 150 per cent. of normal full-load current) flows also through the series field winding thus giving a very great field excitation at starting. The result is a very large starting torque as explained in Art. 125. This characteristic of the series motor adapts it to uses where starting and stopping are frequent but where a steady constant speed is not necessary. Thus the direct-current series motor is used almost universally for electric railways and electric elevators and hoists.

The single-phase alternating-current series motor has the same characteristics as the direct-current series motor and it is coming into use more and more for cranes and hoists where direct-current supply is not available.

The compound motor.—The shunt field winding of the compound motor usually predominates over the series field winding in its magnetizing action. When the series field winding *helps* the magnetizing action of the shunt winding we have what is called *cumulative compounding*, and when the series winding opposes the magnetizing action of the shunt winding we have what is called *differential compounding*. With cumulative compounding the speed of the motor falls off greatly with increase of load. With differential compounding the speed tends to rise with increasing load and actually does rise if the magnetizing action of the series winding is not too weak; but of course the speed drops if the load is excessive.

A use of differential compounding is mentioned at the end of Art. 126, and problem 130 illustrates a use of cumulative compounding.

Reversal of direction of running of a motor is accomplished by reversal of its field connections or by reversal of its armature connections. If connections of field and armature are both reversed the direction of running is left unchanged.

124. Fundamental equation of the direct-current dynamo as applied to a motor.—In the generator the *voltage* is determined by equation (56) inasmuch as all the other quantities in this equation are fixed in value by the conditions under which the generator is used. But in the motor it is the speed n which is determined by this equation, all other quantities in the equation being, as a rule, fixed in value by the conditions under which the motor is used. Therefore the most convenient form of this equation for present purposes is the form in which the speed is expressed in terms of the other quantities.

Let E_t be the electromotive force applied at the armature terminals of the motor (E_t is usually the voltage between the supply mains). This electromotive force is partly used to overcome the resistance R_a of the armature. The part so used is equal to R_aI_a where I_a is the current flowing through the armature. The remainder of E_t is used to overcome the back electromotive force $\Phi Z'n$ in the armature and is equal thereto. Therefore

$$E_t = \Phi Z'n + R_aI_a \tag{57}$$

Solving this equation for n, we have

$$n = \frac{E_t - R_aI_a}{\Phi Z'} \tag{58}$$

where n is the motor speed in revolutions per second, R_a is the resistance of the armature circuit (including, of course, the resistance of the series field winding if the motor is arranged as indicated in Fig. 177 or Fig. 178), I_a is the current flowing through the armature, Φ is the armature flux per field pole, and Z' is the factor $\frac{p}{p'} \cdot Z \div 10^8$ as explained in Art. 115.

125. The torque equation.—Another important motor equation is an expression for the torque exerted on the motor armature by the field magnet. This equation could be derived by considering the side push on each armature wire [see equation (5)], multiplying this side force by its lever arm, namely, the radius of the armature, and adding all such products together; but it is more instructive to derive the expression for torque as follows.

Multiplying both members of equation (57) by I_a, we get

$$E_tI_a = \Phi Z'n \cdot I_a + R_aI_a^2 \tag{59}$$

E_tI_a is the total power delivered to the armature circuit, $R_aI_a^2$ is the power lost in heating the armature circuit, and $\Phi Z'n \cdot I_a$ *is the power expended in turning the armature.* Now the power expended in turning a rotating body is always expressible as $2\pi nT$, where n is the speed of rotation and T is the driving torque. But $\Phi Z'n \cdot I_a$ is expressed in watts, so that $2\pi nT$ must be reduced to watts. If T is expressed in pound-feet, then $8.52nT$* is the power in watts. Therefore we have:

$$\Phi Z'n \cdot I_a = 8.52nT$$

or, solving for T, we get

$$T_{\text{in pound-feet}} = 0.1174\Phi Z'I_a \tag{60}$$

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* Let F be the force in pounds which pulls on the rim of a rotating wheel of which the radius is r feet, and let n be the speed of the wheel in revolutions per second. Then $2\pi r$ is the circumference of the wheel. A point on the circumference moves round the wheel n times per second, and therefore the velocity of the rim is $2\pi rn$ feet per second. Multiplying this velocity by F pounds we get $2\pi nrF$ foot-pounds per second as the rate at which work is done on the wheel. But rF is the torque T exerted on the wheel in pound-feet. Therefore $2\pi nT$ is the power in foot-pounds per second, or $\frac{746}{550} \times 2\pi nT$ is the power in watts, T being expressed in pound-feet.

in which I_a is the armature current in amperes, Φ is the armature flux per field pole expressed, of course, in maxwells or lines, and Z' is the factor $\frac{p}{p'} \cdot Z \div 10^8$ as explained in Art. 115.

The mechanical power actually delivered by the motor is less than $\Phi Z' n I_a$ because of friction and eddy current and hysteresis losses in the rotating armature, the so-called *stray power losses*; and the net available torque at the motor pulley is less than T as given by equation (60) by the amount of torque required to turn the armature and supply the stray power losses.

126. Speed characteristics of the shunt motor.—The speed characteristics of the shunt motor as described in Art. 123 may be most easily explained by first assuming the armature flux Φ to be constant (which it is, except for the small changes due to the magnetizing action of the armature current), and then considering the effects of moderate changes of Φ.

Approximate constancy of shunt motor speed. Equation (58) applies, of course, to any direct-current motor; but in the case of the shunt motor Φ is approximately constant and $R_a I_a$ is small in comparison with E_t, that is to say, only a small portion of the supply voltage E_t is used to overcome armature resistance. Therefore, neglecting $R_a I_a$, equation (58) becomes $n = E_t / \Phi Z'$, so that n is approximately constant.

A slightly different line of argument is as follows: The supply voltage E_t is nearly all used to overcome the back electromotive force $\Phi Z' n$, and therefore it is approximately equal thereto, that is, $E_t = \Phi Z' n$; so that, approximately,

$$n = E / \Phi Z'.$$

Decrease of speed with increase of load, Φ being assumed to be constant. It is most convenient to specify the motor load by giving the value of the armature current I_a; thus I_a is nearly zero at zero motor load, and I_a has a certain value at rated full load. The ordinates of the straight line AB in Fig. 179 represent speeds and the abscissas represent corresponding values of the armature current according to equation (58), Φ being taken as a constant. The line AB shows that the speed of a shunt motor drops in proportion to the armature current I_a as the motor load increases, Φ being assumed to be constant.

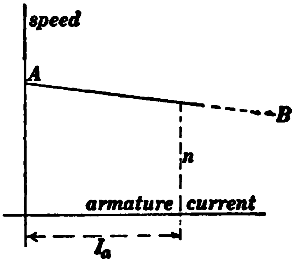

Fig. 179.

Effect of changes of Φ *on shunt motor speed.* When a shunt motor is connected to given supply mains (given value of E_t); the speed of the motor (which is approximately equal to $E_t/\Phi Z'$ as above explained) can be *lowered* by *increasing* Φ, that is by *increasing* the field excitation of the motor; or *raised* by *decreasing* the field excitation of the motor.

Change of Φ *due to magnetizing action of armature current, and its effect on the motor speed.* The magnetizing action of the current in the armature, under usual conditions of motor operation, opposes the magnetizing action of the field current. Therefore the value of Φ in a shunt motor decreases slightly with increasing I_a. This decrease of Φ tends to raise the motor speed as I_a increases, as above explained, whereas the increasing value of R_aI_a tends to decrease the motor speed as I_a increases, as shown in Fig. 179. In most shunt motors the effect of R_aI_a preponderates over the effect of decreasing Φ, and consequently the motor speed falls off with increasing I_a but not to the extent shown in Fig. 179.

The compound motor.—The magnetizing action of the shunt field winding of the compound motor is usually much larger than the magnetizing action of the series winding, and therefore the compound motor may be thought of as a modified shunt motor. With cumulative compounding the value of Φ increases with increase of motor load (increase of armature current I_a) so that the motor speed falls off greatly. With differential compounding the value of Φ decreases greatly with increase of motor load (increase of armature current I_a) so that the speed rises as the load increases; but excessive load must, of course, cause a drop in speed.

When extreme constancy of motor speed is desired, in spite of varying load, the slight falling off of speed of the shunt motor with increasing load may be corrected by a very weak differential compounding.

127. Speed control of the shunt motor.—The term *speed control* refers to the deliberate adjustment of the speed of a motor to any desired value, whereas, *speed regulation* refers to inherent changes of speed due changes of load. For example, the speed of a given shunt motor increases from 1000 revolutions per minute at full load to 1080 revolutions per minute at zero load. The increase of speed is 8 per cent. of full-load speed, and the motor is said to have 8 per cent. speed regulation.

There are three* important methods for controlling the speed of a shunt motor as follows:

(1) *Field-rheostat method of speed control.*—By adjusting a field rheostat the field excitation of a motor can be increased or decreased thus *increasing* or *decreasing* the armature flux Φ, and *decreasing* or *increasing* the speed of the motor as explained in Art. 126. This method of speed control is highly satisfactory in that the motor speed remains nearly constant for all loads for a given position of the field rheostat (given amount of field excitation), but the method is limited as follows: (*a*) It is not feasable to increase the armature flux Φ beyond a certain value on account of the magnetic saturation of the iron in the field magnet, and (*b*) When the field excitation is greatly reduced the magnetizing action of the field current no longer preponder-

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* A method now falling into disuse is to vary the armature flux Φ by introducing an adjustable air gap (preferably the air gap between armature and pole faces) in the magnetic circuit of the machine. Another, the Ward-Leonard method, is to use a motor-driven direct-current generator to supply current at any desired voltage to the variable speed motor, the voltage being controlled by a rheostat in the field circuit of the generator. The field winding of the motor is in this case connected to the primary source of supply, and the variable-voltage supply is connected to the motor armature.

ates over the magnetizing action of the armature current, and excessive sparking takes place at the brushes.

A shunt motor which runs at speed n when its field magnet is near saturation (largest feasible value of Φ) will operate fairly well down to a value of Φ equal to about half of the saturation value, and the corresponding speed is, of course, about $2n$. Sparking at the brushes is to a great extent eliminated by the use of *commutating poles*, or *interpoles*, as explained in Chapter X, and an interpole shunt motor which runs at speed n with largest feasible value of Φ (field magnet near saturation) will operate without serious sparking down to a value of Φ equal to about a quarter of the saturation value, and the corresponding speed is, of course, about $4n$.

When the armature flux is reduced so as to obtain increased speed as above explained, the driving torque T for any given value of I_a is reduced in proportion to the reduction of Φ, according to equation (60). Thus when the motor speed is doubled by halving Φ, the full-load torque (corresponding to full-load value of I_a) is halved.

(2) *Armature-rheostat method of speed control.*—For any given motor load (given value of I_a) any desired portion of the supply voltage E_t may be used up in an adjustable rheostat in the armature circuit. Let R be the fairly large resistance thus placed in the armature circuit (including the resistance of the armature itself). Then RI_a is the electromotive force lost in the rheostat, and the remainder $E_t - RI_a$ overcomes the back electromotive force $\Phi Z'n'$ and is equal thereto. Now at normal full speed, n, the back electromotive force $\Phi Z'n$ is approximately equal to the supply voltage E_t because the armature resistance is small, and when a large part of the supply voltage is used up in the rheostat the remainder

$$E_t - RI_a(= \Phi Z'n')$$

is small and the speed, n', is correspondingly small.

This method of speed control has an unlimited range from normal speed to zero speed, but it is unsatisfactory for two reasons, namely, (*a*) A very considerable amount of power, RI_a^2, is lost in the armature rheostat, and (*b*) For a given adjustment of the armature rheostat (given value of R), the lost electromotive force RI_a varies with the current so that the motor speed varies greatly if its load (the value of I_a) changes.

(3) *Multivoltage method of speed control.*—Figure 180 shows four mains which supply direct-current at 60 volts, 120 volts or 240 volts. A shunt motor may have its shunt field winding permanently connected, say, to the 120 volt mains, and the armature may be arranged to take current at 60 volts, at 120 volts or at 240 volts; then the corresponding motor speeds will be n, $2n$ and $4n$ respectively, n being the speed when the armature is connected to the 60-volt supply mains. To obtain intermediate speeds an adjustable field-rheostat may be used as described under (1). This combination of the field-rheostat method and the multivoltage method gives a very wide range of satisfactory speed control, and it is used to some extent in shops where a large number of variable speed motors are to be operated.

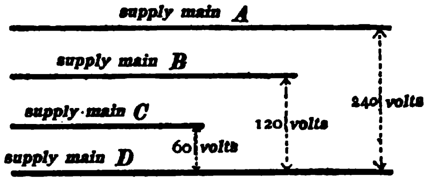

Fig. 180.

128. The shunt motor starting box with automatic overload and dead-line releases.—The most frequent sources of trouble in the operation of a shunt motor are as follows: (*a*) The motor may be greatly overloaded and take excessive current through its armature thus leading to excessive sparking and especially to overheating. It is therefore desirable to provide an automatic device for disconnecting the motor from the supply mains in case of overload. This device is called the *overload release.* (*b*) The motor may be operating satisfactorily and the supply of current may fail, the supply mains becoming dead. When this happens the motor of course stops, but the connections to the supply mains remain intact with all resistance cut out of the armature circuit, so that an excessive current will flow through the armature when the supply of current is again established. It is therefore desirable to provide an automatic device for disconnecting the motor from the supply mains when the supply mains become dead. This device is called the *dead line release.*

The electrical connections of a motor starting box with automatic overload and dead line releases are shown in Fig. 181.

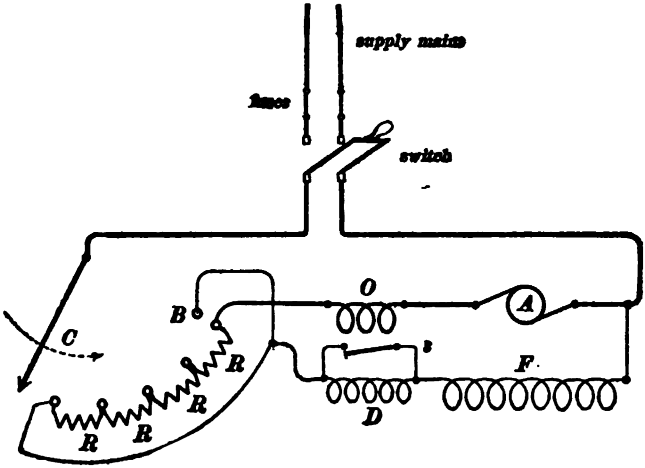

Fig. 181.

The motor armature is represented by *A*, and the shunt field winding by *F*. To start the motor the supply switch is closed, and the rheostat arm *C* is slowly moved in the direction of the dotted arrow until it reaches *B*. The first move of the arm *C* connects the shunt field winding directly to the mains, and at the same time it connects the armature to the mains through the resistances *RR*. The continued movement of the arm cuts the resistances out of the armature circuit. In the particular arrangement shown in Fig. 181 the resistances *RR* are left in series with the field winding as they are cut out of the armature circuit, but this has but little effect upon the field excitation, inasmuch as the resistances *RR* are very small in comparison with the resistance of the shunt field winding, but the resistances *RR* are finally cut out of the field circuit when the arm *C* touches the contact point *B*.

When the rheostat arm *C* is pushed to the right in starting the motor, it stretches a spring, and the rheostat arm is held in the running position by an electromagnet, of which the winding *D* is in series with the field winding *F* of the motor. When the lines become dead, current ceases to flow through *D* and *F*, the electromagnet releases the rheostat arm, and the stretched spring pulls the arm back to the starting position.

The overload release is actuated by an electromagnet of which the winding *O* is in series with the motor armature. The armature* of this electromagnet *O* is held back by a spring, the tension of which is so adjusted that the armature may be moved by a prescribed value of current through *O*. When this value of current is reached, the movement of the armature of *O* actuates a small switch *s* which short-circuits the winding *D*. This causes the electromagnet *D* to lose its magnetism, and release the arm *C* as before.

The resistances *RR*, Fig. 181, are usually very compact and they overheat if they are left in circuit. Therefore when a motor is started the rheostat arm should be moved promptly to the running position, but slowly enough to enable the motor to reach nearly full speed while the arm is being moved.

The automatic motor starter.—A float in a water tank may be arranged to open and close the supply switch of the motor which drives a water pump so that the pump may be automatically stopped when the tank is full and automatically started when the tank is nearly empty. Similarly, an automatic device opens the supply switch of the motor which drives the air pump on a trolley car when the pressure in the air tank reaches a certain value, and closes the switch when the air pressure falls below a certain value. Under such conditions an arrangement must be used to cut out the motor starting resistance automatically after the closing of the supply switch. Such an arrangement is called an *automatic starter.*

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* The armature of an electromagnet is the bar or strip of iron which is attracted by the magnet.

The essential features of a simple two-step starter for a shunt or compound motor of the Rowan Controller Co. are shown in Fig. 182. The shunt field current flows through coil 1 and excites an operating magnet which moves the arms *A* and *B* slowly in the direction of the arrow, making contact first at *a* thus short-circuiting R_1 and a little later at *b* thus short-circuiting R_2.

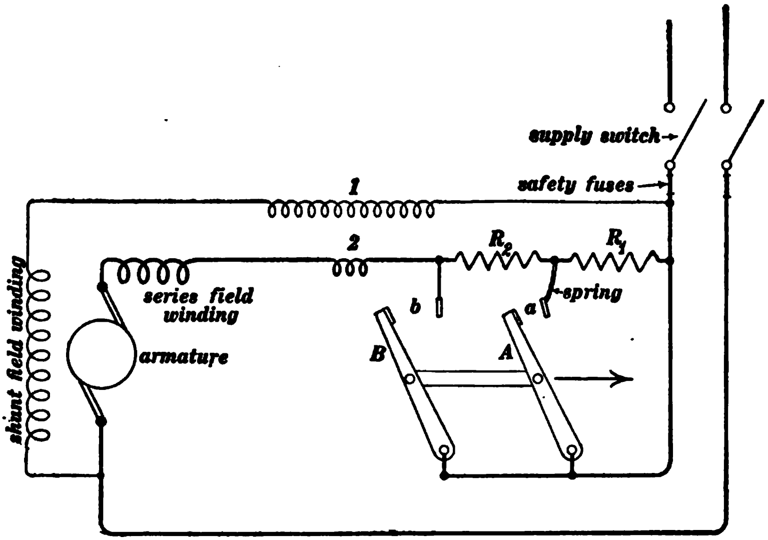

Fig. 182.

A two-step automatic motor starter.

After the motor is in full operation a dead line leaves the above mentioned operating magnet unexcited, and the arms *A* and *B*, thus released, are pulled back to the starting position by a spring which is not shown in the figure.

The coil 2 through which the motor armature current flows is wound on the same iron core as coil 1; the magnetizing action of coil 2 is ordinarily very weak and it opposes the magnetizing action of coil 1 which is usually very strong. After the motor is in full operation an excessive overload increases the current in coil

2, eventually neutralizing the magnetizing action of the current in coil 1, and the arms *A* and *B*, thus released, are pulled back to the starting position as before.

The above description refers to an automatic motor starter for a direct-current motor; a slight modification, however, adapts the device to an alternating-current induction motor. See Chapter XVII.

Automatic motor starters are extensively used in steel mills and shops. Thus, if a motor-driven lathe is to be repeatedly started and stopped as in the cutting of a thread, for example, a great deal of time is saved by the use of an automatic starter. The lathe attendant merely closes the supply switch when he wishes to start the motor and opens the switch when he wishes to stop the motor.

129. Characteristics of the series motor.—The speed and torque characteristics of the series motor are touched upon in Art. 123, for many purposes these characteristics must be com-

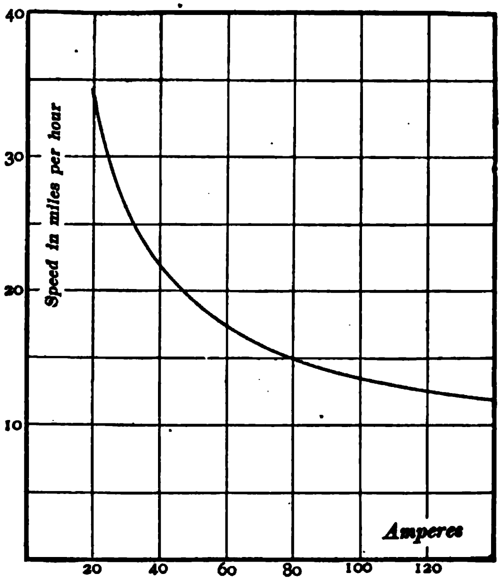

Fig. 183.

pletely determined by tests and represented by plotted curves. Thus the ordinates of the curves in Figs. 183 and 184 represent speeds and torques respectively, and the abscissas represent values of armature-and-field current for a Westinghouse street railway motor (No. 68), voltage between trolley wire and rail being 500 volts, gear ratio being 14 : 68, diameter of car wheels being 33 inches, and the motor being connected between trolley wire and rail with starting resistance all cut out of circuit. In this particular case, however, *speed of car in miles per hour* is used instead of speed of motor in revolutions per minute, and *forward pull on car* (*tractive effort*) is used instead of motor torque, as indicated in the figures.

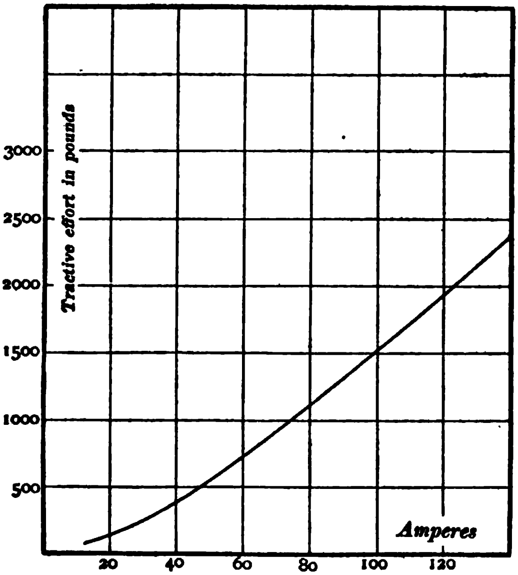

Fig. 184.

130. Example showing the use of characteristic curves.—A 20-ton street car is equipped with two Westinghouse No. 68 series motors of which the characteristic curves (all starting resistance cut out and each motor being connected between trolley wire and rail) are shown in Figs. 183 and 184. It is required to find the full speed (zero acceleration) at which the car will climb a 5.95 per cent. grade. It is also required to find the current taken by each motor, the total power delivered to both motors, the net power represented by the total tractive effort at the given speed, and the efficiency of the car equipment under the specified conditions.

Solution.—A tractive effort of 2,380 pounds would be required to draw a frictionless 20-ton car up a 5.95 per cent. grade, and, allowing 6 pounds of tractive effort per ton of car to overcome friction, we have a total tractive effort of 2,500 pounds, or 1,250 pounds per motor. Therefore, from Fig. 184, we find that a current of 85 amperes is required for each motor, or a total current of 170 amperes (the two motors being in parallel as shown in Fig. 186 with starting resistance all cut out); and the speed corresponding to 85 amperes is found from Fig. 183 to be 14.4 miles per hour. Therefore, the power delivered to the car is 170 amperes × 500 volts = 85 kilowatts, and the power which is represented by the total tractive effort of 2,500 pounds at a speed of 14.4 miles per hour is

$$\frac{2{,}500 \times 14.4 \times 5{,}280}{3{,}600} \times \frac{746}{550} \times \frac{1}{1{,}000} = 71.6 \text{ kilowatts}$$

whence the efficiency is

$$71.6 \text{ kilowatts} \div 85 \text{ kilowatts} = 0.842.$$

131. Starting of the series motor. Series-parallel operation of two similar series motors.—To start a series motor it is connected to the supply mains through an auxiliary resistance (starting resistance), and this resistance is cut out as the motor speeds up with its permanently attached load.

At starting and while the motor speed is low, the supply voltage is much larger than the back electromotive force in the motor armature, and the starting resistance prevents the flow of excessive current. *Under these conditions several similar motors might be connected in series with each other, with reduced starting* *resistance, taking the same current that would be required by a single motor.* Thus two similar series motors connected in series with each other operate satisfactorily at low speeds taking the same current that would be required by one motor alone but developing twice as much effective power and of course twice as much tractive effort. When the motors reach about half of normal* full speed, however, their *combined* back electromotive force is nearly equal to the supply voltage, and each motor must be connected across the supply mains (the two motors connected in parallel with each other) if the speed is to rise to full normal value. The use of two similar motors in this way (motors in series with each other at low speeds and in parallel with each other at high speeds) constitutes what is called the *series-parallel scheme of operation.* Figure 185 shows the connections at starting, as the speed increases the starting resistance is cut out, and at about half-speed it is all cut out. The connections are then quickly changed so as to bring the two motors in parallel with each other with starting resistance in circuit

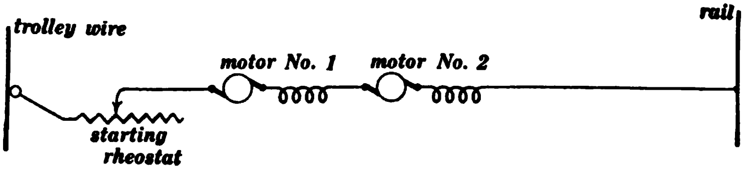

Fig. 185.

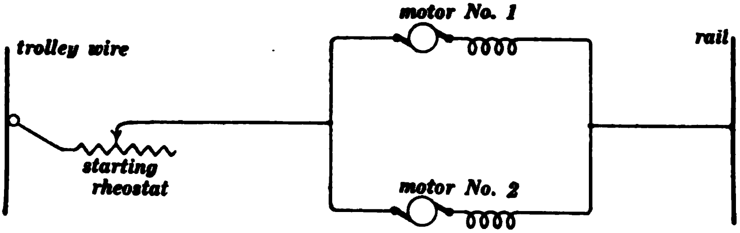

Fig. 186.

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* The speed at full load when both motors are connected across the supply mains as in Fig. 186 with all starting resistance cut out.

as shown in Fig. 186. Then as the speed rises the starting resistance is again cut out. The rather complicated changing of connections and the step by step cutting out of starting resistance are done by turning the handle of a contactor device which is called the *series-parallel controller*.*

132. **Dynamic braking and regenerative braking.**—When an electric railway has long grades it is common practice to brake the cars on down grade by driving the motors as electric generators. In the simplest arrangement the series field windings are connected so as to permit the motors to build up as generators, and the machines thus arranged are short-circuited through the starting resistances. The advantage of this so-called *dynamic braking* over the ordinary mechanical braking is its steadiness and ease of control, and the elimination of the costly wear on brake shoes and wheels.

In *regenerative braking* the motors are driven as generators so as to deliver power back to the trolley line. The arrangement usually employed with two series motors is as follows: Suppose the car or locomotive has been started with the motors in series as in Fig. 185 and brought up to half-speed with starting resistance all cut out, and suppose that it is desired to use regenerative braking at approximately half-speed. Then the connections are as in Fig. 185, except only that the motor field windings are adjacent to each other as shown in Fig. 187. A small auxiliary low-voltage generator C is driven by a small shunt motor (not shown in the figure). The field magnet of C is excited by current taken directly from the trolley, this field excitation is adjusted until the voltage across cd is the same as the voltage across ab, and then the two switches SS are automatically closed; but this closing of SS does not affect the operation of motors 1 and 2 until the field excitation of C

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* The action of the series parallel controller may be completely understood from what is stated above, and it would be perhaps a waste of time and effort to study out the actual diagram of connections on a street car. These connections include an automatic circuit breaker and a motor reversing switch in addition to the series-parallel controller. The motor reversing switch reverses the connections of both motor fields (or of both motor armatures).

is still further increased so as to add to the field current *ii* thus increasing the field excitation of the motors 1 and 2 more and more and increasing their back electromotive forces more and more. The current taken from the trolley is thus decreased to zero, and when the total back electromotive force in the motor armatures becomes greater than the supply voltage a reversed current flows from the rail through armature *C* and through armatures 1 and 2 to the trolley wire, thus returning power to the line. Under these conditions the two dynamos 1 and 2 act as generators, and the mechanical power required to drive them is taken from the car axles thus braking the car as it runs down hill.

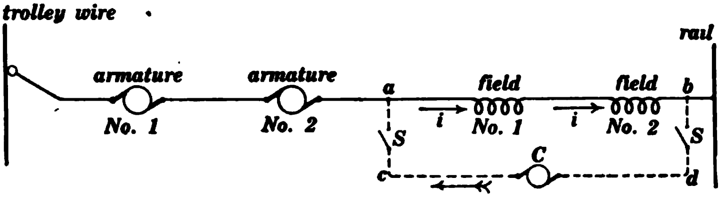

Fig. 187.

Connections for low-speed regenerative braking.

The connections shown in Fig. 187 provide for low-speed regenerative braking; connections for high-speed regenerative braking with the two series motors 1 and 2 in parallel and with all starting resistance cut out are shown in Fig. 188.

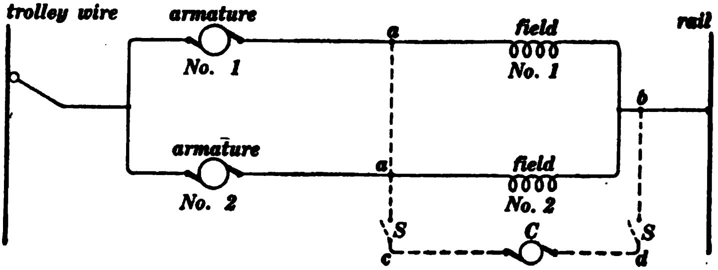

Fig. 188.

Connections for high-speed regenerative braking.

PROBLEMS.

Specifications for problems 120 to 124.—A given shunt motor takes current from 110-volt direct-current supply mains. The terminals of the field winding are connected directly to the supply mains, and the armature is connected to the supply mains through 51.5 feet of cable equivalent to No. 6 Brown and Sharpe copper wire. After the motor has been running for a long time at full load so as to come to its steady running temperature the field current is 3 amperes, with the motor load suddenly reduced to zero the armature current is 3 amperes, and the resistance of the armature including brush contacts and brushes is 0.11 ohm. The full-load armature current is 50 amperes.

120. How much resistance, in addition to armature and connecting cable, must be connected in circuit with the armature in order that the armature current at starting may be 1.5 times normal full-load armature current? How much current would flow through the armature at starting if this additional resistance were not inserted? Ans. 1.34 ohms; 846 amperes.

Note. The resistance of the armature when cold should be used in the solution of this problem, but the answer as given is found by using armature resistance when hot.

121. The armature of the above motor runs at a speed of 967 revolutions per minute at zero load with 3 amperes of armature current as stated. What is the value of the back electromotive force induced in the armature, and at what speed would the armature have to run to make the back electromotive force exactly equal to the supply voltage? Ans. 109.61 volts; 970.44 revolutions per minute.

Note. Assume the armature flux Φ to have a fixed value in solving this problem. The degree of precision required in the answer to this and some of the following problems is beyond the requirements of ordinary practice, but this precision is necessary for the purpose for which the problems are designed. An engineer is usually justified in thinking of the speed of a motor as constant, when it changes, say, from 1000 revolutions per minute at zero load, to 950 revolutions per minute at full load, and then rises, let us say, to 960 revolutions per minute as it becomes hot.

122. Assuming the armature flux to retain its zero-load value Φ as the motor load increases, find the back electromotive force induced in the armature at full load, and find the full-load speed of the motor. Ans. 103.5 volts; 901.3 revolutions per minute.

123. The full-load speed of the motor as calculated in the previous problem is incorrect because the armature flux does not retain its zero-load value Φ as the motor load increases. The full-load speed of the motor, as found by test, is 942.2 revolutions per minute. Find the value of the ratio Φ/Φ', where Φ' is the full-load value of the armature flux. Ans. $\Phi/\Phi' = 1.032$.

124. In the above problems the armature and field are supposed to be at normal running temperature. When the motor is first started its field and armature are cold and their resistances are less than under steady running conditions, and the effect of the reduced field resistance is to cause a slight increase of field excitation. (*a*) The zero-load speed of the motor when cold is 950.2 revolutions per minute and the cold resistance of the armature, including brush contacts and brushes, is 0.09 ohm. What is the value of Φ/Φ', where Φ is the value of the armature flux in problem 121 where the armature and field are hot and Φ' is the value of the armature flux in this problem where the armature and field are cold? (*b*) Find the full-load speed of the cold motor assuming that Φ'/Φ'' is 1.032 as in problem 123, where Φ' is the zero-load value of armature flux when the machine is cold and Φ'' is the full-load value of the armature flux when the machine is cold. Ans. (*a*) 0.98; (*b*) 934.6 revolutions per minute.

125. The motor is assumed to retain its zero load value of Φ at full load as in problem 122. Find the additional resistance which must be inserted in the armature circuit to give a full-load speed of 500 revolutions per minute? In solving this problem the motor is assumed to be at its steady running temperature. To what speed will the motor rise if the motor is unloaded with this additional resistance in its armature circuit? Ans. 0.936 ohm; 941 revolutions per minute.

126. Considering that the armature flux of the above motor is reduced in the ratio of 1.032 to 1 from zero load to full load, according to problem 123, find the additional resistance which must be inserted in the armature circuit to give a full-load speed of 500 revolutions per minute. Ans. 0.9714 ohm.

127. The above motor has its field magnet rewound with wire twice as large in sectional area (half as many turns) so that it will take 6 amperes of current from 55-volt mains and give full normal field excitation (same value of armature flux as in original specifications). The armature winding, however, is not changed so that the full-load value of armature current will be 50 amperes as before. The zero load current in the armature will be about the same as before, let us take it as exactly the same namely, 3 amperes. Find zero-load and full-load speeds, using 0.13 ohm as the total resistance of the armature circuit and assuming the zero-load value of the armature flux to remain unchanged. Ans. 481.8 revolutions per minute; 427.8 revolutions per minute.

128. The motor with its altered field winding is supplied from 55-volt mains, and by test it is found that the field current must be reduced from its normal value of 6 amperes to 2.5 amperes to increase the zero-load speed of the motor from 481.9 revolutions per minute to 967 revolutions per minute. Assume the zero-load armature current to be 3 amperes as before and assuming that the armature flux retains its new zero-load value unchanged, calculate the full-load speed of the motor. Ans. 858.6 revolutions per minute.

129. Find the full-load output of the above motor in horsepower: (*a*) Under normal conditions as originally specified, (*b*) With normal full-load speed reduced to 500 revolutions per minute by inserting resistance in the armature circuit as in problem 125, (*c*) With the altered motor taking current from 55-volt mains as specified in problem 127, and (*d*) Using the altered motor with greatly reduced field excitation as in problem 128. Ans. (*a*) 6.496 horsepower; (*b*) 3.36 horsepower; (*c*) 3.03 horsepower; (*d*) 3.03 horsepower.

Note. Supply voltage multiplied by zero-load armature current gives stray power loss. Full-load armature current multiplied by full-load value of back electromotive force gives total mechanical power developed in turning of armature; subtracting stray power loss from this gives mechanical power output of motor.

130. A shunt motor with a flywheel drives a small rolling mill, and it is desired that the speed drop to 65 per cent. of zero-load speed when a bar goes through the rolls so that the flywheel may give up a large portion of its kinetic energy. It is desired to limit the armature current of the motor to 300 amperes when it drops to 65 per cent. of its zero load speed and by trial it is found that this can be accomplished by increasing the shunt field current of the motor from its normal value of 4 amperes to 9.26 amperes.

?

The motor is of the multipolar type and each shunt field coil has 1500 turns of wire. The zero-load armature current is 15 amperes, and the armature current only flows through the series field winding (long shunt connection). How many turns of wire are required in each coil of a series field winding so as to limit the armature current automatically to 300 amperes by cumulative compounding of the motor field? Ans. 28 turns.

PROBLEMS ON POWER LOSSES AND EFFICIENCY.

131. Find the stray power loss of the motor specified in connection with problems 120 to 124, and find the torque exerted on the armature by the field magnet at zero load under conditions as given in problem 121. Ans. 329 watts; 2.397 pound-feet.

Note. The stray power loss is the friction, hysteresis and eddy current loss in the armature and at a given speed and field excitation it is equal to $E_tI_a - R_aI^2$, where E_tI_a is the total power delivered to the armature at zero load, and $R_aI_a^2$ is the power lost in developing heat in the armature at zero load; I_a here represents the zero-load value of the armature current.

The torque acting on armature to develope 329 watts at the speed of 967 revolutions is found from the equation $329 \text{ watts} = \frac{746}{550} \cdot 2\pi nT$, where T is the torque value in pound-feet and n is the speed in revolutions per second. See foot-note on page 195.

132. Find field loss and stray power loss of above motor under normal conditions. Find armature loss under full load. Find total power intake at full load. Find power output at full load. Find the efficiency of the motor at full load. Ans. 330 watts; 329 watts; 325 watts; 5830 watts; 4846 watts; 83.1 per cent.

Note. The power delivered to the field winding or windings is all lost as heat, and this power is called the field loss. Assume stray power loss to be the same at full load as at zero load. Armature loss is here understood to mean RI_a^2 where R is the total resistance of the armature circuit including connecting cables and I_a is the armature current. The efficiency of a motor is found by dividing the power output by the power intake.

133. Find the efficiency of the above motor for the following values of armature current, 10, 20, 30, 40, 50, 60 and 70 amperes, and plot a curve of which the abscissas represent values of armature current and ordinates represent corresponding values of efficiency.

134. The above motor is used 10 hours each day as follows: 1 hour with 20 amperes in armature, 2 hours with 30 amperes in armature, 6 hours with 50 amperes in armature and 1 hour with 60 amperes in armature. What is the all-day efficiency of the motor?

Note. The all-day efficiency is obtained by dividing the total output of energy during the day by the total intake of energy during the day.

135. The above dynamo is operated as a generator at a speed of 967 revolutions, and the field current is increased to 3.5 amperes to bring the terminal voltage (at the brushes) up to 110 volts with full-load current of 46.5 amperes delivered to the external receiving circuit. The shunt field winding is connected to the brushes of the machine. Find stray power loss on the assumption that it is E'/E times as great as in problem 131, where E' is the total electromotive force induced in the generator armature under conditions here specified and E is the electromotive force induced in the armature in problem 131 (or in problem 121). Ans. 346.6 watts.

136. Find field loss, armature loss, power output and total power required to drive the fully loaded generator of problem 135, and find the efficiency of the generator. Ans. Field loss 385 watts; armature loss 275 watts, power output 5115 watts, power required to drive 6122 watts, efficiency 83.7 per cent.

137. The above shunt motor is used in a test to determine the power required to drive a feed mill. The motor is belted to the mill, pulley sizes being chosen so that the feed mill is driven at its proper speed. The motor is driven from 110 volt mains exactly as stated in the specifications for problems 120 to 124, and the armature current is observed to be 36.4 amperes. Assuming the stray power loss to be the same as at zero load, calculate the power required to drive the feed mill making no allowance for belt losses. Ans. 3503 watts or 4.7 horsepower.

138. A certain shunt motor is tested for efficiency by the Prony brake method and the following data were obtained: speed 1200 revolutions per minute; vertical pull on end of balanced brake arm 18.75 pounds; horizontal distance from center of pulley to line of 18.75-pound pull 28 inches; electromotive force of supply 110 volts; total current delivered to motor 83.6 amperes. Find the efficiency of the motor. Ans. 81.1 per cent.

Note. Use the formula $P = \frac{746}{550} \times 2\pi nT$ for calculating the power in watts absorbed by the brake, n being expressed in revolutions per second, and T being expressed in pound-feet. See footnote on page 195.

139. A certain 6-pole 250-kilowatt 550-volt direct-current generator runs at a speed of 320 revolutions per minute. The commutator is 37.4 inches in diameter. The machine has six sets of carbon brushes, each set has four brushes side by side, each brush has 1.09 square inch of contact area with the commutator, the brush pressure 1.25 pounds per square inch, and the coefficient of sliding friction of the carbon brushes on the copper commutator is 0.1. Find (*a*) Watts lost in brush friction, (*b*) Watts lost in electrical contact resistance between brushes

and commutator at full load. Ans. (*a*) 231.7 watts; (*b*) 909 watts

Note. The full-load current output of the generator is 454.5 amperes and, neglecting shunt field current, this is the current entering the three sets of positive brushes from the commutator.

Under ordinary conditions the electrical contact resistance of carbon brushes in ohms (for one square inch of contact area) is equal to the reciprocal of the current density in amperes per square inch of contact area.

CHAPTER X.

ARMATURE REACTION IN DIRECT-CURRENT DYNAMOS.

133. **Armature reaction.**—The magnetizing action of the current in a dynamo armature is called ***armature reaction***, and it has two distinct effects, namely, (*a*) It crowds over the magnetic lines of force in the gap spaces as indicated in Figs. 190 and 191; this crowding over of the lines of force is ***in the direction of rotation*** in a generator and in the ***opposite direction*** in a motor; and (*b*) It opposes, usually, the magnetizing action of the field winding and reduces the value of the armature flux Φ.

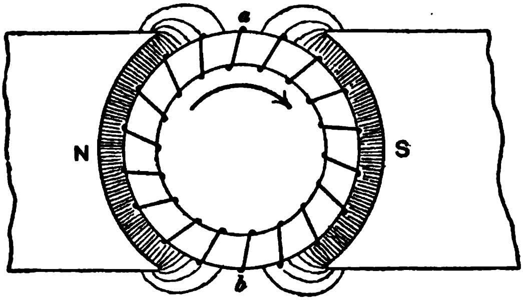

Fig. 189.
Undistorted field when armature current is small.

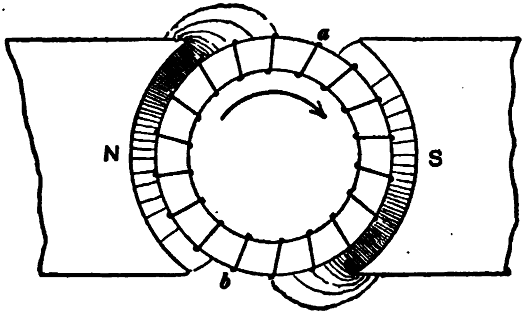

Fig. 190.
Distorted field of generator when armature current is large.

The first effect is called *cross magnetizing action*, and it has an important bearing on sparkless commutation. The second effect, *demagnetizing action*, causes a decrease of voltage of a shunt generator with increase of current output, and it tends to

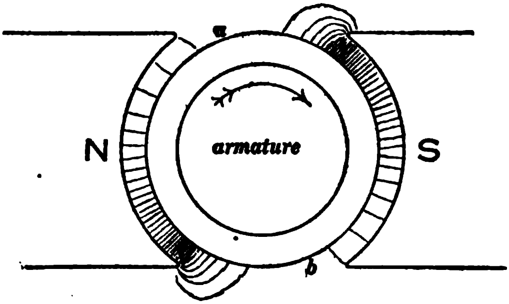

Fig. 191.
Distorted field of motor when armature current is large.

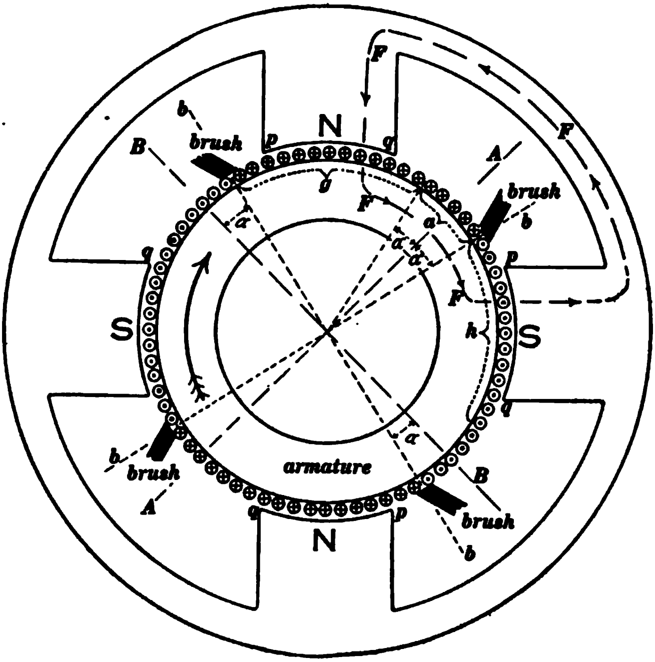

Fig. 192.
This diagram refers to a generator. The direction of rotation, only, need be reversed to make the diagram refer to a motor.

produce an increase of speed of a shunt motor with increase of load as explained in Art. 126.

Demagnetizing action expressed in ampere turns. The lines bb in Fig. 192 are called the *brush axes*, the lines AA and BB are midway between the field poles, and the angle α is called the angle of *forward lead* of the brushes because in Fig. 192 the brushes are shifted forwards (in the direction of rotation of the armature) from the lines AA and BB. If the direction of rotation be reversed in Fig. 192 so as to show motor conditions, the angle α becomes the angle of *backward lead* of the brushes. The direction of current in each armature conductor (each armature wire) is indicated by a dot or cross in Fig. 192.

Consider the armature conductors which fall inside of the dotted loop FF (one of the magnetic circuits of the dynamo). Each of these conductors contributes I ampere-turns of magnetomotive force (plus or minus) to the circuit FF where I is the current in the conductor; but each of the conductors in group g has a corresponding symmetrically located conductor in group h with an equal and opposite current flowing in it, therefore groups g and h neutralize each other in their production of magnetomotive force around the circuit FF. Therefore group a alone produces magnetomotive force around the circuit FF, and this magnetomotive force opposes the magnetomotive force of the field windings. Let the angle α be expressed in radians, then $\alpha/2\pi$ expresses α as a fraction of the entire circumference, and $Z \times 2\alpha/2\pi$ is the number of armature conductors in group a, Z being the total number of armature conductors; and $IZ2\alpha/2\pi$ is the demagnetizing ampere-turns due to group a. That is to say, $IZ2\alpha/2\pi$ is the demagnetizing action of the armature current in ampere-turns, I being the current in each armature conductor. The demagnetizing action is the same on each of the magnetic circuits of the multipolar dynamo.

Cross magnetizing action.—Cross magnetizing action is due almost entirely to the armature conductors which at any instant lie under the pole faces. Thus Fig. 193 shows the armature con-

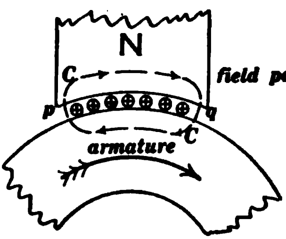

Fig 193.

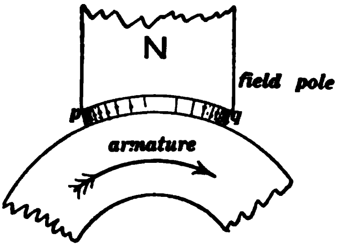

Fig. 194.

ductors which lie under a N-pole face at any instant, and the crosses indicate the currents in the conductors. These conductors produce magnetic flux around the circuit CC thus producing magnetic field in the gap space as indicated by the fine lines in Fig. 194. Now the field winding alone produces a *uniform* field in the gap space in Fig. 194 directed from the pole face into the armature core. Therefore field winding and armature currents work together to produce an intensified field in the gap space under the "trailing" pole tip q, and to produce a weakened field in the gap space under the "leading" pole tip p.

Quantitative expression for weakening of magnetic field under leading pole tips of a generator. Let β be the angular breadth of the pole face in radians Then $\beta/2\pi$ is the breadth of the pole face expressed as a fraction of an entire circumference, $Z \times \beta/2\pi$ is the number of armature conductors under the pole face at any instant, and $IZ \times \beta/2\pi$ is the magnetomotive force in ampere-turns (equal to $IZ\beta/5$ c.g.s units) produced around the circuit CC in Fig. 193 by the band of armature conductors. This magnetomotive force is nearly all concentrated at the two air gaps at p and q; and if we assume that it is *all* concentrated at these air gaps we get $(IZ\beta/5) \div 2l$ as the field intensity at p and q due to the cross magnetization; and, if H is the field intensity in the gap space due to field winding alone, we have

$$\text{net field intensity at } p = H - \frac{IZ\beta}{10l}$$

and

$$\text{net field intensity at } q = H + \frac{IZ\beta}{10l}$$

where l is the distance across the gap space in centimeters, and I is the current in each armature conductor.

134. **The compensating winding.**—The distorting effect of armature reaction (cross magnetization) may be almost entirely eliminated by what is called the *compensating winding* which consists of a layer of carefully insulated wires or conductors embedded in the pole faces as indicated by the groups of small circles NN and SS in Fig. 195*a*. These conductors are parallel to the armature conductors (and parallel to the armature shaft), and they are shown in Fig. 195*a* as having the same spacing as the armature conductors, so that, if the current flowing in each conductor of the compensating winding is the same in value as the current in each armature conductor but opposite in direction,

the combined magnetizing action of the two windings is zero.

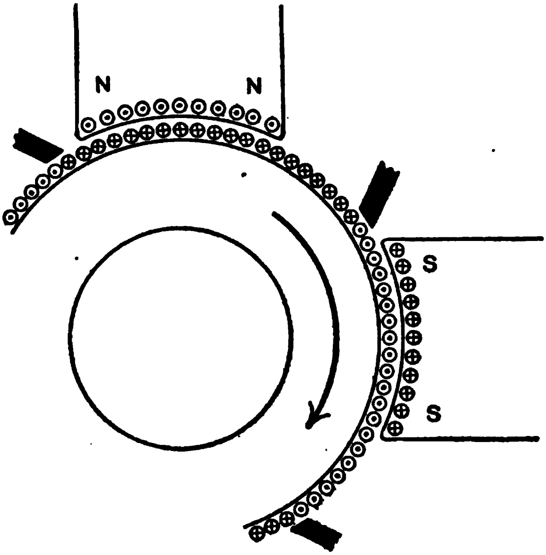

Fig. 195*a*.

Showing essential features of the compensating winding.

The scheme of connecting the conductors of the compensating winding is shown in Fig. 195*b*.

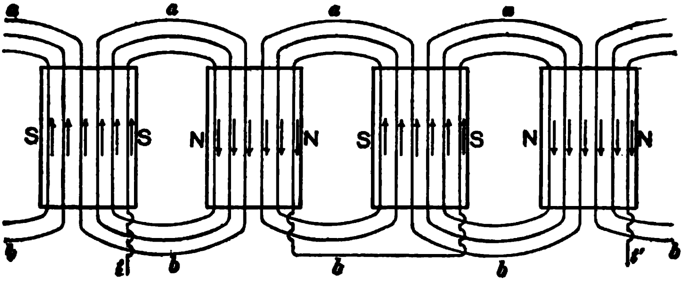

Fig. 195*b*.

The compensating winding is used on large high-voltage direct-current dynamos where every possible precaution must be taken to prevent sparking at the commutator, and the compensating winding is an essential feature of the ordinary direct-current series motor as modified so as to be driven by alternating current.

135. The phenomena of commutation. The non-sparking condition.—A clear idea of the methods used for reducing or eliminating sparking may be obtained by considering what takes place in a section of the armature winding, and at the two commutator bars to which the section is connected when the two bars pass under a brush.

Figure 196 shows a positive brush of a direct-current generator,

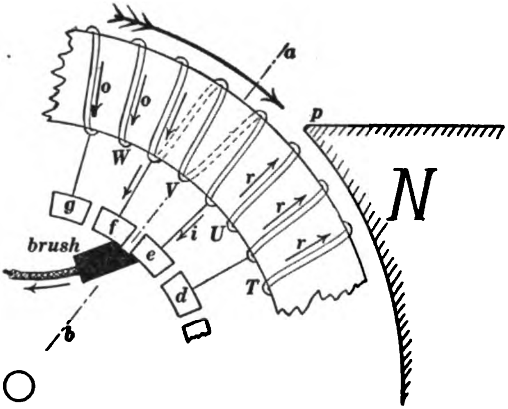

Fig. 196.

it shows a portion of the commutator with its segments or bars *d*, *e*, *f* and *g*, it shows a portion of the armature windings with its sections *T*, *U*, *V* and *W*, and it shows the leading pole tip *p* of one of the field poles. This pole tip *p* is the one under which the field intensity is weakened by cross magnetizing action as explained in connection with Figs. 193 and 194.

Before a given section of the armature winding reaches the brush in Fig. 196, current is flowing in the section as indicated by the small arrows *oo*; and after the given armature section has passed the brush, the current is flowing in the section in a reversed direction as indicated by the arrows *rr*.

Figure 196 shows the armature section *V* short-circuited by the contact of the brush with the two commutator bars *e* and *f*.

This state of affairs lasts for a very short time, and *if an action could be brought about whereby the previously existing current* o *in this section could be reduced to zero and an equal and opposite current* r *established in the section during this short time*, then there would be no tendency whatever for a spark to form between the brush and the bar e at the instant they separate or break contact with each other. There are two actions which tend to produce this so-called sparkless reversal of current in the armature sections as they pass under the brushes as follows:

(*a*) It is to be noted that electromotive forces are induced in all the armature windings except in the neutral axis ab, see Fig. 190. Therefore it is evident that if we give the brush in Fig. 196 a forward lead (in the direction of rotation of the generator armature) from the neutral axis ab, the sections as they are short-circuited by the brush will have electromotive forces induced in them in the *proper direction* to produce sparkless reversal of current; and, if the brush is moved farther and farther forwards, the sections as they are short-circuited will be moving in a stronger and stronger field (see Fig. 190) and the electromotive force induced in a short-circuited section will be greater and greater, so that it is possible to adjust the forward lead of the brush so as to obtain the desired sparkless reversal.

(*b*) The use of high-resistance brushes of carbon tends not only to cause a decrease to zero of the current in a newly short-circuited armature section, but also to establish the desired reversed current. The decrease of the initial current to zero is mainly due to the high resistance of brush contacts and of brush material which form part of the short-circuit; but as the commutator moves beyond the position shown in Fig. 196, the contact resistance between the brush and bar e increases greatly due to decreasing area of contact, and this increasing resistance tends to divert the current i through the section V before the contact between brush and bar e is actually broken.

It is impracticable to depend solely upon the action described under (*a*) for the elimination of sparking, because to do so would require the lead of the brushes to be readjusted with every change of load. Therefore, unless commutating poles are used as explained in Art. 136, the action described under (*b*) must be depended upon. Indeed the sparkless operation of most direct-current motors and generators depends upon the use of carbon brushes.

136. The use of commutating poles. The interpole motor.— The most satisfactory arrangement for producing sparkless reversal of current in the sections of the armature winding at commutation is illustrated in Fig. 197*a*. So-called *commutating poles* or *interpoles*, *nsns*, are placed midway between the field magnet poles, *NSNS*, these commutating poles are excited (magnetized) by windings through which the whole or a definite fraction of the armature current flows. The magnetic field *F* which is thus produced induces an electromotive force in the short-circuited sections, and by adjusting a shunt (like *Cs* in Fig. 173) the field *F* may be made of proper strength to produce sparkless reversal with full load current. Then the field *F* decreases in proportion to any decrease of armature current thus giving sparkless reversal at all loads.

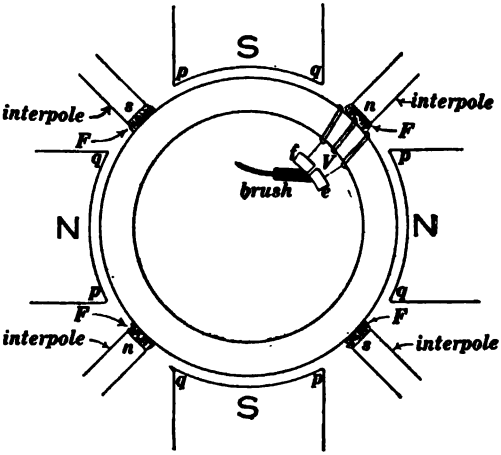

Fig. 197*a*.

Commutating poles or interpoles are now extensively used on direct-current dynamos. Thus the high-voltage generators for supplying direct current at 1500 to 3000 volts for railway service not only have compensating windings but also interpoles, and the interpoles are the more important.

A shunt-motor which is to be run at a wide range of speeds as explained in Art. 127 should be provided with interpoles;

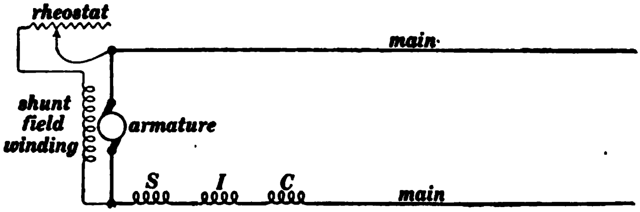

Fig. 197*b*.

Series field winding *S*. Interpole winding *I*. Compensating winding *C*.

and a motor with immovable brushes which is to be repeatedly reversed in its direction of running should be provided with interpoles.

A diagram showing the connections of a compound dynamo with interpoles and compensating windings is shown in Fig. 197*b*.

CHAPTER XI.

DEFINITIONS. HARMONIC ALTERNATING ELECTROMOTIVE FORCE AND CURRENT.

137. The alternating-current ammeter and voltmeter.—An alternating current (or electromotive force) passes repeatedly through similar sets of positive and negative values. Thus the equidistant ordinates of the portions *A* and *B* of the periodic curve in Fig. 198 represent the similar sets of positive and negative values of an alternating current.* The two sets together constitute a *cycle*, and the number of cycles per second is called the *frequency* of the alternating current (or electromotive force).

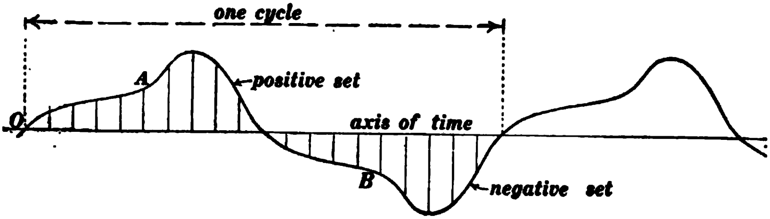

Fig. 198.

In using an ordinary alternating-current ammeter or voltmeter the frequency is so high that the pointer of the instrument stands still at a definite scale reading, so many " amperes " or " volts " as the case may be, and it is important to consider precisely what this constant reading means. It evidently does not mean the average value of the alternating current or electromotive force, because the average value is zero. Indeed a direct-current ammeter or voltmeter of the type described in Art. 16 measures the true average value of a rapidly fluctuating current or electromotive force, and such an instrument gives *zero reading* for an alternating current or electromotive force.

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* This curve represents in fact the current that flows through the primary coil of a transformer when the secondary coil is on open circuit.

To understand the meaning of an *ampere reading* or *voltage reading* on an alternating-current instrument let us consider the electrodynamometer type of ammeter or voltmeter. This instrument consists of a fixed coil and a delicately pivoted coil connected in series with each other, the current which produces the deflection flows through both coils, the fixed coil exerts a force on the pivoted coil which moves the coil and the attached pointer, *and this force is at each instant strictly proportional to the square of the current* for a given position of the pivoted coil

(*a*) Let $\mathcal{I}$ be the constant or direct current which produces a certain deflection or scale reading s on an electrodynamometer type of ammeter. Then the steady deflecting force exerted on the pivoted coil is equal to $k\mathcal{I}^2$, where k is a constant.

(*b*) Let i be the varying value of an alternating current which gives the same steady deflection s. Then the instantaneous value of the force exerted on the pivoted coil is ki^2, and the average value of this force is $k \times$ average i^2.

Now the steady deflecting force under (*a*) must be equal to the average deflecting force under (*b*) because the deflection s is the same. Therefore[*]

$$k\mathcal{I}^2 = k \times \text{average } i^2$$

or

$$\mathcal{I}^2 = \text{average } i^2$$

or

$$\mathcal{I} = \sqrt{\text{average } i^2} \tag{61}$$

But the value of $\mathcal{I}$ is marked on the ammeter scale where the pointer stands under (*a*) above. Therefore $\mathcal{I}$ is the reading s of the ammeter under (*b*). *Therefore, according to equation* (61), *the reading of an alternating-current ammeter gives the square-root-of-the-average-value-of-the-square of the alternating current.*

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[*] The reader should use his calculus to establish the following two propositions, namely, (1) Average $(ax) = a \times$ average x, where a is a constant, and (2) Average $(x + y) =$ average $x +$ average y.

The voltmeter.—A large noninductive* resistance is connected in series with the fixed and pivoted coils of an electrodynamometer instrument so that the instrument may be connected directly across the supply mains. Let R be the value of this resistance including the resistance of the coils of the instrument.

(*a*) Let $\mathcal{E}$ be the constant or direct electromotive force which produces a certain deflection or scale reading s. Then the steady deflecting force exerted on the pivoted coil is $k\left(\frac{\mathcal{E}}{R}\right)^2$.

(*b*) Let e be the varying value of an alternating electromotive force which gives the same steady deflection s. Then† the instantaneous value of the force exerted on the pivoted coil is $k\left(\frac{e}{R}\right)^2$, and the average value of this force is $\frac{k}{R^2} \times$ average e^2.

Now the steady deflecting force under (*a*) must be equal to the average deflecting force under (*b*) because the deflection s is the same. Therefore

$$k\left(\frac{\mathcal{E}}{R}\right)^2 = \frac{k}{R^2} \times \text{average } e^2$$

or

$$\mathcal{E}^2 = \text{average } e^2$$

or

$$\mathcal{E} = \sqrt{\text{average } e^2} \tag{62}$$

But the value of $\mathcal{E}$ is marked on the voltmeter scale where the pointer stands under (*a*) above. Therefore $\mathcal{E}$ is the reading s of the voltmeter under (*b*) above. *Therefore, according to equation* (62), *the reading of an alternating voltmeter gives the*

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* A circuit is said to be non-inductive when its inductance L is negligibly small, and the inductance is negligibly small when the electromotive force $L\frac{di}{dt}$ which is required to make the current change is very small as compared with the electromotive force Ri which is required to overcome resistance See Art. 48.

† The electromotive force e which acts on a circuit of resistance R and inductance L is equal to $Ri + L\frac{di}{dt}$, and if $L\frac{di}{dt}$ is negligible we have $e = Ri$ or $i = e/R$.

square-root-of-the-average-value-of-the-square of the alternating electromotive force.

Effective values of alternating electromotive force and current.—The value of an alternating electromotive force as measured by a voltmeter, the square-root-of-the-average-value-of-e^2, is called the *effective value* or *voltmeter value* of the electromotive force, and it is represented by the letter E. The value of an alternating current as measured by an ammeter, the square-root-of-the-average-value-of-i^2, is called the *effective value* or *ammeter value* of the current and it is represented by the letter I.

Heating effect of an alternating current.—Let i be the value of an alternating current at a given instant, and let R be the resistance of the circuit or coil through which the current flows. Then Ri^2 watts is the rate at which heat is generated in the circuit or coil at the given instant, and $R \times$ average i^2 is the average rate of generation of heat in the circuit or coil. But $I = \sqrt{\text{average } i^2}$ so that I^2 = average i^2. Therefore $R \times$ average $i^2 = RI^2$, or *the average rate of generation of heat in a coil or circuit is found by multiplying the resistance by the square of the effective or ammeter-value of the current.*

138. Harmonic alternating electromotive force or current.—In many cases the curve which represents the varying value of an alternating electromotive force or current is a curve of sines as in Fig. 199. In such a case the electromotive force or current is called a *harmonic alternating electromotive force or current*, or simply, a *harmonic electromotive force or current.* Let f be the

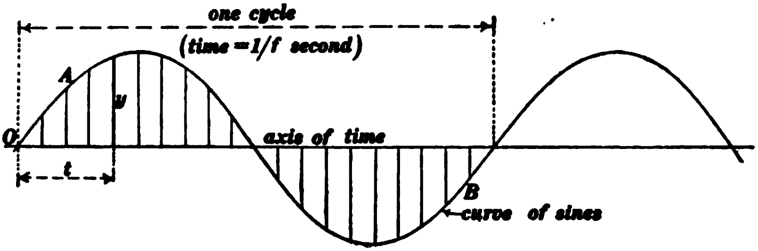

Fig. 199.

frequency (number of cycles per second), then $1/f$ is the so-called period (number of seconds or the fraction of a second per cycle) as indicated in the figure. The abscissas of a curve of sines are most conveniently thought of as angles, and, of course, the angle corresponding to a whole cycle is 360° or 2π radians. Therefore the angle α corresponding to any given time t, reckoned from the point O in Fig. 199, is derived from the proportion $\alpha : 2\pi :: t : 1/f$. Therefore $\alpha = 2\pi f \cdot t$ and the equation to the sine curve in Fig. 199 is

$$e = \mathbf{E} \sin 2\pi f \cdot t \tag{i}$$

where $\mathbf{E}$ is the maximum ordinate (volts) and e is the value of the alternating electromotive force at any instant t; or

$$i = \mathbf{I} \sin 2\pi f \cdot t \tag{ii}$$

where $\mathbf{I}$ is the maximum ordinate (amperes) and i is the value of the alternating current at any instant t. It is customary to represent the factor $2\pi f$ by the Greek letter ω. That is

$$\omega = 2\pi f \tag{63}$$

so that equations (i) and (ii) become

$$e = \mathbf{E} \sin \omega t \tag{64}$$

and

$$i = \mathbf{I} \sin \omega t \tag{65}$$

139. The clock diagram.—Imagine the line OP in Fig. 200 to make f revolutions per second or $2\pi f (= \omega)$ radians per second about the point O in the direction of the curved arrow, and let time be reckoned from the instant that OP coincides with the dotted line (to the right). Then at time t the line will be in the position shown, and Ob will be equal to OP times the sine of ωt. Therefore

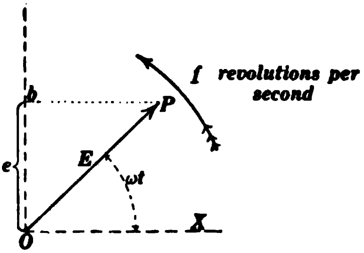

Fig. 200.

If the length of the rotating line OP represents the maximum value $\mathbf{E}$ of a harmonic alternating electromotive force, the projection Ob will at each instant represent the value e of the alternating electromotive force; or

If the length of the rotating line OP represents the maximum value $\mathbf{I}$ of a harmonic alternating current, the projection Ob will at each instant represent the value i of the alternating current.

140. Phase difference.—Consider a harmonic alternating electromotive force e and a harmonic alternating current i of the same frequency. It often happens that the current reaches its maximum value *after* the electromotive force has passed its maximum value. In this case the current is said to *lag behind* the electromotive force in phase. In some cases the current reaches its maximum value *before* the electromotive force reaches its maximum value; and in such a case the current is said to be *ahead of* the electromotive force in phase. When a current is behind an electromotive force in phase, as indicated in Fig. 201, it is called a *lagging current*; when a current is ahead of an electromotive force in phase, as indicated in Fig. 202, it is called a *leading current.*

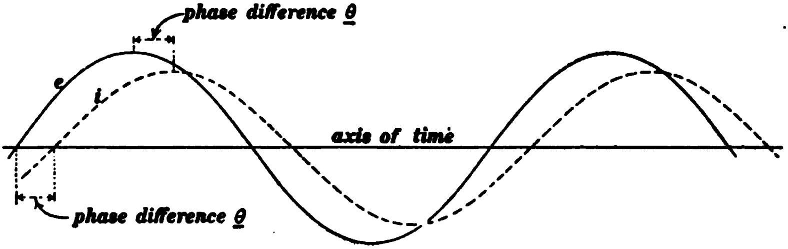

Fig. 201.

Current i behind electromotive force e in phase.

The phase difference in Figs. 201 and 202 is, of course a definite but very small time interval, a definite fraction of a cycle, and it corresponds also to a definite angle θ (a definite fraction of 2π radians). The current in Fig. 201 is θ radians behind the electromotive force in phase, and in Fig. 202 the current is θ radians ahead of the electromotive force in phase.

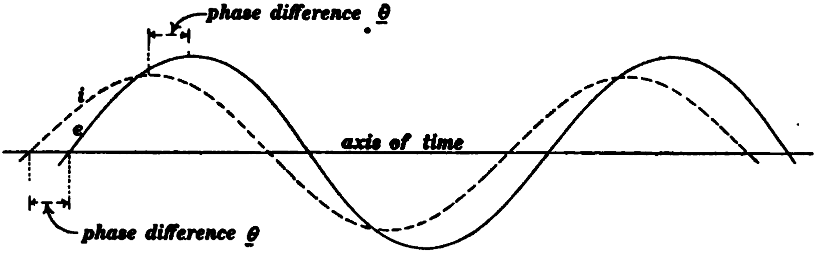

Fig. 202.

Current i ahead of electromotive force e in phase.

Phase difference in the clock diagram.—Figure 203 is a clock diagram corresponding exactly to Fig. 201, the alternating current which is represented by the varying projection of the rotating line **I** is θ radians behind the electromotive force which is represented by the varying projection of the rotating line **E**; and Fig. 204 is a clock diagram corresponding exactly to Fig. 202.

If we reckon time from the instant that the line **E** coincides with OX, then the angle **E**OX will be $2\pi ft$ or ωt radians, the angle **I**OX in Fig. 203 is $\omega t - \theta$, and the angle **I**OX in Fig. 204 is $\omega t + \theta$. Therefore we have

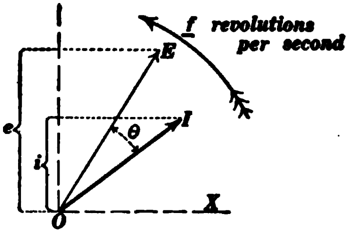

Fig. 203.

Corresponding to Fig. 201. Current behind voltage in phase.

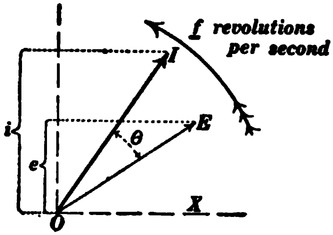

Fig. 204.

Corresponding to Fig. 202. Current ahead of voltage in phase.

$$\left.\begin{array}{lr}\text{a given harmonic electromotive force} & e = \mathbf{E}\sin\omega t\\ \text{a current lagging } \theta^\circ \text{ behind } e & i = \mathbf{I}\sin(\omega t - \theta)\\ \text{a current leading } \theta^\circ \text{ ahead of } e & i = \mathbf{I}\sin(\omega t + \theta)\end{array}\right\} \quad (66)$$

Remark.—Consider a rotating clock-diagram vector whose varying projection represents a harmonic alternating electromotive force or current. We shall hereafter speak of such a vector simply as *representing* the electromotive force or current.

141. Relation between maximum value and effective value of harmonic alternating electromotive force or current.—The effective value of an alternating electromotive force or current, as measured by a voltmeter or ammeter, is the square-root-of-the-average-value-of-the-square of the electromotive force or current as explained in Art. 137; and the maximum value of a harmonic alternating electromotive force or current is the quantity $\mathbf{E}$ in equation (64) or the quantity $\mathbf{I}$ in equation (65) as explained in Art. 138.

The effective value of a harmonic alternating electromotive force (or current) is equal to its maximum value $\mathbf{E}$ *(or* $\mathbf{I}$*) divided by the square root of* 2. That is

$$E = \frac{\mathbf{E}}{\sqrt{2}} \tag{67}$$

$$I = \frac{\mathbf{I}}{\sqrt{2}} \tag{68}$$

in which E and I represent effective values as indicated by a voltmeter or ammeter.

These relations may be established by reducing the expression, (effective value)^2 $= \frac{1}{2\pi}\int_0^{2\pi} \mathbf{E}^2 \sin^2 x$, but it is more instructive to derive these relations as follows: Let $e = \mathbf{E} \sin \omega t$, then $e^2 = \mathbf{E}^2 \sin^2 \omega t$, and average e^2 = average $(\mathbf{E}^2 \sin^2 \omega t)$ $= \mathbf{E}^2 \times$ average $(\sin^2 \omega t)$; therefore it is necessary only to find the average value of the sine squared of the uniformly growing angle ωt. Now in general

$$\sin^2 \omega t + \cos^2 \omega t = 1 \tag{i}$$

so that

$$\text{average } (\sin^2 \omega t) + \text{average } (\cos^2 \omega t) = 1$$

but the sine and the cosine of a uniformly growing angle pass similarly through identical sets of values during a cycle, and therefore, since ωt is a uniformly growing angle, we have

$$\text{average } (\sin^2 \omega t) = \text{average } (\cos^2 \omega t)$$

Hence from equation (i) we have $2 \times$ average $(\sin^2 \omega t) = 1$ or average $(\sin^2 \omega t) = \frac{1}{2}$. Therefore average $e^2 = \mathbf{E}^2 \times$ average $(\sin^2 \omega t) = \frac{\mathbf{E}^2}{2}$, or effective value $= \sqrt{\text{average } e^2} = \mathbf{E}/\sqrt{2}$.

Two principles are used, without proof, in this discussion, namely, (1) Average $(ax) = a \times$ average x, where a is a constant, and (2) Average $(x + y) =$ average $x +$ average y. If the student cares to go into the calculus discussion of this subject it will be instructive to establish these two relations.

Use of effective values instead of maximum values in the clock diagram.—According to Art. 139 the lengths of the vectors or lines in a clock diagram represent maximum values **E** and **I**; but the ratio of maximum values to effective values is constant according to equations (67) and (68) and therefore it is permissible to think of the clock-diagram vectors as representing effective values. Indeed this is always done when the clock-diagram is used to show the relation between several alternating electromotive forces or currents. Thus the harmonic alternating electromotive forces c, a and b in Fig. 205*a* *as measured by a voltmeter* are proportional to the lengths of the vectors C, A and B in Fig. 205*b*; and the harmonic alternating currents c, a and b in Fig. 206*a* *as measured by an ammeter* are proportional to the lengths of the vectors C, A and B in Fig. 206*b*.

142. Addition of harmonic alternating electromotive forces of the same frequency.—Consider two alternators A and B connected in series with each other as shown in Fig. 205*a*. Let a and b be the values of the electromotive forces of machines A and B respectively at any instant, and let c be the value

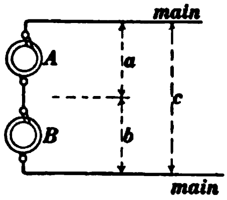

Fig. 205*a*.

a, b and c represent instantaneous values of electromotive force and $c = a + b$.

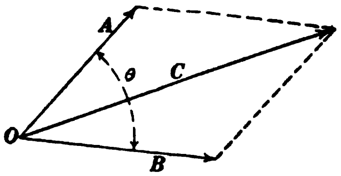

Fig. 205*b*.

Lengths of A, B and C represent voltmeter values of a, b and c.

at the same instant of the electromotive force between the mains. Then $c = a + b$ at each instant. If a and b have the same frequency f (and therefore a definite phase difference θ), *then* c *will be a simple harmonic electromotive force of frequency* f as represented by the clock-diagram vector C in Fig. 205*b*. To make this evident let us think of A, B and C as representing maximum values so that their projections will represent instantaneous values, but the projection of the diagonal C is at each instant equal to the sum of the projections of the sides A and B of the parallelogram. Therefore $c = a + b$, where c, a and b are the projections of C, A and B.

Resolution of a harmonic alternating electromotive force into harmonic parts.—Any given harmonic alternating electromotive force c (represented by the clock-diagram vector C in Fig. 205*b*) may be resolved into two harmonic alternating electromotive forces a and b (represented by the clock-diagram vectors A and B in Fig. 205*b*).

143. Addition of harmonic alternating currents of the same frequency.—Consider two alternators A and B connected in parallel with each other as indicated in Fig. 206*a*. Let a and b be the values of the currents delivered by machines A and B respectively at any instant and let c be the value at the same instant of the current in the mains. Then $c = a + b$ at each instant. If a and b have the same frequency f (and therefore a definite phase difference θ), *then* c *will be a simple harmonic*

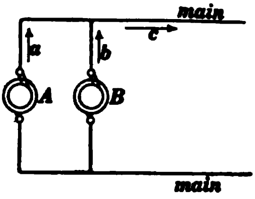

Fig. 206*a*.

a, b and c represent instantaneous values of current and $c = a + b$.

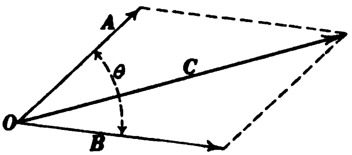

Fig. 206*b*.

Lengths of A, B and C represent ammeter values of a, b and c.

current of frequency f as represented by the clock-diagram vector C in Fig. 206*b*.

Resolution of a harmonic alternating current into parts.—Any given harmonic alternating current c (represented by the clock-diagram vector C in Fig. 206*b*) may be resolved into two harmonic alternating currents a and b (represented by the clock-diagram vectors A and B in Fig. 206*b*.

144. Examples of resolution into parts.—(*a*) *Dividing of an alternating electromotive force between two coils in series.* Figure 207*a* shows two coils, or units of any kind, connected in series between alternating-current supply mains. The supply voltage c . divides into two parts a and b such that $c = a + b$ at each instant; but from Fig. 207*b* it is evident that the voltmeter value of c is *not* equal to the sum of the voltmeter values of a and b, unless the phase difference θ is zero.

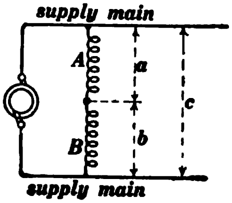

Fig. 207*a*.

a, b and c represent instantaneous values of electromotive force and $c = a + b$.

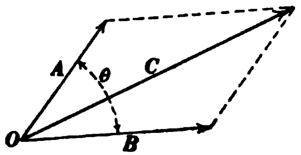

Fig. 207*b*.

Lengths of A, B and C represent voltmeter values of a, b and c.

(*b*) *Dividing of an alternating current between two coils in*

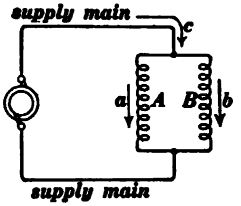

Fig. 208*a*.

a, b and c represent instantaneous values of current and $c = a + b$.

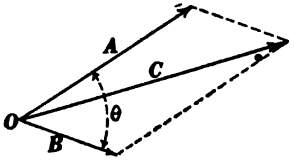

Fig. 208*b*.

Lengths of A, B and C represent ammeter values of a, b and c.

parallel. Figure 208*a* shows two coils, or units of any kind, connected in parallel with each other between alternating-current supply mains. The total current c divides into two parts a and b such that $c = a + b$ at each instant; but from Fig. 208*b* it is evident that the ammeter value of c is *not* equal to the sum of the ammeter values of a and b, unless the phase difference θ is zero.

145. Instantaneous and average values of power.—Let e be the value at a given instant of the electromotive force acting on a receiving circuit, and let i be the value of the current at the same instant; then ei is the power in watts which is being delivered to the receiving circuit at the given instant, and the average value of ei is the average power delivered to the receiving circuit. This *average power* is usually called, simply, the *power* delivered to the receiving circuit, and it is ordinarily *not* equal to the product EI where E is the voltmeter value of the electromotive force and I is the ammeter value of the current.

Some idea of the pulsating character of the instantaneous power ei as delivered by alternating-current supply mains may be obtained from Figs. 209, 210 and 211. Negative values of the power ei mean power returned to the supply mains by the

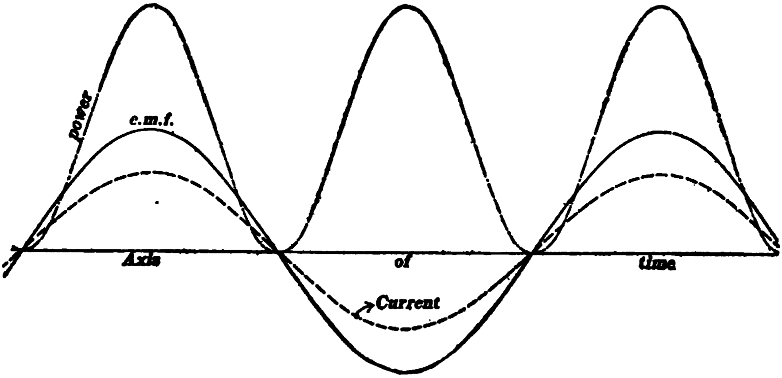

Fig. 209.

Electromotive force e and current i in phase with each other. Average power in this particular case is equal to EI.

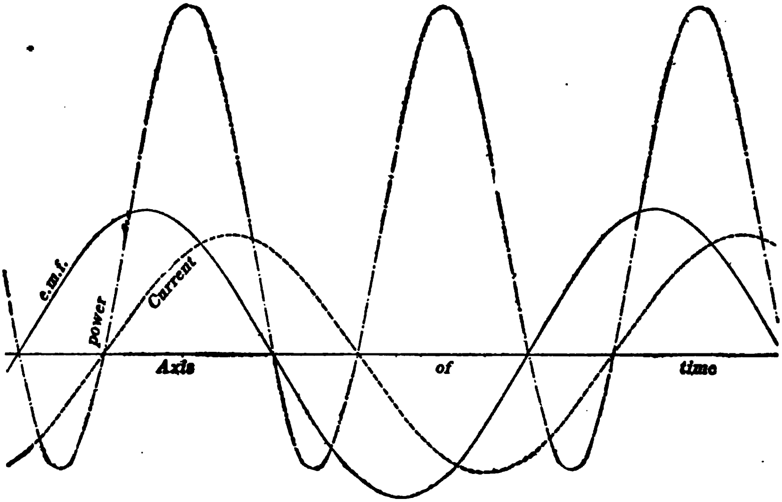

Fig. 210.

Current i is 60° (one sixth of a cycle) behind electromotive force e in phase. Average power in this particular case is $\frac{1}{4}EI$.

receiving circuit. The relations in Fig. 211 are exactly analogous to the relation between the alternating velocity i of a vibrating weight and the force e exerted on the weight by a supporting spring (friction being negligible). During a quarter of a cycle

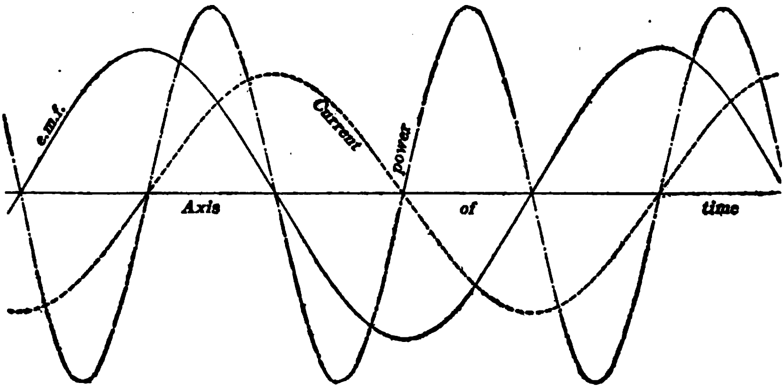

Fig. 211.

Current i is 90° (one quarter of a cycle) behind electromotive force e in phase. Average power in this particular case is zero.

the spring pushes on the weight in the direction in which it is moving and *the spring does work on the weight* (power ei positive); and during the next quarter of a cycle the push of the spring opposes the motion of the weight and the *weight does work on the spring* (power ei negative).

146. The wattmeter.—Power taken from alternating-current supply mains is nearly always measured by means of a *wattmeter* which consists of a delicately pivoted coil of fine wire A connected directly across the supply mains in series with a large non-inductive* resistance R, and a stationary coil of coarse wire B connected in series with the receiving circuit L as indicated in Fig. 212. The force action between the coils A and B moves the pivoted coil A, and the pointer which is attached to the pivoted coil plays over a divided scale.

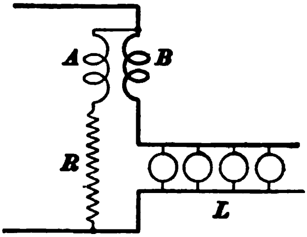

Fig. 212.

(*a*) Let s be the scale reading of the instrument when the instrument is connected to direct-current supply mains (connections the same as in Fig. 212) with constant voltage $\mathcal{E}$ across the mains and with constant current $\mathcal{I}$ flowing through coil B and the receiving circuit. The force exerted on the pivoted coil A is then equal to $k \cdot \frac{\mathcal{E}}{R} \cdot \mathcal{I}$, where k is a constant.

(*b*) Let the instrument give the same scale reading s when it is connected to alternating-current supply mains as indicated in Fig. 212, let e be the value of the alternating electromotive force at a given instant, and let i be the value at the same instant of the alternating current flowing through coil B and the receiving circuit. Then the value of the current in coil A at the given instant is e/R, the instantaneous value of the force exerted on the pivoted coil is $k \cdot \frac{e}{R} \cdot i$, and the average value of this force is $\frac{k}{R} \times$ average ei. But since the deflection of the instrument is the same as under (*a*) above this average force must be equal to the steady force $k \cdot \frac{\mathcal{E}}{R} \cdot \mathcal{I}$. Therefore

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* A circuit of which the inductance is negligibly small is called a non-inductive circuit. See second footnote on page 228.

$$\frac{k}{R} \times \text{average } ei = k \cdot \frac{\mathcal{E}}{R} \cdot \mathcal{I}$$

or

$$\text{average } ei = \mathcal{E}\mathcal{I}$$

But average ei is the power delivered by the alternating current mains, and $\mathcal{E}\mathcal{I}$ is the steady direct-current power which gives the same deflection. Therefore *the wattmeter indicates alternating-current power correctly when it has been calibrated to read direct-current power, the circuit AR being noninductive.*

147. Power factor of a receiving circuit.—Let E be the voltmeter value of the alternating electromotive force between the supply mains, let I be the ammeter value of the alternating current which is delivered to a receiving circuit, and let P be the power delivered to the receiving circuit as measured by a wattmeter. Then EI is usually larger in value than P, and the quotient P/EI is called the *power factor* p of the receiving circuit. That is

$$p = \frac{P}{EI} \tag{69}$$

The power factor of a non-inductive circuit is unity.—This may be shown as follows: Let e be the instantaneous value of the electromotive force and i the instantaneous value of the current. Then $i = e/R$ and $ei = e^2/R$, so that

$$\text{average power} = \text{average } ei = \frac{1}{R} \times \text{average } e^2$$

but E^2 = average e^2, so that

$$\text{average power} = \frac{E^2}{R} \tag{i}$$

Furthermore $Ri = e$ or $R^2i^2 = e^2$, so that

$$R^2 \times \text{average } i^2 = \text{average } e^2$$

or

$$R^2I^2 = E^2 \tag{ii}$$

where E is the voltmeter value of the electromotive force and I is the ammeter value of the current. Using equation (ii) we may eliminate R from equation (i) and we get

$$\text{average power} = EI$$

and therefore, according to equation (69), the power factor p is equal to unity.

148. The induction type watt-hour meter.—The energy delivered to the customers of an electric power and lighting plant is measured by the *watt-hour meter*, and as alternating-current service is coming to be almost universal the induction type of

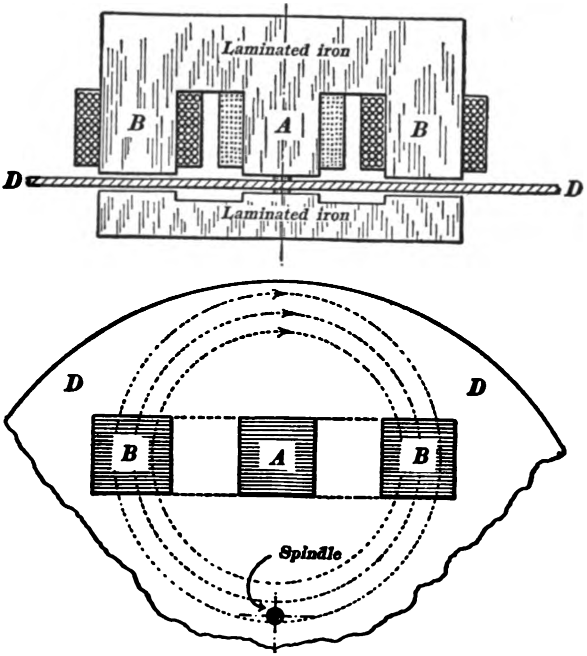

Fig. 213.

watt-hour meter is almost universally used. The essential features of this instrument are shown in Fig. 213. A thin disk of aluminum or copper DD is mounted on a delicately pivoted spindle, and the disk is acted upon by the three laminated magnet poles ABB as indicated. The pole or lug A is wound with fine wire which is connected directly across the alternating-current supply mains (resistance of coil A and its connecting wires is very small). The poles or lugs BB are wound with coarse wire and this winding is connected in series with the customer's receiving circuit. Under these conditions the disk turns at a rate which is proportional to the rate of delivery of energy to the customer so that the number of revolutions made by the disk in a month, say, is proportional to the energy delivered.*

Explanation of action of induction type of watt-hour meter. Resistance of coil A being negligible a sufficient alternating flux must pass through lug A to induce in coil A a back electromotive force at each instant equal to the alternating supply voltage e; this alternating flux in passing through the disk induces an alternating electromotive force ke along the dotted lines in Fig. 213; and this electromotive force ke produces an alternating current equal to hke along the dotted lines in the disk in Fig. 113, where h and k are proportionality constants (the current paths in the disk are approximately non-inductive). The magnetic field under the lugs BB is proportional to the alternating current i which flows through the coils BB, the current hke in flowing under the lugs BB causes the magnetic field under the lugs to push sidewise on the disk material through which the current hke flows, and this side push is proportional to i and to hke; therefore the torque which drives the disk is at each instant proportional to ei and the average driving torque is proportional to average ei. The rotation of the disk is hindered by passing between the poles of an auxiliary permanent steel magnet called a *damping magnet*, and thus the hindering torque is proportional to the speed of the disk. But driving torque and hindering torque balance each other, very nearly, and therefore the speed of the disk is proportional to the driving torque or to ei.

149. Expressions for average power P in case of harmonic alternating electromotive force and current.—The discussion in Arts. 145 to 148 is not restricted to harmonic electromotive force and current, it is entirely general. When electromotive force and current are harmonic the average power may be expressed by the equation

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* See discussion of this simple case of integration in footnote on page 148.

$$P = \frac{\mathrm{EI}}{2} \cdot \cos \theta \tag{70}$$

or, using the values of **E** and **I** from equations (67) and (68), we get

$$P = EI \cos \theta \tag{71}$$

where P is the average power, E is the voltmeter value of the electromotive force, I is the ammeter value of the current, and θ is the angular phase difference between electromotive force and current.

By comparing equation (71) with equation (69) it may be seen that, in the case of harmonic electromotive force and current, the power factor of a circuit is equal to $\cos \theta$.

It is to be noted that $-\theta$ may be substituted for $+\theta$ in equations (70) and (71) without changing the sign of P and without changing its value. That is to say, for a given value of phase difference the average power is the same whether the current be lagging or leading.

Derivation of equation (70). Equation (70) may be established by reducing the expression average $ei = \frac{\mathrm{EI}}{2\pi} \int_{\omega t=0}^{\omega t=2\pi} \sin \omega t \sin (\omega t - \theta) \cdot d(\omega t)$, but it is more instructive to proceed as follows: Using equations (66), we have

$$ei = \mathrm{EI} \sin \omega t \sin (\omega t - \theta)$$

or, expanding $\sin (\omega t - \theta)$, we get

$$ei = \mathrm{EI} \cos \theta \cdot \sin^2 \omega t - \mathrm{EI} \sin \theta \cdot \sin \omega t \cos \omega t$$

so that, taking average values, we have

$$P = \text{average } ei = \mathrm{EI} \cos \theta \times \text{average } (\sin^2 \omega t) - \mathrm{EI} \sin \theta \times \text{average } (\sin \omega t \cos \omega t)$$

But the average value of $\sin^2 \omega t$ is equal to $\frac{1}{2}$ as explained in Art. 141, and the average value of $\sin \omega t \cos \omega t$ is zero because it passes through identical sets of positive and negative values in a similar way. Therefore

$$P = \text{average } ei = \mathrm{EI} \cos \theta \times \tfrac{1}{2}$$

150. Power component of current and wattless component of current.—Figure 214 represents a harmonic alternating current

I which is $\theta°$ behind the harmonic supply voltage E in phase. The component $I \cos \theta$ is often called the *power component* of I, and the component $I \sin \theta$ is often called the *wattless component* of I. Similarly, the electromotive force E may be resolved into components parallel to I and perpendicular to I, namely, $E \cos \theta$ and $E \sin \theta$, and these are called the *power component* of E and the *wattless component* of E respectively.

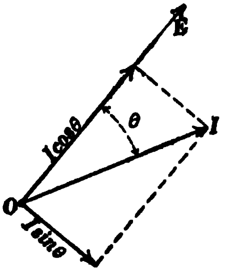

Fig. 214.

151. The voltage-current relation. Special cases.—The most important equation in the elementary theory of alternating currents is the equation expressing the harmonic electromotive force required to maintain a given harmonic alternating current

$$i = \mathrm{I} \sin \omega t \tag{72}$$

in a given circuit. To establish the voltage-current equation in its general form it is necessary to consider the following special cases.

(*a*) *Electromotive force required to maintain the given current in a non-inductive receiving circuit of resistance R.*—The required electromotive force, e, is equal to Ri at each instant. Therefore, using equation (72), we have

$$e = R\mathrm{I} \sin \omega t \tag{a}$$

This electromotive force is in phase with i, and its maximum value is equal to $R\mathrm{I}$, where I is the maximum value of the given current.

(*b*) *Electromotive force required to maintain the given harmonic current* [*see equation* (72)] *in a circuit of inductance L but of zero resistance.*—In this case the electromotive force is used only to make the current increase and decrease, and it is therefore at each instant given by the equation $e = L\frac{di}{dt}$, according to Art. 48. Therefore, differentiating equation (72), we get $\frac{di}{dt} = \omega \mathrm{I} \cos \omega t$, but $\cos \omega t = \sin (\omega t + 90°)$. Therefore $\frac{di}{dt} = \omega \mathrm{I} \sin (\omega t + 90°)$, so that the required electromotive force is

$$e = \omega L\mathrm{I} \sin (\omega t + 90°) \tag{b}$$

that is to say, the required electromotive force is 90° ahead of i in phase [see equations (66)] and its maximum value is equal to $\omega L\mathrm{I}$.

(*c*) *Electromotive force required to maintain the given harmonic current " through " a condenser of which the capacity is* C *farads.* This problem is greatly simplified by first considering the electromotive force required to make the charge q of the condenser vary harmonically so that we may have:

$$q = \mathrm{Q} \sin \omega t \tag{i}$$

The required electromotive force is at each instant equal to q/C according to equation (21) of Art. 55. Therefore the required electromotive force is

$$e = \frac{q}{C} = \frac{\mathrm{Q}}{C} \cdot \sin \omega t \tag{ii}$$

But the varying charge of the condenser involves a flow of current, namely, $i = \frac{dq}{dt}$. Therefore, differentiating equation (i) we get $i = \frac{dq}{dt} = \omega \mathrm{Q} \cos \omega t = \omega \mathrm{Q} \sin (\omega t + 90°)$, so that

$$i = \omega \mathrm{Q} \sin (\omega t + 90°) \tag{iii}$$

Equations (iii) and (ii) evidently give us a solution of our original problem because i as expressed by equation (iii) may be thought of as a *given* harmonic current, and e as given by equation (ii) is the electromotive force required to maintain the current i through the condenser. (*c*) *The required electromotive force is* 90° *behind* i *in phase and its maximum value is* Q/C *or* $\omega \mathrm{Q}/\omega C'$ *or* $\mathrm{I}/\omega C$ where $\mathrm{I}(= \omega \mathrm{Q})$ is the maximum value of the given current as expressed by equation (iii). It is desired, however, to get an expresison for the electromotive force required to maintain the original given current as expressed by equation (72). Therefore, making use of the above statement (*c*), and keeping in mind the expression for phase lag in equations (66), we get

$$e = \frac{\mathrm{I}}{\omega C} \sin (\omega t - 90^\circ) \qquad (c)$$

as the electromotive force required to maintain the current $i = \mathrm{I} \sin \omega t$ through a condenser of capacity C.

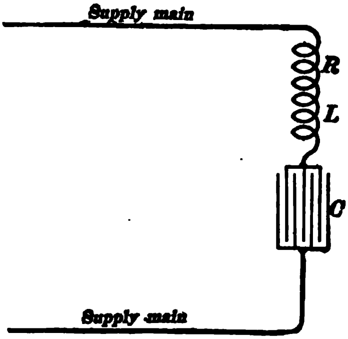

Fig. 215.

152. Voltage-current relation. General case.—*The electromotive force required to maintain a given harmonic current* [see equation (72)] *in a circuit of resistance R and inductante L and containing a condenser of capacity C, as shown in Fig.* 215. This electromotive force is the sum of the electromotive forces expressed by equations (*a*), (*b*) and (*c*) above. Therefore the required electromotive force is

$$e = R\mathrm{I} \sin \omega t + \omega L\mathrm{I} \sin (\omega t + 90^\circ) + \frac{\mathrm{I}}{\omega C} \sin (\omega t - 90^\circ) \quad (d)$$

and this equation may be most easily reduced to its simplest form by using the clock diagram of Fig. 216 in which the three

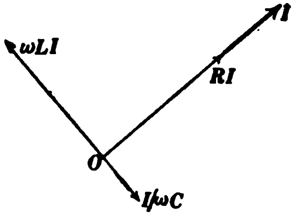

Fig. 216.

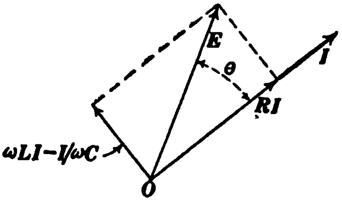

Fig. 217.

parts a, b and c of the required electromotive force are represented in value and in phase by the clock-diagram vectors $R\mathbf{I}$, $\omega L\mathbf{I}$ and $\mathbf{I}/\omega C$ respectively. The resultant of these three vectors is evidently the vector $\mathbf{E}$ in Fig. 217 from which we get

$$\mathbf{E}^2 = (R\mathbf{I})^2 + \left[\left(\omega L - \frac{1}{\omega C}\right)\mathbf{I}\right]^2$$

so that

$$\mathbf{E} = \mathbf{I}\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

whence, putting

$$X = \omega L - \frac{1}{\omega C} \tag{73}$$

and using equations (67) and (68), we get

$$\left.\begin{aligned} E &= I\sqrt{R^2 + X^2} \\ \text{or} \quad I &= \frac{E}{\sqrt{R^2 + X^2}} \end{aligned}\right\} \tag{74}$$

and from Fig. 217 we get

$$\tan\theta = \frac{X}{R} \tag{75}$$

That is to say, the electromotive force required to maintain a given alternating current I (ammeter value) in the circuit shown in Fig. 215 has a voltmeter value E which is given by equation (74) and it leads the current in phase by the phase angle θ which is given by equation (75). Equations (74) and (75) are together exactly equivalent to equation (d), and they are very much simpler to use.

The quantity X as given by equation (74) is called the *reactance* of the circuit and the quantity $\sqrt{R^2 + X^2}$ is called the *impedance* of the circuit. Reactance and impedance are both expressible in ohms.

153. Special cases of voltage-current relation.—*Case* (e). *Receiving circuit having resistance* R *and inductance* L *but not containing a condenser* (*a simple inductive circuit*). In this case the clock diagram of Fig. 216 reduces to Fig. 218, and equations (73), (74) and (75) become

$$X = \omega L \tag{76}$$

or

$$\left.\begin{aligned} E &= I\sqrt{R^2 + X^2} \\ I &= E/\sqrt{R^2 + X^2} \end{aligned}\right\} \tag{77}$$

$$\tan\theta = \frac{X}{R} \tag{78}$$

The current in a simple inductive circuit lags behind E in phase as indicated in Fig. 218, and the angle of phase lag approaches 90° (one quarter of a cycle) when R is very small.

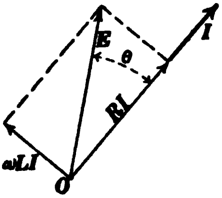

Fig. 218.

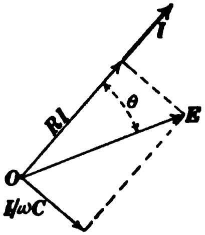

Fig. 219.

This is exactly analogous to the following: An alternating mechanical force e is exerted on a weight causing it to vibrate to and fro. Let us consider force e as positive when towards the right, and let us consider the velocity i of the weight as positive when towards the right. Then maximum positive force e is exerted on the weight when it is farthest towards the left, and maximum positive value of i occurs one quarter of a cycle later.

Case (f). Receiving circuit having resistance R and containing a condenser of capacity C farads. In this case the clock diagram of Fig. 216 reduces to Fig. 219, and equations (73), (74) and (75) become

$$X = -\frac{1}{\omega C} \tag{79}$$

or

$$\left.\begin{aligned} E &= I\sqrt{R^2 + X^2} \\ I &= E/\sqrt{R^2 + X^2} \end{aligned}\right\} \tag{80}$$

$$\tan\theta = \frac{X}{R}\ \text{(negative)} \tag{81}$$

The current in this case is ahead of E in phase as indicated in Fig. 219, and the angle of phase advance approaches 90° (one quarter of a cycle) when R is very small. This is exactly analogous to the following: A flat spring is fixed in a vise, and one takes hold of the free end of the spring and moves it to and fro. At a given instant the end of the spring has its maximum velocity towards the operator and a quarter of a cycle later the operator will be exerting his maximum pull.

154. Electric resonance.—When an alternating mechanical force of frequency f is exerted on a pendulum, the pendulum soon settles to a *steady state of oscillation at frequency* f, and this steady state of oscillation will be most violent if the frequency f of the alternating force happens to coincide with the frequency of free oscillation of the pendulum. This phenomenon is called *resonance*, and the circuit of Fig. 215 shows an exactly analogous phenomenon which is called *electric resonance.*

A harmonic alternating electromotive force of *given value* (as measured by a voltmeter) will produce a maximum of alternating current (as measured by an ammeter) in a circuit like Fig. 215 when the frequency is such as to make

$$X = \omega L - \frac{1}{\omega C} = 0$$

or

$$\omega = \sqrt{\frac{1}{LC}} \tag{82}$$

This expresses the critical frequency in radians per second; and, since $\omega = 2\pi f$, we have

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

where f is the critical frequency in cycles per second, or the frequency of free oscillation of the circuit in Fig. 215. [See equation (29) of Art. 63.]

At the critical frequency the reactance X is zero, and equations (74) and (75) reduce to

$$E = RI$$

or

$$I = E/R$$

and

$$\tan \theta = 0$$

That is to say, at the critical frequency the reactance of the condenser $(-1/\omega C)$ annuls the reactance of the inductance (ωL) and the voltage-current relation *after the full current is established* is the same as if the circuit contained resistance only.

The very sharply defined frequency at which resonance occurs is shown in Fig. 220. The ordinates of the curve represent the ammeter values of the current produced by a harmonic electromotive force of 200 volts (voltmeter value) in a circuit like Fig. 215 in which $R = 2$ ohms, $L = 0.352$ henry, and $C = 20$ microfarads, and the abscissas represent various frequencies. At very low frequencies the value of $1/\omega C$ is very great, and the cur-

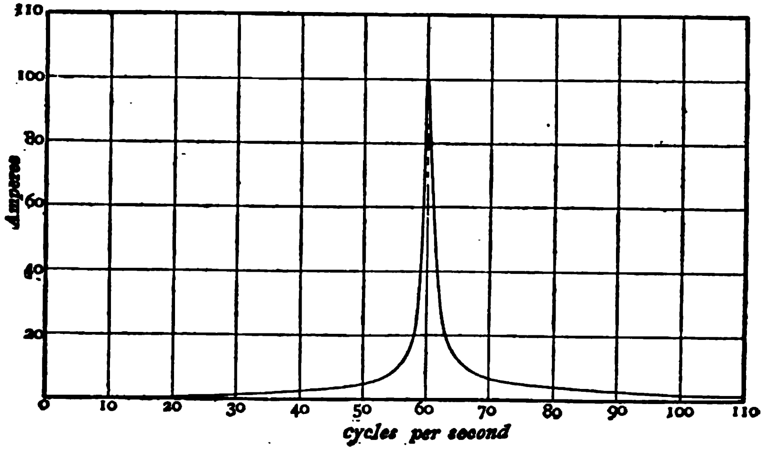

Fig. 220.

rent is limited by the condenser reaction; at the critical frequency of 60 cycles per second, the current is equal to E/R (= 100 amperes ammeter value); and at very high frequencies the value of ωL is very great, and the current is limited by the inductance reaction.

155. Multiplication of electromotive force by resonance.—Figure 221 represents a circuit like that shown in Fig. 215 except that the resistance R is supposed to be separated from the inductance L. When resonance exists $(\omega L = 1/\omega C)$ the current $I = E/R$, and the electromotive forces across inductance and condenser, namely, ωLI and $I/\omega C$ may both be very much larger than the supply voltage E. Thus when $E = 200$ volts at 60 cycles per second, $R = 2$ ohms, $L = 0.352$ henry and $C = 20$ micro-farads; then $\omega L = 1/\omega C$, $I = E/R = 100$ amperes, $\omega LI = 13{,}270$ volts, and $I/\omega C = 13{,}270$ volts.

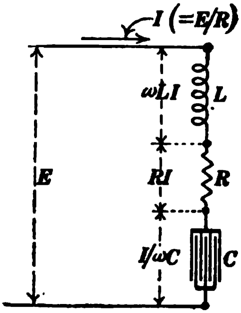

Fig. 221.

A strip of window glass has a small weight attached to one end and the other end is clamped in a vise; the strip of glass thus acts as a flat spring and when started the weight oscillates back and forth at a certain frequency f. A very small alternating force of frequency f is exerted on the weight by touching it with the finger in proper rhythm, the oscillations become more and more violent, and the strip of glass breaks showing that the forces developed in the resonating system may be much larger than the alternating force exerted on the system from the outside.

156. Multiplication of current by resonance.—When an inductance and a condenser are connected in parallel with each other as shown in Fig. 222, the total current I delivered to the two divides into two parts I_1 and I_2 both of which may be very much larger in value than I. The current relations are shown in Fig. 223, in which the vector OE represents the alternating electromotive force across the branch-points *a* and *b*, the line OI_1 represents the current in *L*, the line OI_2 represents the current in *C*, and the line *OI* represents the total current *I*.

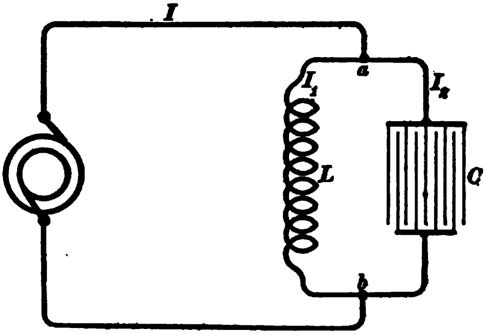

Fig. 222.

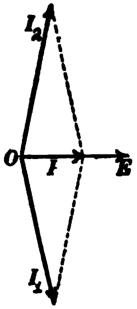

Fig. 223.

Although *L* and *C* are in *parallel* with each other in Fig. 222 in their relation to the supply mains, they are in series with each other in the closed circuit *LaCb*, and multiplication of current by resonance is due to the surging of current back and forth in this closed circuit.

The multiplication of current by resonance may be strikingly shown by connecting very small low-voltage glow lamps in the main circuit and in the two branches in Fig. 222. Then if $\omega L = 1/\omega C$ the two lamps in the branch circuit will be heated to full brilliancy while the single lamp through which the undivided current flows will not be perceptibly heated.

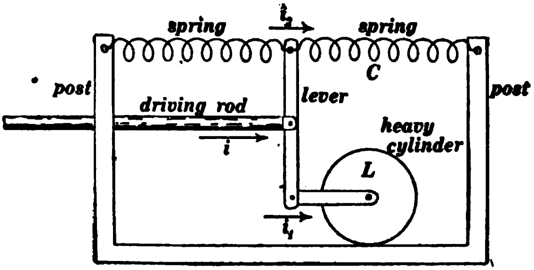

Fig. 224.

The multiplication of alternating current by resonance may be most easily understood in terms of the mechanical analog as follows:

The to and fro velocity i of the driving rod of Fig. 224 divides up into the two velocities i_1 and i_2 of the ends of the lever.	The main current i in Fig. 222 divides up into two currents i_1 and i_2 in the two branches L and C.
The velocity i of the driving rod is at each instant equal to *half* the algebraic sum of the velocities i_1 and i_2 of the ends of the lever.	The current i in the main circuit is at each instant equal to the *whole* algebraic sum of the currents i_1 and i_2 in the two branches.
At a critical frequency the to and fro motion of the driving rod sets up a see-saw motion of the lever, and the to and fro velocities of the ends of the lever i_1 and i_2 may both be much larger than i.	At a critical frequency the main current i sets up a "see-saw" (a surging of current back and forth around the closed loop $LaCb$), and the branch currents i_1 and i_2 may both be much larger than i.

157. Circuits in series. Voltage drop in a transmission line.—A transmission line is of course in series with the circuit to which it delivers current, and a consideration of the problem of voltage drop in a transmission line will furnish an example of the general problem of circuits in series.

(*a*) *Voltage drop in a transmission line which delivers current to a non-inductive receiving circuit.*—Let OI, Fig. 225, represent the current flowing through the transmission line and the receiving circuit. The voltage E_1 at the terminals of the receiv-

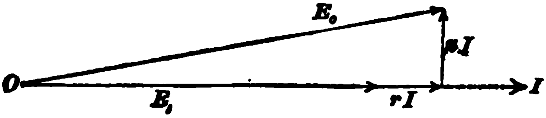

Fig. 225.

ing circuit is in phase with I, and the generator voltage E_0 is the vector sum of E_1, rI and xI; where rI is the electromotive force used to overcome the line resistance r, and xI is the electromotive force used to overcome the line reactance x. An examination of the figure shows that in this case (receiving circuit non-inductive) the line resistance produces a difference in *value* between E_0 and E_1, whereas the line reactance produces a difference in *phase* between E_0 and E_1.*

(*b*). *Voltage drop in a transmission line which delivers current to a highly inductive receiving circuit of small resistance.*—Let OI, Fig. 226, represent the current flowing through the transmission line and the receiving circuit. The voltage E_1 at the terminals of the receiving circuit is nearly 90° ahead of I in phase as shown, and the generator voltage E_0 is the vector sum of E_1, rI and xI as before. In the present case, however, the line resistance produces a difference in *phase* between E_0 and E_1, and the line reactance produces a difference in *value* between E_0 and E_1.†

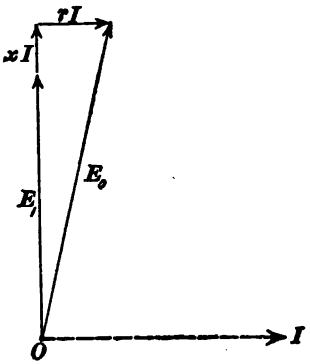

Fig. 226.

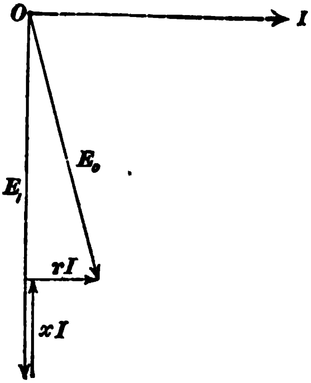

Fig. 227.

(*c*) *Voltage drop in a transmission line which delivers current*

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* This statement is made on the assumption that xI is small as compared with E_1.

† This statement is made on the assumption that rI is small as compared with E_1.

to a condenser or any receiving circuit having negative reactance, but having small resistance.—This case is shown in Fig. 227, and E_0 is the vector sum of E_1, rI and xI as before; but, E_1 and xI are nearly opposite in direction, inasmuch as E_1 is nearly 90° behind I under the assumed conditions. Therefore in this case the line resistance produces a difference in *phase* between E_0 and E_1 and the line reactance produces a difference in *value* between E_0 and E_1, **making** E_1 **greater than** E_0.

(*d*) *Voltage drop in a transmission line which delivers current to a receiving circuit whose power factor* (*cos* θ) *has any given value.*—Let the line OI, Fig. 228, represent the current flowing through the transmission line and the receiving circuit. Let the line E_1 represent the voltage across the receiving circuit, θ being the phase difference between I and E_1 as shown. Let r be the resistance and x the reactance of the transmission line, and let E_0 be the voltage at the generator. Lay off rI parallel to OI, and xI at right angles to OI. Then the difference between the values of E_1 and E_0 is the required transmission line drop.

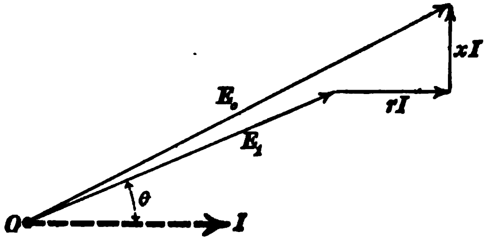

Fig. 228.

The value of $\sqrt{r^2I^2 + x^2I^2}$ is called the *impedance drop*, and the difference between the numerical values of E_0 and E_1 is called simply the (*voltage*) *drop* in the line.

158. Circuits in parallel.—The general problem of circuits in parallel is somewhat complicated if one attempts to find algebraic expressions for the combined resistance and reactance of two or more circuits in parallel. Two simple cases of this problem are, however, of considerable interest, namely, (*a*) the problem of finding the power factor of two receiving circuits in parallel, the power factor of each and the current delivered to each being given, and (*b*) the problem of compensating for lagging currents.

(*a*) *Power factor of receiving circuits in parallel.*—Let the line OE, Fig. 229, represent the voltage across the two receiving circuits. Let I_1 be the current delivered to receiving circuit number one and $\cos\theta_1$ its power factor, and let I_2 be the current delivered to receiving circuit number two and $\cos\theta_2$ its power factor. Lay off the clock diagram in Fig. 229 carefully to scale, and draw the diagonal I as shown. The angle θ between E and I is then the angle whose cosine is the required power factor of the two receiving circuits in parallel.

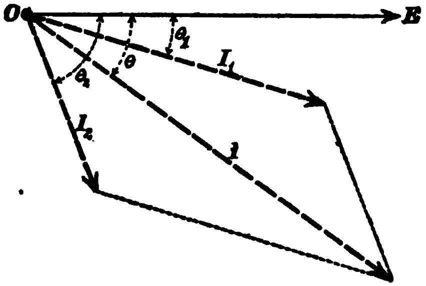

Fig. 229.

(*b*) *Compensation for lagging currents.*—When a transmission line delivers current to an inductive receiving circuit, power is delivered over the line to the receiving circuit during the time that the product ei is positive, and power is delivered back over the line from the receiving circuit to the generator during the time that the product ei is negative (compare Art. 145). This backward flow of energy from receiving circuit to generator represents an essentially unnecessary service of the transmission line, and it is desirable, and in some cases feasible, to reduce this backward flow of energy by connecting a condenser or something which is equivalent to a condenser (such as an over-excited synchronous motor) in parallel with the receiving circuit.

Let OE, Fig. 230, represent the voltage at the terminals of the receiving circuit, and let OI represent the lagging current delivered to the receiving circuit. Let R be the resistance of the receiving circuit and X its reactance. Then

$$\tan\theta = \frac{X}{R}$$

and

$$\sin\theta = \frac{X}{\sqrt{R^2 + X^2}}$$

The component Ob of the current I is equal to $I \sin\theta$ or $IX/\sqrt{R^2 + X^2}$. But I is equal to $E/\sqrt{R^2 + X^2}$, so that the component Ob is equal to $EX/(R^2 + X^2)$. If a condenser of capacity C is connected across the transmission line at the receiver end, then the current flowing into the condenser will be 90° ahead of E, or parallel to Oc, Fig. 230, and its value will be equal to E divided by the condenser reactance $1/\omega C$, or to $E\omega C$; and if this current is numerically equal to Ob, the sum of I and Oc will be in phase with E. That is, the receiving circuit and the condenser together will take current in phase with E, and the instantaneous value of power delivered over the transmission line will never be negative. The capacity of the condenser required to produce this result is given by the relation above mentioned, namely, $E\omega C = EX/(R^2 + X^2)$, which gives

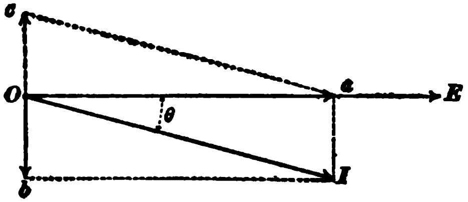

Fig. 230.

$$C = \frac{X}{\omega(R^2 + X^2)}$$

PROBLEMS.

140. A device is arranged to connect a voltmeter to direct-current supply mains and disconnect it repeatedly, connection never reversed. The voltmeter is connected for 0.01 second and

disconnected for 0.04 second, over and over. The supply voltage is 110 volts. Find the reading of the voltmeter (*a*) When it is of the direct-current type, and (*b*) When it is of the alternating-current type. Ans. (*a*) 22 volts; (*b*) 49.2 volts.

141. A receiving circuit takes 50 amperes from 110-volt alternating-current supply mains, and the power delivered to the receiving circuit, as measured by a wattmeter is 4620 watts. What is the power factor of the receiving circuit?

142. The electromotive force e in Fig. 209 is harmonic and its maximum value is 155.6 volts. The current i is harmonic and its maximum value is 141.4 amperes. Find (*a*) voltmeter value of e, (*b*) ammeter value of i, (*c*) average value of ei, and (*d*) maximum value of ei. Ans. (*a*) 110 volts; (*b*) 100 amperes; (*c*) 11,000 watts; (*d*) 22,000 watts.

143. The electromotive force e in Fig. 210 is harmonic and its maximum value is 311.2 volts, and the current i is harmonic and its maximum value is 212.1 amperes. Find (*a*) voltmeter value of e, (*b*) ammeter value of i, (*c*) average value of ei. Ans. (*a*) 220 volts; (*b*) 150 amperes; (*c*) 16,500 watts.

144. A receiving circuit takes 200 amperes from alternating supply mains, as measured by an ammeter, the supply voltage is 220 volts as measured by a voltmeter, and the power delivered to the receiving circuit is 38.9 kilowatts as measured by a wattmeter. Assuming electromotive force and current to be harmonic calculate the phase difference between voltage and current and calculate the values (ammeter values) of power component and wattless component of the current. Ans. 28°; 176.6 amperes power component, 93.9 amperes wattless component.

145. Two alternators A and B are connected in series. The electromotive force of A is 1,100 volts, and the electromotive force of B is 1,200 volts. The electromotive force of A is 90° ahead of the electromotive force of B in phase. (*a*) What is their combined electromotive force? The two alternators give a current of 125 amperes which lags 30° behind their resultant electromotive force in phase. (*b*) What is the power output of each alternator? Ans. (*a*) 1,628 volts; (*b*) output of A 29,780 watts, output of B 146,470 watts.

146. The electromotive force of alternator A, problem 145, is 135° ahead of the electromotive force of alternator B in phase. A current of 120 amperes flows through both alternators, lagging 30° behind their resultant electromotive force. What is the output of each alternator. Ans. Output of A 30,800 watts negative, output of B 122,770 watts positive.

Note. Negative output means intake as a motor.

147. Two coils A and B are connected in series across 110-volt alternating-current mains. The voltage across A is 76 volts and the voltage across B is 80 volts. What is the phase difference between the voltages across A and B, assuming all of the voltages to be harmonic? Ans. 90° 21′.

Note. It is impossible to determine from the data of this problem which of the voltages, A or B, is ahead of the other in phase.

148. A non-inductive rheostat of which the resistance is 120 ohms is connected in series with an alternating-current fan motor to 220-volt alternating supply mains. The voltage across the 120-ohm rheostat is 125 volts, and the voltage across the fan motor is 128 volts. Find (*a*) the current flowing through the circuit, (*b*) the phase difference between the current and the voltage across the fan motor, and (*c*) the power delivered to the fan motor. Ans. (*a*) 1.04 amperes, (*b*) 59° 10′, (*c*) 68.34 watts.

Note. It is impossible from the data of this problem to tell whether the voltage across the 120-ohm rheostat is ahead of or behind the voltage across the fan motor in phase. As a matter of fact, the voltage across the 120-ohm rheostat is in phase with the current, and the voltage across the fan motor is ahead of the current in phase.

149. An alternator delivers 200 amperes of current to glow lamps, and 75 amperes to start an induction motor. The power factor of the motor while starting is 0.3. Find the total current on the assumption that both currents are harmonic. Ans. 233.7 amperes.

150. A harmonic alternating current, maximum value 100 amperes, frequency 60 cycles per second, flows through a circuit consisting of a non-inductive resistance of 2 ohms, a resistanceless inductance of 0.003 henry, and a condenser of which the capacity is 0.00006 farad, all connected in series. Draw a clock diagram representing the phases of the electromotive forces across the resistance, across the inductance and across the condenser, respectively, and calculate the effective value of each. Ans. 141.4 volts across the resistance, 79.97 volts across the inductance, and 3,126 volts across the condenser.

151. (*a*) Calculate the reactance value of 0.12 henry at 60 cycles per second. (*b*) Calculate the reactance value of a 20-microfarad condenser at 60 cycles per second. (*c*) Calculate the reactance of both in series. (*d*) At what frequency is the reactance value of both in series equal to zero? Ans. (*a*) 45.2 ohms, (*b*) − 132.6 ohms, (*c*) − 87.39 ohms, (*d*) 102.7 cycles per second.

152. A coil of wire of negligible resistance is connected across 110-volt 60-cycle supply mains and the current in the coil as measured by an ammeter is 10.46 amperes. What is the inductance of the coil? Ans. 0.0279 henry.

Note. Electromotive force and current assumed to be harmonic in this and in all of the following problems.

153. A coil of wire of which the resistance is 2.1 ohms is connected to 110-volt 60-cycle supply mains, and the current in the coil as measured by an ammeter is 10.25 amperes. What is the inductance of the coil?

154. A 20-micro-farad condenser is connected across 110-volt 60-cycle mains. Calculate the ammeter value of the current. Ans. 0.829 ampere.

155. An inductance of 0.08 henry and a condenser of 25 microfarads capacity are connected in series, the resistance of the circuit being 7 ohms. (*a*) What is the critical frequency of the circuit, that is to say, at what frequency does resonance occur? (*b*) What is the effective value of the electromotive force across the condenser terminals when an effective electromotive force of

110 volts at the critical frequency acts upon the circuit? Ans. (*a*) 112.5 cycles per second, (*b*) 888.9 volts.

156. A harmonic electromotive force of 1,100 volts (effective) produces a harmonic current of 100 amperes (effective) in a given circuit, the current being 20° behind the electromotive force in phase. Calculate the resistance, reactance, and impedance of the circuit. Ans. Resistance 10.34 ohms, reactance 3.76 ohms, impedance 11 ohms.

Note. In all of the following problems effective or voltmeter values of electromotive force, and effective or ammeter values of current are to be understood.

157. A harmonic electromotive force of 110 volts produces a current of 15 amperes in a receiving circuit of which the power factor is equal to 0.7. Find the values of resistance, reactance, and impedance. Ans. Resistance 5.13 ohms, reactance ± 5.24 ohms, impedance 7.33 ohms.

Note. From the data of this problem it is impossible to determine whether the current is ahead of or behind the electromotive force in phase. It is desirable always in specifying the value of a power factor of a receiving circuit to state whether the current leads or lags, thus one would speak of a 70 per cent. leading power factor or a 70 per cent. lagging power factor, as the case may be.

158. A transmission line of which the resistance is 4 ohms and the reactance is 3 ohms delivers 100 amperes to a non-inductive receiving circuit, the electromotive force across the terminals of the receiving circuit being 10,000 volts. (*a*) What is the value of the generator voltage, and (*b*) what is the phase difference between generator voltage and the voltage across the receiving circuit? Ans. (*a*) 10,404.3 volts, (*b*) 1° 39′.

159. If the transmission line specified in problem 158 were to deliver 100 amperes of current at 10,000 volts to a condenser, what would be the value of the generator voltage? Ans. 9,703.1 volts.

160. The transmission line specified in problem 158 delivers 100 amperes of current at 10,000 volts to a receiving circuit of which the power factor (lagging) is 0.7. What is the value of the generator voltage? Ans. 10,494.6 volts.

161. Two condensers, each of negligible resistance, have capacities of 0.5 and 0.05 microfarad respectively. The two condensers are connected in series to 1,100-volt alternating-current mains. What is the electromotive force between the terminals of each condenser? Ans. 100 volts, and 1,000 volts respectively.

162. An alternator delivers current to two inductive receiving circuits in parallel. One receiving circuit takes 20 amperes and its power factor (lagging) is 0.9, and the other receiving circuit takes 25 amperes and its power factor (lagging) is 0.7. What is the power factor of the combination? Ans. 0.801.

163. An alternator delivers 100 amperes at 1,100 volts and 60 cycles per second to an inductive receiving circuit of which the power factor is 0.85. What capacity condenser would be required to compensate for lagging current? What number of leaves of paraffined paper 22 by 27 centimeters would be required for this condenser, thickness of paraffined paper being 0.08 centimeter, allowing 1 centimeter margin beyond the tinfoil? Take inductivity of paraffined paper equal to 2. Ans. 127.0 microfarads, 114,800 leaves.

Note. This condenser would have a volume of about 400 cubic feet and its cost would be excessive. It is impracticable to use a condenser for supplying wattless current. The only practicable device for this purpose is the over-excited synchronous motor.

CHAPTER XII.

POLYPHASE SYSTEMS.

159. The single-phase system and the polyphase system.—The single-phase system consists essentially of a simple alternator delivering alternating current to a single circuit. The polyphase system consists essentially of two or more electrically distinct (but not mechanically distinct) alternators delivering two or more distinct alternating currents over two or more distinct transmission lines to two or more electrically distinct receiving circuits; the several alternating currents being of the same frequency and having fixed phase relations with each other. The polyphase system has come into use chiefly on account of the requirements of the polyphase induction motor and synchronous converter.

160. The two-phase alternator.—Figure 231 shows the arrange-

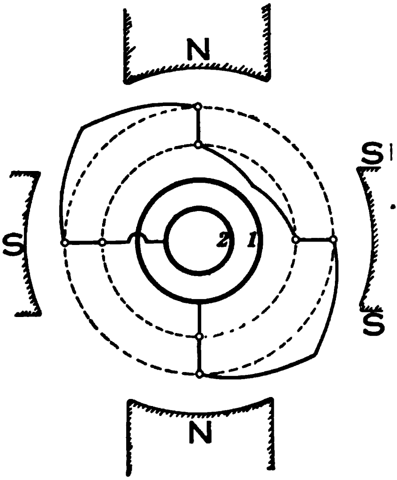

Fig. 231.

Single-phase alternator.

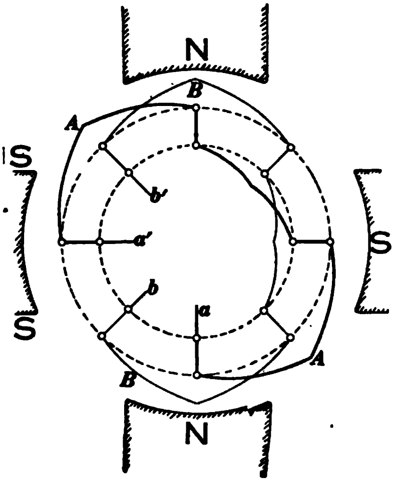

Fig. 232.

Two-phase alternator.
4 collector rings not shown.

ment of a very simple type of single-phase alternator, exactly like Fig. 58, and as in Fig. 58 the dotted circles represent front and back ends of the armature, the short and heavy radial lines represent the armature conductors (parallel to the armature shaft of course), the curved lines *B* represent cross connections at the back end of the armature, the curved line *F* represents a connection across the front end of the armature, and the straight lines *CC* are the connections to the two collector rings which are represented by the two small black circles. The field magnet poles are in fact very close to the armature but they are shown widely separated from the armature in Fig. 231 to give room for showing the back connections. Figure 232 is diagram exactly like Fig. 231 but showing a two-phase alternator with two distinct armature windings *A* and *B*, the terminals *aa'* of winding *A* being connected to two collector rings and the terminals *bb'* of *B* being connected to two collector rings, four collector rings in all.

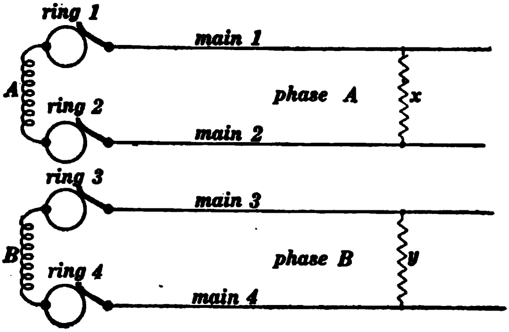

Fig. 233*a*.

Conventional diagram of the four-ring, four-wire arrangement of the two-phase system. Receiving circuits are represented by *x* and *y*.

This arrangement is shown conventionally in Fig. 233*a*, the *four-ring, four-wire arrangement* of the two-phase system.

One collector ring can be used as a common terminal for the two armature windings *A* and *B*, and a common return wire may be used for the two phases as indicated in Fig. 233*b*;

this is the *three-ring, three-wire arrangement* of the two-phase system. The four-ring four-wire arrangement of the two-phase system is generally used in practice.

The two-phase clock diagram.—Figure 232 shows the armature conductors of the A winding in the middle of each field pole (electromotive force of A winding at its maximum value), and the conductors of the B winding midway between the field poles (electromotive force of B winding at its zero value). Therefore the electromotive force of the A winding is a quarter of a cycle (90°) ahead of or behind the electromotive force of the B winding, and the two electromotive forces A and B may be represented by the clock-diagram vectors A and B in Fig. 234. If the receiving circuits are exactly alike then the currents x and y (see Fig. 233) are the same in value and they lag equally behind the corresponding electromotive forces A and B, as shown by the clock-diagram vectors x and y in Fig. 234. When the receiving circuits x and y are alike the system is said to be *balanced*.

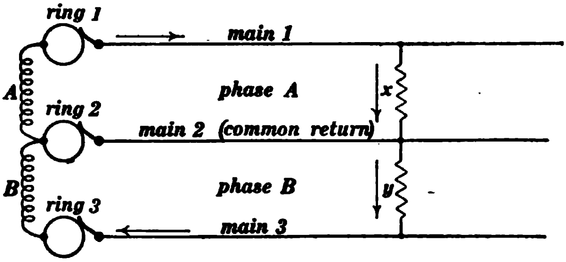

Fig. 233*b*.

Conventional diagram of the three-ring, three-wire arrangement of the two-phase system. Receiving circuits are represented by x and y.

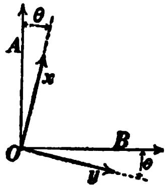

Fig. 234.

Two-phase clock diagram.

161. The three-phase alternator.—Figure 235 is a diagram exactly like Figs. 231 and 232 but showing a three-phase alternator with three distinct armature windings A, B and C, and in the most general case each terminal a, a', b, b', c and c' is connected to a collector ring, six collector rings in all. However, one collector ring can be used as a common terminal for the three armature windings A, B and C, and a common return wire may be used for the three phases as indicated in Fig. 236.

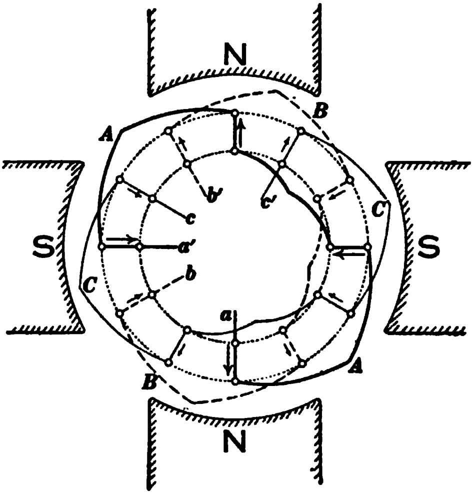

Fig. 235.

Three-phase alternator. Collector rings not shown.

This is the *four-ring, four-wire arrangement* of the three-phase system. The three currents x, y and z come together in the common return wire.

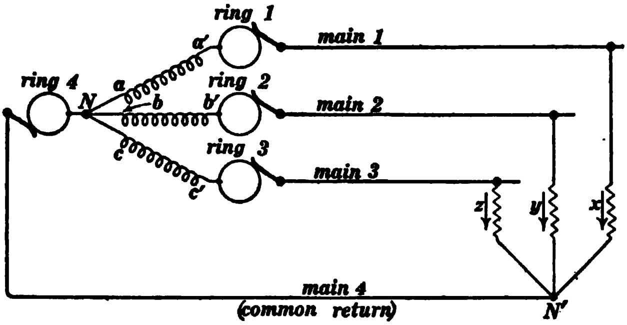

Fig. 236.

Conventional diagram of the four-ring four-wire arrangement of the three-phase system. Receiving circuits are represented by x, y and z.

The three armature windings in Fig. 235 are alike and they are spaced so that the electromotive forces which are induced in the windings are equal and 60° apart in phase as represented by the clock-diagram vectors A, B and C in Fig. 237. Let us assume that the three receiving circuits are alike, then the three currents x, y and z are the same in value and they lag equally behind the respective electromotive forces A, B and C as indicated in Fig. 237. Under these conditions the system is said to be balanced, and it is evident that the current y is at each instant equal to the algebraic sum of x and z, because the vector y is the diagonal of the parallelogram of which the vectors x and z are the sides. Therefore, if the connections bb' of the B winding in Fig. 236* are reversed, the current in main 2 and receiving circuit y will be reversed as represented by the vector y' in Fig. 237, and the algebraic sum of the currents x, y and z (which will be the current in main 4 in Fig. 236) will be zero at all times. Therefore, with similar receiving circuits (balanced system) and with connections of winding B reversed as stated, there will be no current at all in main 4 so that return main 4 and ring 4 may be dispensed with. The

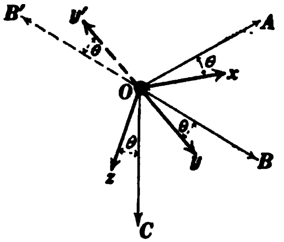

Fig. 237.

Hereafter the vectors B' and y' will be used to represent voltage and current of phase B, but prime marks will be omitted as in Fig. 238.

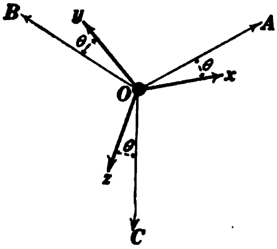

Fig. 238.

Conventional three-phase clock diagram.

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[*] The lettering in Figs. 235 and 236 show the conditions *after* this reversal of connections. That is to say, the lettering and connections in Figs. 235 and 236 correspond to the clock diagram of Fig. 238.

three winding terminals a, b and c are connected together at what is called the *neutral junction* N, the three winding terminals a', b' and c' are connected to the three collector rings and thence to the three mains—three mains only being used, and any two of the mains may be thought of as the return for the other one. We thus have the commonly used *three-ring, three-wire arrangement* of the three-phase system.

Figure 239*a* is a winding diagram of a 4-pole, three-phase alternator, the three armature windings A, B and C being connected together at the neutral junction N and to three collector rings as above explained. This arrangement constitutes what is called the Y-scheme of connections.

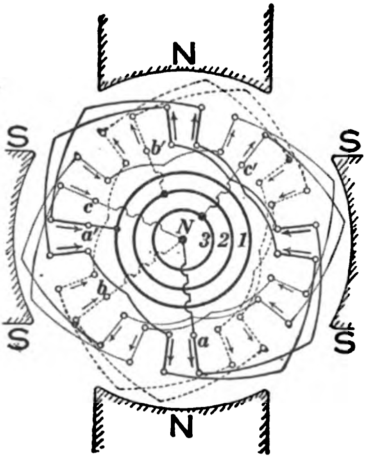

Fig. 239*a*.

Y-connected three-phase armature. Arrows represent currents. Total current flowing to N is zero.

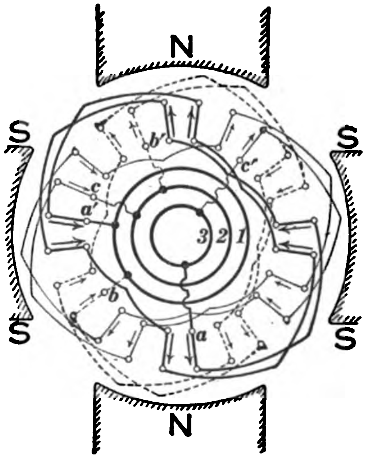

Fig. 239*b*.

Δ-connected three-phase armature. Arrows represent electromotive forces. Total electromotive force around Δ is zero.

162. The Y-scheme of connections and the Δ-scheme of connections.—When main 4 and collector ring 4 (see Fig. 236) are dispensed with as above explained, we have what is called the Y-scheme of connections as shown in Fig. 239*a*. A conventional diagram of a Y-connected three-phase alternator armature is shown in Fig. 240.

It is evident that the algebraic sum of the instantaneous values of the electromotive forces *A*, *B* and *C*, Fig. 238, is always

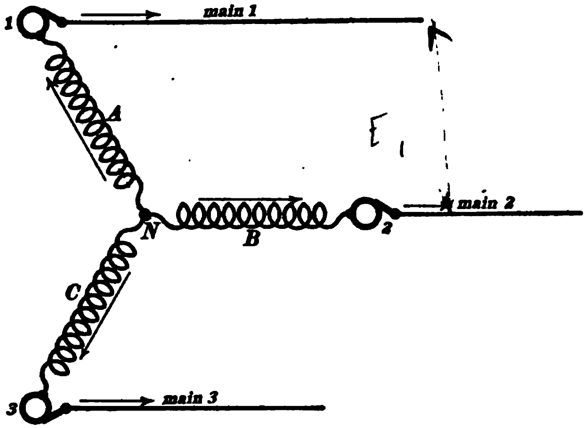

Fig. 240.

Conventional diagram of Y-connected three-phase armature. The arrows do not represent currents; they represent *directions* in which currents and electromotive forces are considered positive.

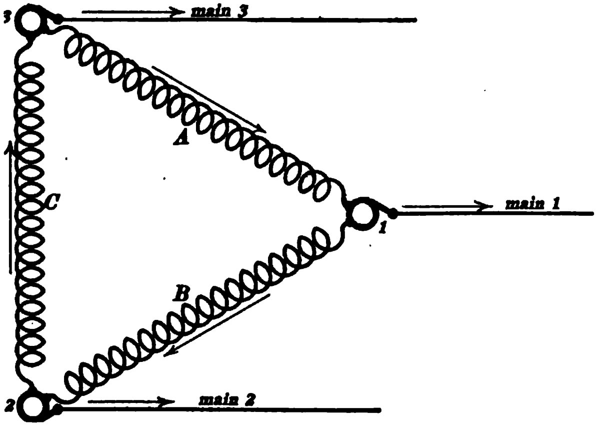

Fig. 241.

Conventional diagram of Δ-connected three-phase armature. The arrows do not represent currents; they represent *directions* in which currents and electromotive forces are considered positive.

zero. Therefore, if the three armature windings are connected to the three collector rings as indicated in Fig. 241 and as shown in Fig. 239*b*, there will be no tendency* for short-circuit current to flow around the closed circuit or mesh or Δ formed by the three armature windings. This arrangement is called the Δ-scheme of connections.

Connections of similar receiving circuits.—When main 4 is dispensed with in Fig. 236, the three similar receiving circuits are connected together at what is called a neutral junction N', and the three receiving circuits are said to be Y-connected. Figure 242 is a conventional diagram of three Y-connected receiving circuits, A, B and C.

Receiving circuits may be connected to three-wire three-phase mains as indicated in Fig. 243.

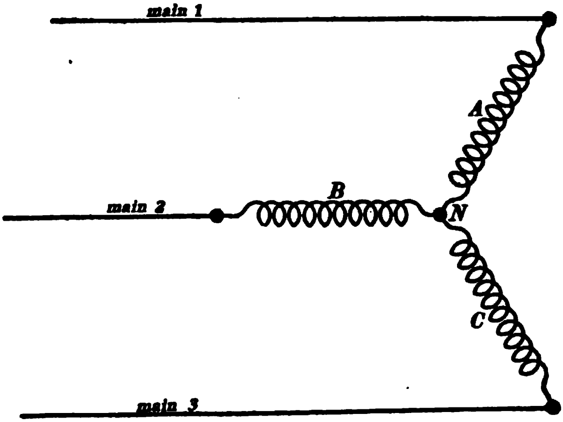

Fig. 242.

Y-connected receiving circuits.

163. Electromotive force and current relations in the Y-scheme of connections.

(1) **Referring to the generator.** (*a*) *Electromotive force relations.*—Let E_1, E_2 and E_3 be the electromotive forces from main 1 to main 2, from main 2 to main 3, and from main 3 to

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* When the electromotive forces are harmonic.

main 1, respectively, see Fig. 240. From this figure it is evident that a positive electromotive force in winding A produces an electromotive force from main 1 to main 2; that is a positive electromotive force in winding A contributes positively to E_1;

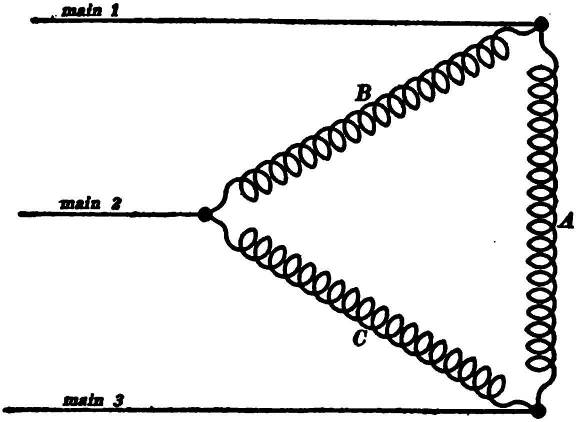

Fig. 243.

Δ-connected receiving circuits.

whereas a positive electromotive force in winding B contributes negatively to E_1. Therefore E_1 is equal to $A - B$ (vector difference). Similarly E_2 is equal to $B - C$, and E_3 is equal to $C - A$. These relations are shown in Fig. 244, and from this figure we see that *the value of the electromotive force between any pair of mains is* $\sqrt{3}$ *times the electromotive force in each Y-connected armature winding.*

(*b*) *Current relations.*—In the Y-scheme of connections the currents in the mains are identical to the currents in the respective windings. This is evident from Fig. 240.

(2) **Referring to the receiving circuits.** (*a*) *Electromotive force relations.*—Given the electromotive forces, E_1, E_2 and E_3 between the mains as shown in the clock diagram, Fig. 244, then the electromotive forces acting on the respective Y-connected receiving circuits are represented by the lines A, B and C. Therefore *the value of the electromotive force acting on*

each Y-connected receiving circuit is equal to the electromotive force between mains divided by $\sqrt{3}$.

(*b*) *Current relations.*—In the Y-scheme of connections the current in each main is identical to the current in each receiving circuit.

164. Electromotive force and current relations in the Δ-scheme of connections.

(1) **Referring to the generator.** (*a*) *Electromotive force relations.*—In the Δ-scheme of connections the electromotive forces between mains are identical to the electromotive forces developed by the respective armature windings. This is evident from Fig. 241.

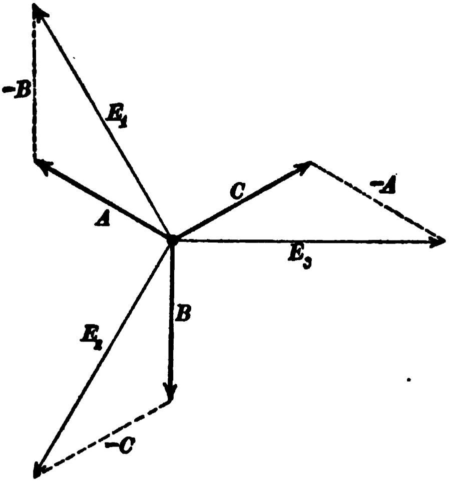

Fig. 244.

(*b*) *Current relations.*—Let I_1, I_2 and I_3 be the currents in mains 1, 2 and 3 respectively, each being considered positive when it flows away from the generator as shown by the arrows in Fig. 241, and let *a*, *b* and *c* be the currents in the respective windings, *A*, *B* and *C*. From Fig. 241 it is evident that a positive current in winding *A* produces a positive current in main 1, and that a positive current in winding *B* produces a negative current in main 1. Therefore I_1 is equal to $a - b$ (vector difference). Similarly, I_2 is equal to $b - c$, and I_3 is equal to $c - a$. These relations are shown in Fig. 245, and

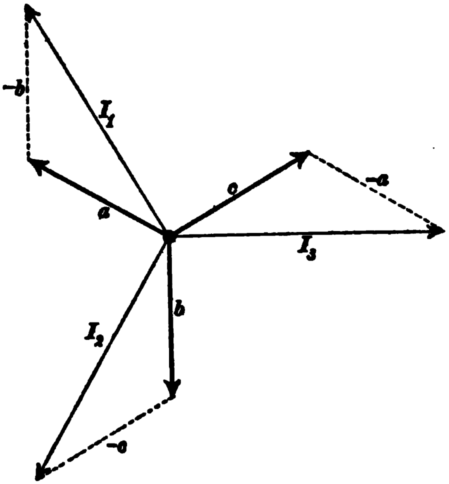

Fig. 245.

from this figure we see that *the value of the current in each main is* $\sqrt{3}$ *times the current in each* Δ*-connected armature winding.*

(2) **Referring to the receiving circuits.** (*a*) *Electromotive force relations.*—In the Δ-scheme of connections the electromotive forces acting on the receiving circuits are identical to the electromotive forces between the respective pairs of mains.

(*b*) *Current relations.*—Given the currents a, b and c in the respective receiving circuits (see Fig. 245), then the currents in the respective mains are represented by the lines I_1, I_2 and I_3. Therefore, *the value of the current in each* Δ*-connected receiving circuit is equal to the current in each main divided by* $\sqrt{3}$.

165. Power in polyphase systems.—When the receiving circuits are unlike (system unbalanced) a different amount of power is, in general, delivered to each, and the system must be treated as two or three independent single-phase systems. The only case in which the power relations in a polyphase system can be treated on a simpler basis than that corresponding to independent single-phase systems, is the case in which the system is balanced. In this case there are two important matters to be considered: (*a*) The constancy of power, and (*b*) the value of the power.

(*a*) *Constancy of power in balanced polyphase system.*—In the single-phase alternator the instantaneous value ei of the power pulsates as explained in Art. 145, whereas a polyphase generator delivers a perfectly steady flow of energy, that is, a steady power, to balanced receiving circuits. In this case the power delivered to each receiving circuit of course pulsates, but the pulsations are so related that the sum does not pulsate. An interesting consequence of this steady flow of power in a balanced polyphase system is that, whereas, the driving torque of a single-phase alternator must pulsate except in so far as the pulsations of torque are averaged out by the fly-wheel effect of the rotating armature, the driving torque of a polyphase generator is perfectly steady when it delivers current to balanced receiving circuits. Moreover a single-phase motor of any kind is driven by a pulsating torque, whereas a polyphase motor of any kind is driven by a steady torque.

(*b*) *Value of power in balanced polyphase system.*—If A is the value of the electromotive force across one of the receiving circuits of an n-phase system, a the value of the current in the receiving circuit, and $\cos\theta$ the power factor, then $Aa\cos\theta$ is the power delivered to the receiving circuit, and the total power delivered to the n similar receiving circuits is

$$P = nAa\cos\theta \tag{a}$$

If E is the electromotive force between mains of a three-wire three-phase system, and if I is the current in each main, then the total power delivered by the mains is

$$P = \sqrt{3}EI\cos\theta \tag{b}$$

in which $\cos\theta$ is the power factor of each receiving circuit. This form of the equation is confusing for the reason that the voltage E, the current I, and the power factor do not all refer to a given circuit. The form (*a*) of the equation is therefore to be preferred.

166. Measurement of power in polyphase systems.—When a polyphase system is unbalanced, the power delivered by each phase must be measured exactly as in the case of an ordinary single-phase system (see Art. 146). When a polyphase system is balanced, the measured power delivered by one phase may be multiplied by the number of phases to give the total power, but exact similarity of the various receiving circuits is never realized in practice and therefore a considerable error may be made if the total power is assumed to be nP, where P is the measured power delivered by one phase and n is the number of phases. Therefore it is usually considered necessary to measure the actual total power.

Best method for measuring power approximately by means of a single wattmeter.—It is usually very inconvenient to disconnect the current coil of a wattmeter and transfer it quickly from one main to another, but it is always very easy to change the connections of the voltage coil. The best method therefore for using a single wattmeter for measuring the power delivered to approximately balanced receiving circuits by a three-wire system (three-phase) is to connect the current coil in a given main (say main 1), with one terminal of the voltage coil connected permanently to that main, and take the readings of the wattmeter (*a*) when the other terminal of the voltage coil is connected to main 2, and (*b*) when the other terminal of the voltage coil is connected to main 3. The sum of the two readings a and b gives a fairly accurate value for the total power. In making these measurements, however, the wattmeter deflection may be reversed when connections are changed from (*a*) to (*b*); in this case the connections of the voltage coil must be *reversed* to obtain reading b, and the total power becomes $a - b$.

Use of two wattmeters for measuring accurately the power delivered by any three-wire system.—The total power delivered by any three-wire system may be measured by two wattmeters with their current coils placed in two of the mains, and their voltage coils connected from each of these two mains to the third main. Thus Fig. 246 shows two wattmeters connected for measuring the power delivered by a three-wire three-phase system. The receiving circuits may be either Δ-connected or Y-connected.

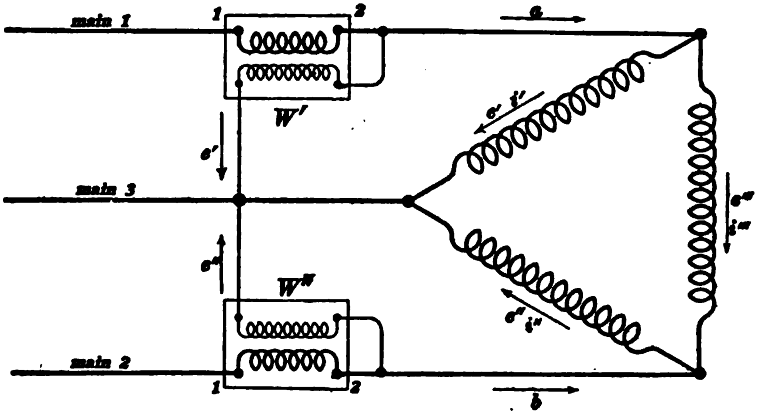

Fig. 246.

In some cases the two wattmeter readings are to be added, and in other cases the lesser reading must be subtracted from the greater to give the total power delivered. To determine whether the readings are to be added or subtracted, disconnect receiving circuit $e'''i'''$ in Fig. 246 for a moment. Under these conditions both wattmeters must indicate delivered power, so that if either wattmeter deflection is reversed by disconnecting $e'''i'''$, then the original reading of that wattmater was negative. If it is not practicable to disconnect the receiving circuit $e'''i'''$, then the two wattmeters may be interchanged making the connections of each what they would be in Fig. 246 if the paper were creased along main 3 and one half of the diagram folded over the other. If either wattmeter reading is reversed by this interchange of connections, then the original reading of that meter was negative.

To show that the sum of the two wattmeter readings gives the total power. Let the positive directions in the mains 1 and 2 and in the three receiving circuits be chosen as shown by the arrows in Fig. 246. These directions are chosen symmetrically with reference to the two wattmeters in order that each wattmeter reading may be considered as positive when it represents delivered power. Let e', e'' and e''' represent the instantaneous values of the voltages, let i', i'' and i''' represent the instantaneous values of the currents in the receiving circuits, and let a and b represent the instantaneous values of the currents in mains 1 and 2 as shown. Then, taking proper account of signs we have:

$$a = i' + i'''$$
$$b = i'' - i'''$$

The reading W' of the upper wattmeter is equal to the average value of the product of the current a in its current coil and the voltage e' across its voltage coil. That is:

$$W' = \text{average } ae'$$

and similarly

$$W'' = \text{average } be''$$

Substituting the above values of a and b in the expression for $W' + W''$ we have:

$$W' + W'' = \text{average } e'i' + \text{average } e''i'' + \text{average } (e' - e'')i'''$$

but $e' - e''$ is equal to e''' so that:

$$W' + W'' = \text{average } e'i' + \text{average } e''i'' + \text{average } e'''i'''$$

The three-wire watt-hour meter.—The simple watt-hour meter for measuring the total energy delivered to a receiving circuit is connected in a manner exactly similar to the connections of the simple wattmeter as indicated in Fig. 212. The three-wire watt-hour meter which is extensively used to measure energy delivered to a user of polyphase power, is essentially two watt-hour meters combined and having one moving element; and a three-wire watt-hour meter is connected as shown in Fig. 246.

167. Relative copper economy of two-phase and three-phase transmission lines.—A given amount of power can be transmitted over a certain distance with an indefinitely small amount of copper if no limit is set to the voltage that can be used. There is, however, a limit to the voltage that can be used satisfactorily, and it is customary therefore to compare the copper economy of two transmission systems on the basis of equal voltages. This basis of comparison is slightly misleading, however, as between the three-wire three-phase transmission line and the three-wire two-phase line, because a three-wire two-phase line with E volts (effective) between the middle wire and each outside wire has $\sqrt{2}E$ volts (effective) between outside wires, whereas in a three-wire three-phase line with E volts between any two wires there is no higher voltage than E anywhere.

For given percentage loss of power or for given percentage drop of voltage the relative weights of copper required for given voltage E are as follows:

(*a*) Three-wire two-phase line with voltage E between middle and either outside wire.	1.00
(*b*) For four-wire two-phase line with voltage E between each of the two pairs of wires.	1.36
(*c*) For three-wire three-phase line with voltage E between any two wires.	1.03

As stated above this comparison is unfair as between (*a*) and (*c*), and the three-wire three-phase line must be considered as most economical. In fact most long transmission lines are three-wire three-phase lines.

PROBLEMS.

164. The distance from center to center of adjacent poles of an alternator measured on the surface of the armature is 10.4 inches. How far apart must two armature conductors be placed so that there may be a phase difference of 55° in the electromotive forces induced in the respective conductors? Ans. 3.18 inches.

165. A common return wire is used for the two currents of a two-phase system as shown in Fig. 233*b*. The system is balanced and each current is equal to 100 amperes. What is the current in the common return wire? Ans. 141.4 amperes.

166. The electromotive force of each phase, problem 165, is

500 volts. What is the electromotive force between the outside wires in Fig. 233*b*? Ans. 707 volts.

167. The electromotive force developed in each winding of a 2-phase alternator is 100 volts. What is the electromotive force given by the two windings connected in series? Ans. 141 volts.

168. The electromotive force developed in each winding of a 3-phase alternator is 100 volts. What is the electromotive force developed by the three windings in series? Ans. 200 volts or zero.

169. The armature windings of a three-phase alternator are Y-connected to the collector rings and each winding developes an electromotive force of 250 volts (voltmeter value). What is the voltmeter value of the electromotive force between mains?

170. Three similar receiving circuits are Δ-connected to 3-phase mains, the electromotive force between each pair of mains being 110 volts. The power delivered to the three circuits is 150 kilowatts and the power factor of each circuit is 0.90. What is the current in each circuit and in each main? Ans. 505 amperes, 875 amperes.

171. Three similar receiving circuits are Y-connected to the 3-phase mains of problem 170; the total power delivered is 150 kilowatts and the power factor of each circuit is 0.90. What is the current in each circuit and in each main, and what is the electromotive force between the terminals of each receiving circuit? Ans. 875 amperes, 875 amperes, 63.5 volts.

172. A given 3-phase alternator is provided with six collector rings so that it may be connected, Δ or Y, for experimental purposes. The electromotive force developed by each winding is 100 volts. Find (*a*) the current in each receiving circuit, (*b*) the current in each main, (*c*) the voltage between mains, and (*d*) the current in each armature winding, when the alternator is Δ-connected to the mains, and the mains are Y-connected to three similar receiving circuits, each one of which has a resistance of 4 ohms and a reactance of 3 ohms. Ans. (*a*) 11.55 amperes, (*b*) same as (*a*), (*c*) 100 volts, (*d*) 6.67 amperes.

173. Find (*a*) the current in each receiving circuit, (*b*) the current in each main, (*c*) the voltage between mains, and (*d*) the current in each armature winding, when the generator specified in problem 172 is Y-connected to the mains and the mains are Δ-connected to the given receiving circuits. Ans. (*a*) 34.64 amperes, (*b*) 60 amperes, (*c*) 173.21 volts, (*d*) 60 amperes.

CHAPTER XIII.

THE ALTERNATOR AS A GENERATOR. THE GENERATING STATION AND THE SUBSTATION.

168. Field excitation of the alternator.—As stated in Art. 40, the field magnet of an alternator is always excited by direct current, and this direct current is nearly always supplied by a small direct-current generator which is called the *exciter*. This exciter has shunt field windings, and two rheostats are generally used, one in circuit with the shunt field winding of the exciter and the other in the field circuit of the alternator. The electromotive force of the alternator can be controlled by adjusting either of these rheostats.

169. Electromotive force of an alternator.—The equation expressing the electromotive force of a direct-current dynamo in terms of armature flux and speed (the so-called fundamental equation) is comparatively simple and *it serves as a basis for the discussion of the important matters of speed regulation and speed control of direct-current motors.* The fundamental equation of the alternator has no such important use, and therefore the theory of the electromotive force of an alternator need not be fully developed.

(*a*) The frequency of the alternating electromotive force is equal to np where n is the speed of the alternator in revolutions per second and $2p$ is the number of field magnet poles. This relation is explained in Art. 41.

(*b*) The shape of the electromotive force curve of an alternator depends upon the distribution of the magnetic flux under the pole faces and upon the distribution of the armature conductors as illustrated in Figs. 247, 248 and 249. It is desirable, especially if current is delivered to a long transmission line, to have the electromotive force curve of an alternator approximate very closely to a curve of sines as indicated in Fig. 249; that is to say, it is desirable that the electromotive force of an alternator should be very nearly harmonic. Indeed electromotive forces

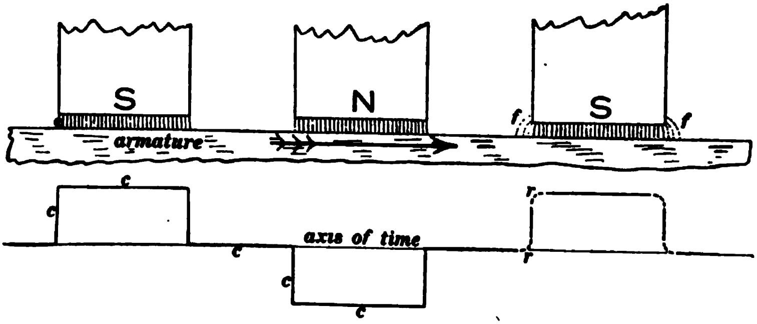

Fig. 247.

One armature conductor per field pole as in Fig. 231. The fringe of the magnetic field *ff* causes a rounding of the corners of the curve as shown by the dotted lines *rr*.

and currents are assumed to be harmonic in most practical calculations.*

170. Standard frequencies.—An alternating-current transformer or motor is always designed for a particular frequency

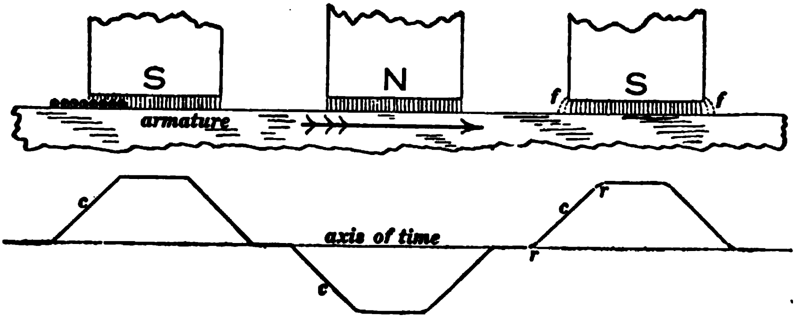

Fig. 248.

Band of armature conductors per field pole. The fringe of the magnetic field *ff* causes a rounding of the corners of the curve as shown by the dotted lines *rr*.

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* A very complete discussion of the causes and consequences of non-harmonic electromotive forces and currents is given in Franklin's *Electric Waves*, The Macmillan Co., 1909; and in a later volume of this revised treatise on the *Elements of Electrical Engineering*.

and therefore certain standard frequencies have been adopted in practice. These frequencies are 25 cycles per second and 60 cycles per second. The lower frequency is preferable for operating single-phase commutator motors such as are used on the New York, New Haven and Hartford Railway; whereas the higher frequency is preferable for operating lamps, and a 60-cycle transformer is cheaper than a 25-cycle transformer of the same rating. The 60-cycle induction motor is, perhaps, somewhat cheaper than the 25-cycle induction motor, and the use of commutating poles or interpoles has brought the 60-cycle synchronous converter (rotary converter) nearly up to the 25-cycle converter in reliability of operation, so that the lower frequency is not now considered necessary for operating synchronous converters. The lower frequency is, however, much more satisfactory for very long transmission lines, because it involves much less voltage drop due to the inductance (or reactance) of the line.

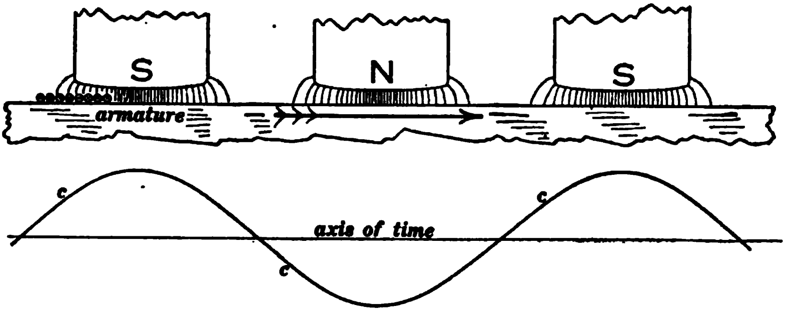

Fig. 249.

Same as Fig. 248 but with pole faces rounded. In this case the electromotive force is very nearly harmonic.

171. Voltage regulation and voltage control.—The voltage of an alternator usually falls off with increase of load (rises with decrease of load); and the rise of voltage from full load to zero load, expressed as a percentage of full-load voltage, is called the *percentage regulation.*

Low percentage regulation (small rise of voltage from zero load to full load or small drop of voltage from zero load to full load) is desirable because all power plants are intended to supply current at constant voltage, but *it is very undesirable in very large modern plants because a big drop of voltage tends to prevent excessive flow of current in case of accidental short circuit.* In many cases the short-circuit current is kept within safe limits by inserting an auxiliary inductance coil in the main circuit of the alternator (the so-called *power-limiting reactance*), and this inductance necessarily causes poor voltage regulation. The common practice in large plants is, therefore, to provide a switchboard attendant to keep the station voltage at the desired value by adjusting the field rheostats. The *Tirrill regulator* is an automatic device for controlling the station voltage by means of field rheostats.

172. Alternator ratings.*—The permissible current output of an alternating-current generator is determined solely by the heating of the machine, and a given alternator might be rated by specifying the number of phases, the frequency at a chosen speed, the voltage it is intended to give, and the permissible current output. It is customary, however, to specify the product of voltage and current instead of permissible current. Thus a two-phase, 25-cycle, 5000-volt, 1000-ampere-per-phase alternator would be rated at 5 million volt-amperes or 5000 kilovolt-amperes per phase. The product of volts and amperes is not actual power output unless the power factor of the receiving circuit is zero. Adequate rating of a large alternator is vitally dependent upon ventilation; air is blown through specially designed ducts in the armature and field structure. In case of a short-circuit in the armature or field, the insulating material is sometimes ignited and a damper is usually arranged to cut off the ventilation to avoid rapid combustion of insulating materials and great consequent damage in case of short-circuit.

173. The generating station.—Before entering into a discussion of the various kinds of alternating-current transformers, converters, and motors it is helpful to consider the essential features of the generating station and of the substation, although some of these features cannot be appreciated until the principles which are developed in the following chapters are understood. Thus the operation of alternating-current generators in parallel and the method of starting an additional generator (the so-called *synchronizing* of the additional machine) are explained in the chapter on the synchronous motor (the ordinary alternator operating as a motor).

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* See Standardization Rules of the American Institute of Electrical Engineers for full information concerning this important topic of rating.

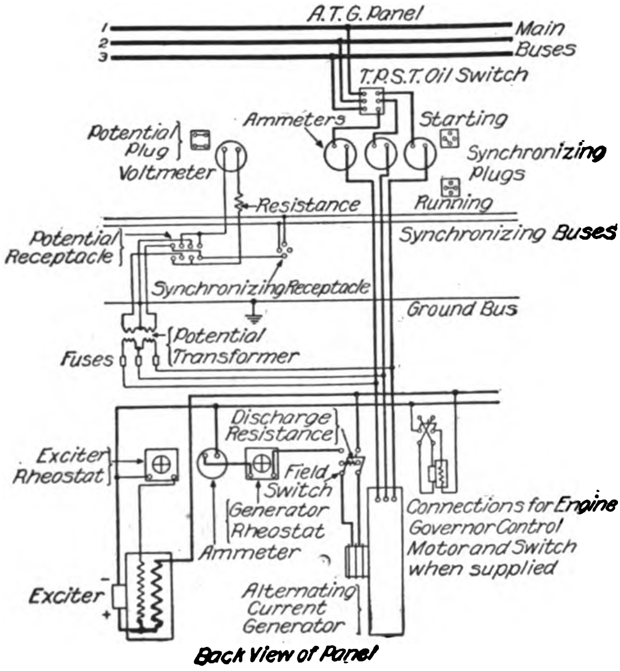

Fig. 250.

Figure 250 shows the connections for operating a number of alternating-current three-phase generators in parallel. The main busses, the synchronizing busses, the ground bus, and the exciter busses run the entire length of the switch-board which has a panel for each alternating-current generator (a *generator panel*), and each of the alternating-current generators is connected to the various busses exactly as shown in the figure.

(*a*) The generator as represented is of the revolving field type, and the field exciting current is led into the alternator field windings through two collector rings as shown. The alternator armature is stationary and the three minute circles represent the armature terminals.

(*b*) The exciter is represented as having a shunt field winding with a field rheostat. The exciter brushes are connected to the exciter busses, and the exciting current for each alternator is taken from the exciter busses through an ammeter and a controlling rheostat as shown.

The field switch is arranged so as to insert a moderately high resistance in the field circuit before actually breaking the circuit. This resistance is called a " discharge resistance."

(*c*) A single alternating voltmeter is provided on each generator panel and arranged so that it can be connected through a " potential transformer " across any two armature leads so as to indicate the voltage of any one of the three phases. The " potential transformer " in its simplest form is a small transformer used to step-down a high voltage in a known ratio so that it can be measured by a low-voltage voltmeter. The diagram shows two potential transformers for stepping down all three phases. The use of two transformers in this way is explained in Art. 203.

The " potential receptacle " and the " potential plug " constitute a switch arrangement *which cannot be wrongly used* for connecting the voltmeter to any one of the phases as stated.

(*d*) Each phase has its individual ammeter inserted in an alternator lead between the alternator and the main busses. It is usually desirable, however, to use a " current transformer " with each ammeter, that is to say, a highly insulated transformer with its primary coil connected in an alternator lead and its secondary coil connected to an ammeter. When this transformer has the same number of turns in its primary and secondary coils, the secondary current as indicated by the ammeter is sensibly equal to the current in the lead.

(*e*) A triple pole oil-break switch is connected in the alternator leads as shown, and "tripping coils" are inserted in the leads so that excessive current in any lead or leads will release a mechanism which will open the oil switch very quickly in case of serious short-circuit.

Provision is usually made for retarding the tripping of the main oil switches in case of a short circuit so that smaller or sectional circuit breakers or fuses out in the system may open and eliminate the short circuit without putting the whole station out of service.

(*f*) There are two "synchronizing plugs" as indicated, and

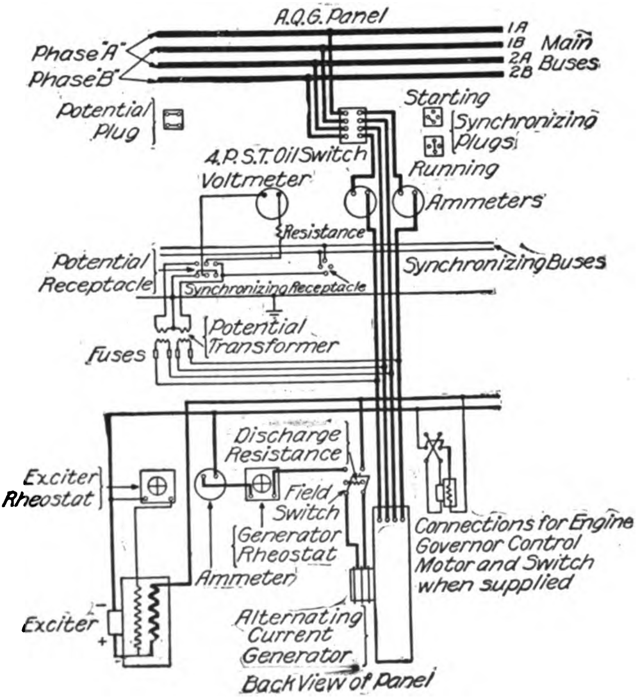

Fig. 251.

the "running plug" is in the "synchronizing receptacle" of every machine in full operation. When an additional alternator (with its oil switch open, of course) is to be put in operation, its "starting plug" is inserted in its "synchronizing receptacle," its driving engine is started and brought up to speed, and the speed is then carefully adjusted by manipulating the switch of the governor-control motor, and when the two necessary conditions are satisfied, as explained in Art. 176, the oil switch is closed, the engine governor is set free, and the "running plug" is put in place of the "starting plug" in the "synchronizing receptacle."

Figure 251 shows the connections for operating a number of alternating-current quarter-phase (two-phase) generators in parallel. This figure shows the four-wire arrangement inasmuch as each phase has a separate pair of bus bars. The above description of Fig. 250, very slightly modified, applies to Fig. 251.

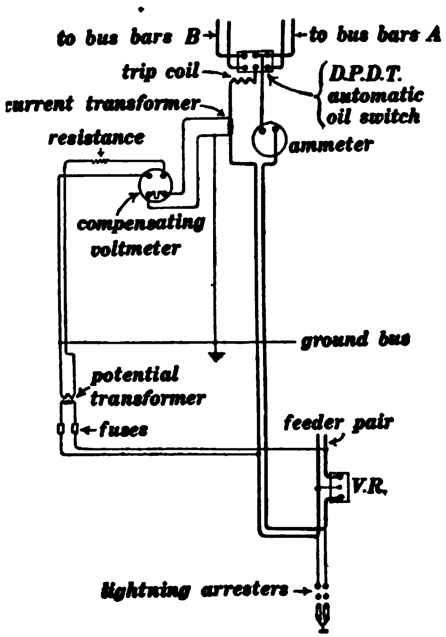

Fig. 252.

Feeder panels. — The wires which lead out from a generating station to a substation or to a group of customers are called *feeders*, and the switches and ammeters and other controlling apparatus pertaining to a set of feeders are usually arranged on a switch-board panel. Thus Fig. 252 shows the arrangement of a single-phase feeder pair. The lightning arresters are connected near the point where the feeders leave the building, and sometimes a voltage regulator V.R. is connected as indicated. In this particular instance a double-pole double-throw oil switch is arranged so that the given pair of feeders can be connected to one pair of bus bars *A* or to the other pair of bus bars *B*. The diagram shows a special arrangement of the voltmeter for indicating *not* the voltage across the feeders at the station but the voltage across the feeders at the substation or at the customer's end. The voltage regulator is a transformer with its fine wire primary coil connected across the busses or feeders, and its secondary coil of coarse wire connected in the feeder circuit so that the electromotive force induced in the secondary coil is added to (or, it may be subtracted from) the bus-bar voltage thus raising (or lowering) the voltage across the pair of feeders. The secondary coil of the voltage regulator is arranged, most simply, so that any desired number of turns of its secondary coil can be connected in the feeder circuit at will.

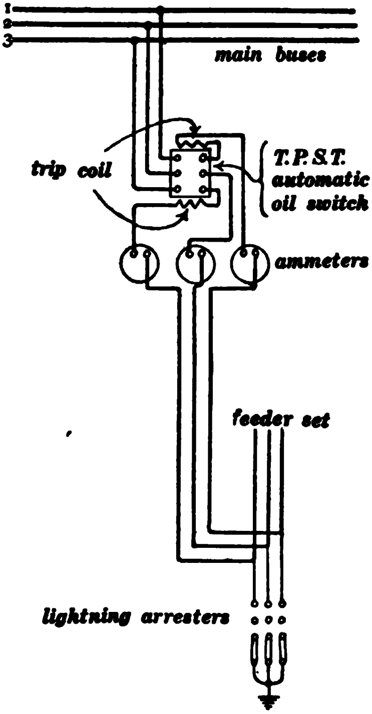

Fig. 253.

Figure 253 shows the arrangement of a set of three-wire three-phase feeders. The diagram shows the feeders arranged to take current from one set of busses only, and no voltage regulators are shown. The trip coils of the automatic oil switch are arranged to operate by a short-circuit between any two of the three feeder wires.

174. **The substation.**—Power is usually delivered by a large

power station to a transmission line or lines at very high voltage, and certain apparatus must be used at the other end of the transmission line for converting the high-voltage alternating power into a form suitable for use.

(*a*) In the simplest case the transmitted power is merely reduced in voltage (with a corresponding increase in volume of current) by step-down transformers as explained in Art. 43. A group* of transformers used in this way is called a transformer substation, and such substations are often out-of-doors.

(*b*) In many cases the alternating current delivered by the transmission line has a frequency of 25 cycles per second and it is sometimes desirable to convert to a 60-cycle frequency which is much better for lighting and somewhat better for small motors. The more usual arrangement for this purpose is as follows: A group of step-down transformers delivers power at reduced voltage to a 25-cycle synchronous motor which in turn drives a 60-cycle alternating-current generator. Another arrangement which is sometimes used is briefly described in Art. 214.

(*c*) In many cases power delivered by the transmission line is to be used in the form of direct current. In this case step-down transformers deliver the polyphase alternating currents to a synchronous converter (rotary converter) which in its turn delivers direct current. Another arrangement, not so generally used, is to drive a synchronous motor as under (*b*) and use this synchronous motor for driving an ordinary direct-current generator.

175. The lightning arrester.—At every point where a line wire enters a transformer or any other device, or wherever a line wire enters a building a lightning arrester should be installed as indicated in Fig. 89 and as explained in Art. 52. A spark gap between the wire and ground and a choke coil back of the spark gap as in Fig. 89 constitute the essential features of the lightning arrester, but a lightning discharge across the spark gap in Fig. 89 short-circuits the generator *G* and some device is necessary to prevent the continued flow of excessive short-circuit current.

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* This refers to a polyphase system.

(*a*) The short-circuit current in case of a direct-current line is nearly always eliminated by a magnetic blow out arranged to act on the arc across the spark gap in Fig. 89 as explained in Art. 17.

(*b*) In case of alternating-current lines of moderately high voltage, the spark gap in Fig. 89 is a series of very short air gaps between blocks of brass. The lightning discharge passes easily to ground over such a series of gaps, whereas 1000 volts (effective) of alternating electromotive force cannot maintain a flow of alternating current across three 3/64 inch gaps in series.

(*c*) All high-voltage alternating-current transmission lines where they enter a piece of apparatus or a building are equipped with aluminum valve arresters. A number of deep bowls of sheet aluminum are nested together but insulated from each other, and the spaces between are partly filled with a solution of ammonium phosphate as shown in Fig. 254. This stack of bowls is set upright in a cylinder which is filled with oil, the bottom bowl being connected to a metal base and the top bowl being connected to a metal terminal; and the whole arrangement is then inserted between the spark gap in Fig. 89 and the ground. The full voltage of a 50,000-volt transmission line cannot maintain a current through, say, a one-hundred-bowl unit of this kind, but the passage of a lightning discharge to ground is not perceptibly obstructed.

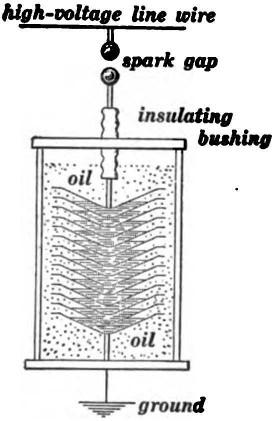

Fig. 254.

CHAPTER XIV.

THE ALTERNATOR AS A MOTOR.

176. The synchronous motor. Operation of alternating-current generators in parallel.—Let A' and A'', Fig. 255, be two alternating-current generators delivering current to supply mains a and b at a frequency of *exactly* 60 cycles per second, and B is an alternator which is to be put into operation so as to deliver power to mains ab as a generator operating in parallel with A' and A'', or so as to take power from mains ab as a motor. One armature terminal of B is connected to main b, a switch s is arranged to connect its other terminal to main a, the primary coil of a small step-down transformer T is connected across the switch, and an ordinary glow lamp L is connected to the secondary coil of T as shown.

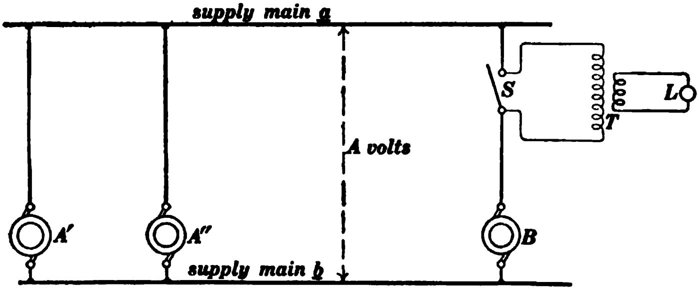

Fig. 255.

The field magnet of B is excited from an outside source of direct current and machine B is started and brought up to full speed by an outside source of power. The lamp L will then pulsate in brightness.* By adjusting the speed of machine

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* These pulsations are due to the fact that at one moment the *sum* of supply voltage A and machine voltage B acts on the primary of transformer T, and the next moment (after B has gained or lost half a cycle as compared with A) the *difference* of voltages A and B acts on the primary of T. This matter is easily understood with the help of the clock diagram of Fig. 257.

B it can be brought more and more nearly to the exact frequency of the supply voltage, namely, 60 cycles per second, as indicated by slower and slower pulsations of the lamp L. When the lamp pulsates *very slowly* alternator B is said to be *in synchronism* with the supply voltage, and *under these conditions the switch* s *may be closed* **when the lamp becomes dark** without excessive or even perceptible flow of current through the armature winding of B. If the outside source of power used for starting B is a large steam engine, the steam may now be turned on full, and machine B will operate as a generator in parallel with A' and A'', delivering its share of power to the supply mains; or, if the outside source of power is disconnected, machine B will continue to run as a motor taking power from the supply mains. In both cases machine B remains in exact synchronism with machines A' and A''. Let the position which the rotating armature of B would have at any instant if it were neither delivering power to nor taking power from the supply mains be called its *mean position* at that instant; when steam is turned on the driving engine pushes the rotating armature of B a certain angular distance ahead of this mean position and machine B delivers power to the supply mains; or, if machine B is loaded as a motor, its armature falls a certain angular distance behind this mean position and it takes in power from the supply mains. When an alternator is acting as a motor in this way it is called a *synchronous motor.*

The usual arrangement of a lamp as a synchronoscope is shown in Fig. 256. This figure shows exactly the "synchronizing" connections of a machine (or machines) already in operation and a machine which is being started in a generating station (see Art. 173*f*), and it is equivalent to the arrangement shown in Fig. 255 except that by reversing either coil of either transformer (T or T') the lamp may be at its *maximum brightness* when the conditions are right for the closing of the switches s and s'. The lamp synchronoscope has been to a great extent displaced in practice by several types of dial synchronoscope, in which a pointer or hand rotates at a speed equal to $f - f'$, where f is the frequency of machine A and f' is the frequency of machine B in Fig. 256. When this hand rotates very slowly the two machines are in synchronism, and then when it points in a certain marked direction the conditions are proper for closing the switches s and s'.

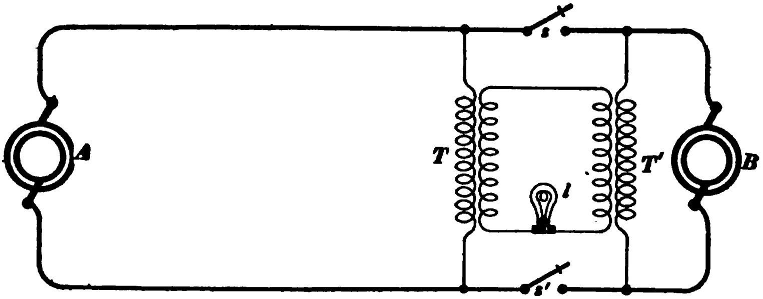

Fig. 256.

The polyphase synchronous motor.—The above discussion refers explicitly to a single-phase alternator B giving power to or taking power from single-phase supply mains. Consider a polyphase alternator B with its separate armature windings separately connected to the separate pairs of polyphase supply mains; the above discussion applies to one phase of such an arrangement, that is, to one armature winding of B one pair of lead wires and one pair of supply mains. In starting a polyphase machine the procedure is exactly as above described except that the switches s and s' are arranged to make connection for all phases simultaneously when they are closed; the synchronoscope (lamp or dial type, as the case may be) is inserted in one phase only, exactly as shown in Fig. 256.

177. The clock diagram of the synchronous motor.—A polyphase alternator B takes power from polyphase supply mains and operates as a synchronous motor. Let us consider one only of the phases, that is to say, one pair of supply mains between which the voltage is A, one pair of leads from these mains to the motor, and one armature winding of B to which this pair of leads is connected; and let R be the resistance and L the inductance of this receiving circuit. Let the clock-diagram vector A, Fig. 257, represent the supply voltage A, and let the clock-diagram vector B represent the induced electromotive force in the particular armature winding of machine B which is under consideration. These two electromotive forces A and B act on the same circuit and they are together equivalent to the electromotive force which is represented by the clock-diagram vector E.

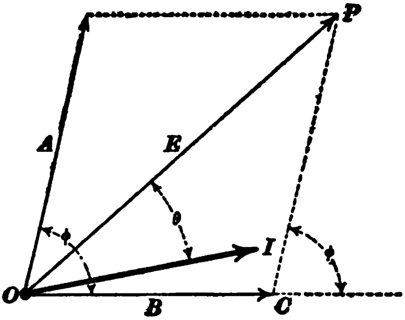

Fig. 257.

If machine B is absolutely in synchronism with the supply voltage the phase angle ϕ remains unchanged, but a momentary decrease (or increase) of speed of machine B causes the phase difference ϕ to increase (or decrease), and *every possible position of the point* P *in Fig.* 257 (*for various values of the phase difference* ϕ) *lies on the dotted circle of Fig.* 258.

When machine B is a generator the projection OQ in Fig. 258 represents the *electrical power developed* in each phase of the armature winding of B; and when machine B is a motor the projection OQ is towards e from O and it represents the *mechanical power developed* by each phase of the armature winding in turning the armature. The electrical power developed in the armature of machine B in the first case is less than its electrical power output because of the RI^2 loss in its armature winding; and the mechanical power developed in turning the armature of B in the second case is less than the belt output of B as a motor because of stray power losses (friction, eddy current and hysteresis losses in its armature); stray power losses are ignored in this discussion in order that we may speak in simplest terms of the mechanical power developed in the armature as the mechanical output of the motor, or the motor load.

Proof that OQ, *Fig.* 258, *represents motor load per phase when it is in the direction* Oe. Let $\omega (= 2\pi f)$ be the frequency in radians per second. Then from equations (73), (74) and (75), we have

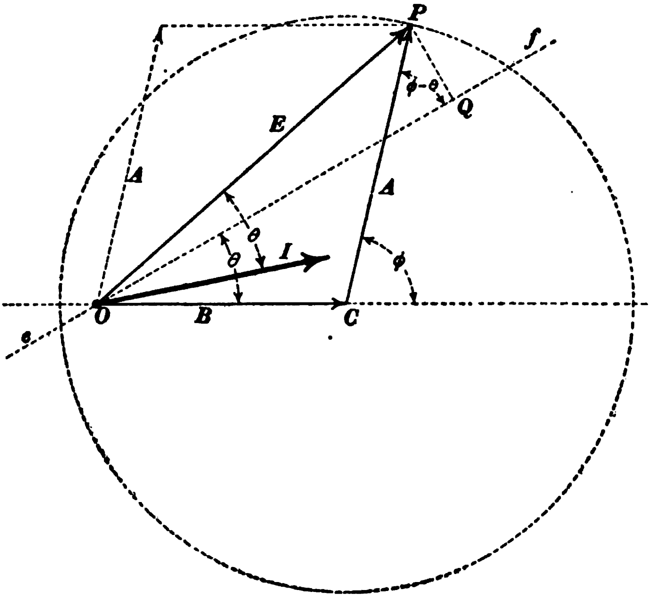

Fig. 258.

$$\tan \theta = \frac{\omega L}{R} \tag{i}$$

and

$$I = \frac{E}{\sqrt{R^2 + \omega^2 L^2}} \tag{ii}$$

where E is the voltage E in Fig. 258 and I is the current in one armature winding. Now, according to equation (71) the electrical power developed by one phase of machine B is $BI \cos (BI)$ where (BI) is the phase angle between B and I, and when this angle is greater than 90° its cosine is negative and of course BI cos (BI) is negative; and this negative value of BI cos (BI) is the portion of power delivered by the supply mains which is used to overcome the back electromotive force B in one phase of the armature winding, and this power all reappears as mechanical power in the turning of the armature as explained in Art. 125. The line ef is drawn through O making with OC the angle θ as shown. The angle between B and E is equal to $(BI) + \theta$. This angle between B and E is also equal to the sum of the angles POf and $fOC(= \theta)$. Therefore $(BI) + \theta = POf + \theta$, or $POf = (BI)$. Consequently

$$\overline{OQ} = \overline{OP} \cos (BI) = E \cos (BI) = I\sqrt{R^2 + \omega^2 L^2} \times \cos (BI)$$

or

$$I \cos (BI) = \frac{\overline{OQ}}{\sqrt{R^2 + \omega^2 L^2}}$$

or, multiplying both members of this equation by B, we get

$$BI \cos (BI) = \frac{B}{\sqrt{R^2 + \omega^2 L^2}} \times \overline{OQ} \tag{83}$$

Motor and generator regions on the circle diagram of Fig. 258.—Draw a line through O at right angles to ef as shown by ss' in Fig. 259. Then when the point P is anywhere in the region sMs' the projection of OP is in the direction from O towards e and machine B takes in power from the supply mains as a motor. When the point P is anywhere in the region to the right of ss' machine B delivers power to the supply mains as a generator. The maximum motor load (per phase) is represented by the maximum negative value of OQ, namely, DM in Fig. 259.

Circle diagram for case in which the voltage B of the synchronous motor is larger than the supply voltage A.—Figure 260 is a diagram entirely similar to Fig. 259 except that machine voltage B is larger than supply voltage A. The terminal voltage of the machine is, of course, equal to A because the terminals of the machine are connected directly to the supply mains; but portions of the supply voltage are used to overcome the resistance R and the reactance ωL of the armature winding and the remainder of the supply voltage overcomes the back electromotive force B and is equal thereto and this *remainder*

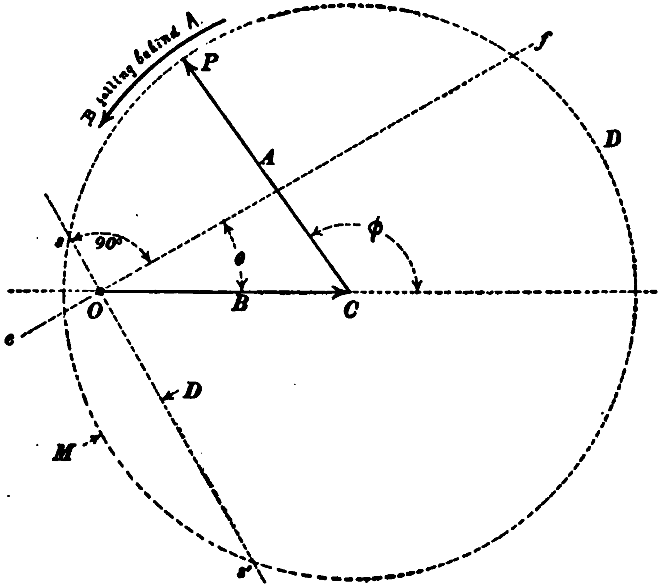

Fig. 259.

is larger than the whole of the supply voltage because of the phase of the reactance drop, ωLI. See Art. 157*c*. The maximum motor load as represented by DM in Fig. 260 is evidently smaller than the maximum motor load in Fig. 259, although the scale factor in equation (83), namely, $B/\sqrt{R^2+\omega^2L^2}$, is increased when B is increased.

The line Oss' in Fig. 260 (at right angles to ef) would not cut the circle at all if the angle θ were much smaller; therefore the possibility of operating machine B as a motor when its voltage B is larger than the supply voltage A depends upon the inductance L or reactance ωL of the circuit inasmuch as $\tan\theta = \omega L/R$ according to equation (75).

If an outside inductance is connected in series with each phase of the machine B then the voltage across the armature terminals of B will be larger than the supply voltage.

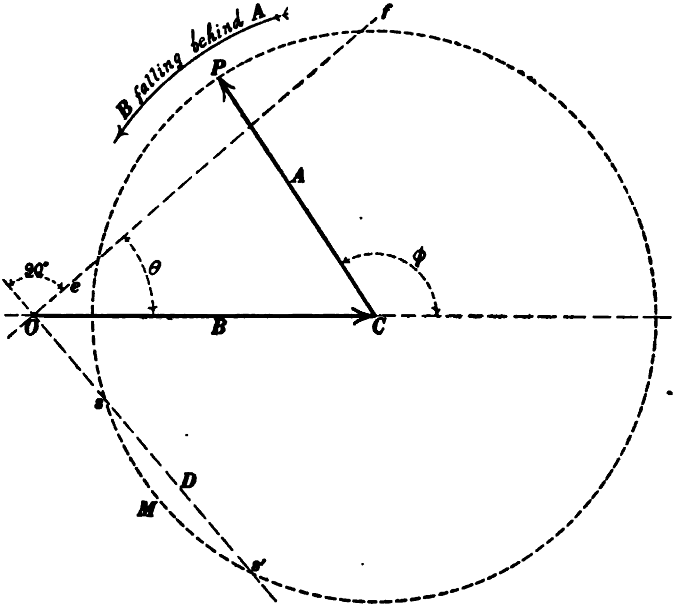

Fig. 260.

178. Speed-torque characteristics and stability of running of a synchronous motor.—The synchronous motor is usually started by induction motor action as explained in Art. 182, and the torque is very small at all speeds below full synchronous speed. Therefore the load is usually disconnected from a synchronous motor at starting. Indeed the synchronous motor, as most extensively used, drives a direct-current generator, and this generator does not deliver any current, except its own field current and field current for the synchronous motor, while the synchronous motor is being started and brought up to speed.*

When a synchronous motor has been started and brought up to full synchronous speed it runs at absolutely* synchronous speed if its load is constant. Increase of load causes the motor speed to drop very slightly for a small fraction of a second, the motor then falls further behind the supply voltage in phase and comes quickly back to exact synchronous speed. The motor acts in this way from zero load up to a certain critical value of overload, and at this critical value of overload the motor " falls out of synchronism " and comes to a standstill.

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* Similar conditions prevail when a synchronous converter is being started by induction motor action. See Arts. 182 and 187.

This behavior of the synchronous motor is explained as follows: Suppose machine B to be running steadily as a synchronous motor with constant load and with the point P standing anywhere between s and M in Fig. 259. A sudden increase of *load* causes a momentary decrease of speed of machine B which moves P towards M and the motor *intake* increases more and more.

Machine B continues to lose speed, however, up to the instant that *intake* becomes equal to *load*. Then, as machine B continues to fall behind supply voltage A in phase, the *intake* exceeds the *load* and the speed of B rises, and, of course, machine B is farthest behind A in phase when it reaches exact synchronous speed. Then, *intake* being much greater than *load*, the speed of B continues to rise, reaching a maximum at the instant that *intake* becomes again equal to *load*. Then, as machine B continues to get ahead of A in phase, *load* exceeds *intake* and the speed of B falls, and, of course, machine B is farthest ahead of A in phase when exact synchronous speed is again reached; and so on. This backward and forward movement of machine B constitutes hunting oscillations. See Art. 179.

When increase of *load* carries the point P to M then further increase of *load* carries P beyond M and causes a decrease of *intake*. Thus the speed of the machine must continue to decrease (because its *load* is greater than its *intake*); the point P moves farther and farther beyond M, the *intake* decreases more and more (becoming eventually an *output* when P passes s'), and the speed of the machine B drops quickly to zero.

While the speed of machine B is dropping to zero the point P may pass several times around the circle in Fig. 259; while passing from s to M to s' the machine takes in power from the mains, but not nearly enough to make up for the great output of power by machine B while P was traversing the right hand portion of the circle from s' around to s.

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* Hunting oscillations are here ignored. See Art. 179.

179. Hunting oscillations of the synchronous motor.—Imagine a synchronous motor B running steadily in exact synchronism with supply voltage A and with the point P standing still anywhere between s and M in Fig. 259 or 260, and let the position of the armature of B *at any instant* under these steady conditions of running be called *its mean position* for the given load. A sudden increase or decrease of load causes the point P to shift to a new position in Fig. 259 or 260, and gives to the armature a new *mean position*, but in changing to the new mean position the armature always overshoots, as it were, and oscillates back and forth through its new mean position.* These oscillations are called *hunting oscillations*, and sometimes these oscillations not only persist but increase more and more in amplitude until the point P (which is carried forwards and backwards by the oscillations) is carried far into the region of unstable running (between M and s' in Figs. 259 and 260) and then the motor drops out of synchronism and comes to a standstill as explained in Art. 178.

The prevention of troublesome hunting depends upon two things, namely, (*a*) The elimination of periodic disturbances like the pulsations of a reciprocating steam engine; if such pulsations happen to be in rythm with the hunting oscillations of the alternating-current generator in its relation to other generators connected in parallel with it, serious hunting is produced; and (*b*) By providing for an action which dampens the hunting oscillations as ordinary friction dampens the oscillations of a pendulum.

Two damping arrangements are used, namely, (1) A heavy fly-wheel may be loosely attached to the synchronous motor shaft so as to slip (with considerable friction) as the motor speed rises and falls with hunting; and (2) Heavy copper bars may be imbedded in the pole faces of the field magnet of a synchronous motor, these bars being parallel to the armature shaft and the projecting ends of the bars being welded to two copper end-rings as indicated in Fig. 261. The action of these so-called

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* Let it be understood that these oscillations are superposed on a steady state of forward motion of the armature.

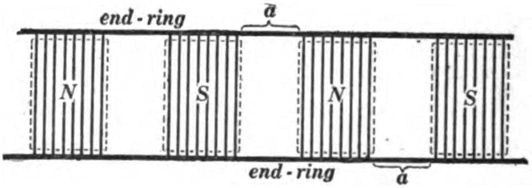

Fig. 261.

The dotted lines represent pole faces. The portions *aa* of the end rings are useful for induction motor starting as explained in Art. 182.

damping grids or *damping frames* is as follows: When the field of a dynamo (alternating-current dynamo or direct-current dynamo) is distorted as shown in Fig. 190 the pull of the lines of force on the armature opposes the motion of the armature; and when the field of a dynamo is distorted as shown in Fig. 191 the pull of the lines of force on the armature helps the motion of the armature. As the speed of a synchronous motor increases and decreases during hunting oscillations the torque exerted on the armature increases and decreases,* the flux from a pole face shifts forwards and back over the pole face, this shifting flux sweeps across the copper bars of the damping frame thus inducing currents in these frames which dampen the hunting oscillations.

The hunting oscillations of a synchronous motor are often damped quite effectively by the machine which is driven by the motor, especially if a belt drive is used. The synchronous converter (rotary converter) is essentially a synchronous motor as explained in the next chapter, and it is especially prone to hunting oscillations. Therefore damping frames are nearly always embedded in the pole faces of a synchronous converter.

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* In case of the synchronous converter, the torque is positive while the speed is increasing and negative while the speed is decreasing.

180. The excitation characteristic of the synchronous motor. Conditions which give maximum efficiency.—A peculiarity of the synchronous motor when B is less than A as in Fig. 259, is that, starting at zero load, *the current intake of the motor decreases with increase of load* until the point P, in Fig. 259, reaches the line CO. This is evident when we consider that the resultant electromotive force OP is a minimum when P is on the line CO or, in other words, when A and B are in exact opposition in phase. This peculiarity of the synchronous motor may be stated in another way, namely, *when the load on the motor is fixed in value there is a certain value of B (certain degree of field excitation of machine B) for which the current I is a minimum and for which the RI^2 loss is a minimum, supply voltage A being fixed in value.* This value of B which enables the synchronous motor to carry its given load with minimum value of I gives the maximum efficiency for given load and given value of A, and therefore the field excitation of a synchronous motor should always be adjusted (a different adjustment is necessary for each motor load) to give minimum I.

The curve which shows the variation of current intake I of a synchronous motor (per phase) with exciting current in the field windings of the motor for a given motor load is called the *excitation characteristic* of the motor for the given load. This curve shows a sharply defined minimum value of I corresponding to a certain value of the field current.

181. The synchronous condenser. Use of the over-excited synchronous motor as a compensator for lagging current.—A synchronous motor of which the voltage B is greater than the supply voltage A as represented in Fig. 260 is called an *over-excited* synchronous motor. The clock diagram of Fig. 262 is the clock diagram of Fig. 260 with the point P at s, and therefore Fig. 262 refers to an over-excited synchronous motor at zero load. A careful study of Fig. 262 will make clear the fact that the current I delivered to each phase of the over-excited synchronous motor is ahead of the supply voltage A

in phase, about 70° ahead in the figure. Therefore the wattless component of I (at right angles to the supply voltage A) is a *leading* wattless current such as is required for annulling a *lagging* wattless current as explained in Art. 158*b*. For example, alternating supply mains may supply lagging currents to operate induction motors, and by connecting a synchronous motor across the mains at the receiver end (that is where the induction motors are connected) and making the necessary adjustment of its field excitation the total current delivered at the receiver end of the line may be brought into phase with the supply voltage across the receiver end of the line. Thus the combined power factor of all the receiving apparatus is unity. The over-excited synchronous motor is therefore often called a *power factor corrector.*

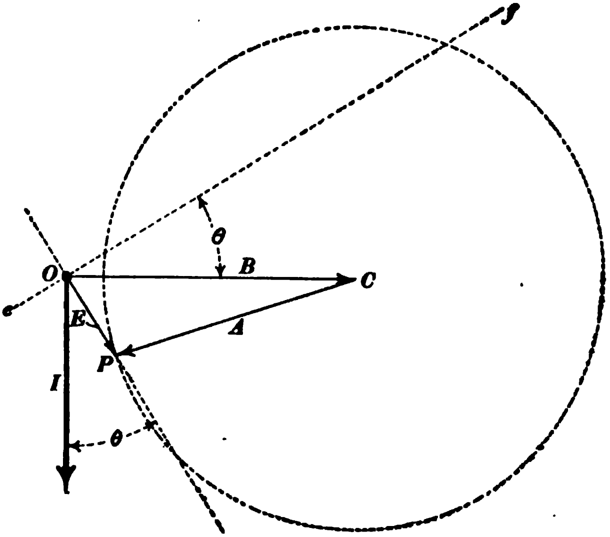

Fig. 262.

The power factor corrector should be used at the receiver end of a transmission line so as to relieve the line of the unnecessary service involved in the carrying of the wattless current required by a low power factor (lagging) receiver, and of course relieve the generator of the necessity of supplying the wattless current. The flow of the wattless current means additional RI^2 losses in the line and in the alternator armature; it also means ordinarily a slightly greater variation of the supply voltage with load (poor voltage regulation).

182. Induction-motor starting of the polyphase synchronous motor.—The armature windings of a polyphase alternator are identical to the stator windings of a polyphase induction motor, and the damping grids or frames which are described in Art. 179 are essentially the same thing as the so-called squirrel cage rotor winding of an induction motor.* Therefore, with field magnet unexcited, a small " induction motor " torque action is exerted on the armature of a polyphase alternator by its field structure when its armature is connected to polyphase supply mains.

To connect the full normal supply voltages (two-phase or three-phase) to the synchronous motor armature at starting is objectionable because of the excessive flow of current. Therefore an auxiliary set of small step-down transformers is† used to deliver starting currents (polyphase) at, say, 1/4, 1/2 and 3/4 of normal supply-voltage value. This set of small transformers is called a *starting compensator.*

If the synchronous motor drives its own exciter (small direct-current generator for supplying field current), this exciter will come into action only when the motor reaches nearly full speed, and in this case the motor field circuit need not be open at starting. To start the motor and bring it up to synchronism and in step (meaning into the phase relation which is indicated by darkness of lamp L in Fig. 255) it is only necessary to supply polyphase starting currents to its armature windings and raise the supply voltages step by step by means of the starting compensator, the last voltage step being to full normal value of supply voltages.

This method of starting, however, developes only a very small starting torque and it cannot be used to start a synchronous motor with a connected load. A synchronous motor driving a direct-current generator can be quite satisfactorily started by induction motor action, and the synchronous converter which is essentially a synchronous motor can be satisfactorily started by induction motor action, but the field windings of a synchronous converter usually consist of many turns of fine wire and it is necessary to use what is called a field "break up" switch as explained in Art. 187.

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* The polyphase induction motor is described in Chapter XVII.

† Always arranged for what is called the autotransformer connections.

PROBLEMS.

174. The supply voltage *A* in Fig. 255 is 1200 volts, the voltage of machine *B* is 1100 volts, and the step-down ratio of the transformer *T* is 10 : 1. Find the voltage acting on the lamp *L* (*a*) When *A* and *B* work together on the primary coil of *T*, (*b*) When *A* and *B* are exactly opposed to each other in their action on the primary coil of *T*, and (*c*) When the phase difference between *A* and *B* is ± 90°, Ans. (*a*) 230 volts; (*b*) 10 volts; (*c*) 163 volts.

175. The voltages of machines *A* and *B* in Fig. 256 are 1000 volts and 950 volts respectively, and the transformers *T* and *T'* have the same step-down ratio, namely, 10 : 1. Find the voltage acting on the lamp *l* (*a*) When voltages of machines *A* and *B* work together with respect to the circuit *AsBs'*, (*b*) When the voltages of machines *A* and *B* exactly oppose each other with respect to the circuit *AsBs'*, and (*c*) when the voltages of machines *A* and *B* are 90° apart in phase. Ans. (*a*) 195 volts or 5 volts; (*b*) 5 volts or 195 volts; (*c*) 138 volts.

176. The value of supply voltage *A* in Fig. 255 is 1000 volts, the voltage of machine *B* is 950 volts, the frequency is 60 cycles per second, each receiving circuit has a resistance of 0.2 ohm and an inductance of 0.0003184 henry. Each inch of lines *A* and *B* represent 200 volts (voltmeter value, of course). Find the amount of power represented by each inch of *OQ*. Ans. 814.5 kilowatts.

177. Find current in each phase of machine B when it is running stably at zero load, neglecting stray power losses; data as in problem 176. Ans. 418 amperes.

178. Find the load per phase which will reduce the current (from 418 amperes at zero load) to a minimum value and find this minimum value of current; data as in the previous problem. Ans. 174.6 kilowatts; 214 amperes.

179. Find the greatest possible synchronous motor intake of machine B per phase; data as in previous problem. Ans. 755 kilowatts.

Note. In all of the above problems the demagnetizing action of armature current is ignored, whereas it is by no means negligible in its effects. In this particular problem severe overload conditions are involved and the magnetizing or demagnetizing action of armature current is large.

180. The voltage of each phase of machine B is increased by manipulation of field rheostat until machine will barely run as a synchronous motor at zero load (stray power loss assumed to be zero). Find the value to which B is increased. Ans. 1166 volts.

181. The voltage of machine B is adjusted to give minimum current (maximum efficiency) for given motor load on B of 200 kilowatts per phase. Find the value of the current in each phase, and find the value of voltage of each phase of B. Ans. 208.7 amperes, 958.6 volts.

Note. When voltage of B is adjusted to give minimum current for given load on B the current in each phase comes into phase with the supply voltage A as indicated in Fig. 263. The power per phase taken from the supply mains is AI and this must be equal to $RI^2 + 200{,}000$ watts, whence I may be determined.

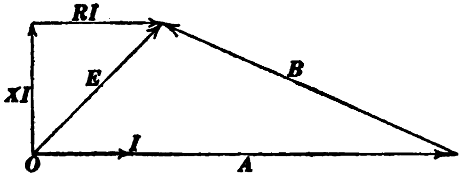

Fig. 263.

182. An alternator B has an electromotive force of 1,100 volts, a resistance of 1 ohm, and a reactance of 0.58 ohm. The machine is driven as a synchronous motor with zero load, from 1,000-volt mains. (*a*) What is the value of the current? (*b*) What is the component of this current which is 90° ahead of the supply electromotive force in phase? (*c*) What capacity of condensers would take the same amount of wattless leading current from the 1,000-volt mains at a frequency of 60 cycles per second? Ans. (*a*) 211.4 amperes, (*b*) 206.6 amperes, (*c*) 548 microfarads.

CHAPTER XV.

THE CONVERSION OF ALTERNATING CURRENT INTO DIRECT CURRENT.

183. The conversion of alternating current into direct current.—It is frequently necessary to utilize in the form of direct current the power which is delivered by an alternator, or, in other words, to convert alternating current into direct current. Thus, if the power delivered by an alternator is to be used for charging storage batteries it must be used in the form of direct current. If power, which is transmitted to a distance in the form of alternating current, is to be used to drive direct-current motors, such as are extensively employed in electric railway operation, it must be converted from alternating current to direct current.

The methods for converting alternating current to direct current are briefly as follows:

(*a*) *By means of the rectifying commutator.* This is essentially a reversing switch driven by a synchronous motor and adjusted to reverse the connections of the receiving circuit at each reversal of supply voltage. This rectifier is used in some installations to rectify the high voltage output of a small step-up transformer for supplying the steady (one-direction) voltage required in the Cottrell process which is described in Art. 81. A small rectifier of this type, driven by a synchronous vibrating device, is extensively used to rectify the very low voltage output of a step-down transformer for charging a small group of storage batteries.

(*b*) *The vacuum tube valve* or rectifier is described in Art. 85.

(*c*) *The mercury-arc rectifier* is extensively used for handling the small amount of power, a very few kilowatts, required for charging storage battery sets.

(*d*) *The motor-generator set,* usually consisting of a synchronous motor driving a direct-current generator, is used for converting large amounts of power when the greatest freedom of control of the voltage of the direct-current generator is desired.

(*d*) *The synchronous converter* is nearly always used for converting large amounts of power from alternating current to direct current.

184. The mercury-arc rectifier.—Figure 264*a* represents a glass bulb *DD* containing a pool of mercury *C* and a graphite or iron electrode *A′*. The bulb is highly exhausted and contains only mercury vapor. A very high electromotive force is required to start a current in either direction through the mercury vapor, but when the current is once established *in the direction represented by the arrows* (*A′* being an anode and *C* being a cathode), it can be maintained by an electromotive force of 10 or 15 volts. Under these conditions a jet of mercury vapor *N* appears to issue from a point on the surface of the mercury. This jet is called the *cathode blast* or the *negative blast.**

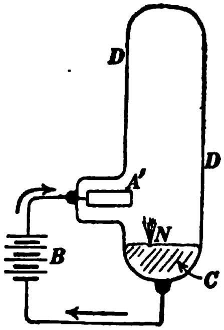

Fig. 264*a*.

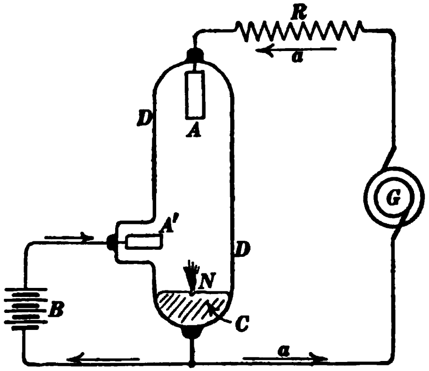

Fig. 264*b*.

Figure 264*b* represents a glass bulb, exactly like that which is shown in Fig. 264*a*, with a battery *B* (or any source of direct current) maintaining a cathode blast *N*, and an alternating-current generator *G* connected, as shown, to a graphite or iron electrode *A* and to the mercury cathode *C*. Under these conditions, current can flow readily in the direction of the arrows *aa*, but it cannot flow in the opposite direction (unless the electromotive force of the generator *G* amounts to many thousands of volts). That is to say, the bulb *DD* with its auxiliary battery *B* for maintaining the cathode blast *N* acts like a valve, permitting current to flow in one direction but not permitting current to flow in the other direction. Such an arrangement is called a *mercury-arc rectifier.*

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* See a paper on "The Electric Arc," by C. P. Steinmetz, *Transactions of the International Electrical Congress*, Vol. 2, pages 710 to 730, St. Louis, 1904; also a paper on "The Mercury Arc Rectifier," by C. P. Steinmetz, *Transactions of the American Institute of Electrical Engineers*, Vol. 24, pages 371 to 393, 1905; and a paper by P. D. Wagoner, read before the *National Electric Light Association* at Denver, Colo., in June, 1905.

The mercury-arc rectifier is arranged in practice as shown in Fig. 265. In this arrangement the mercury-vapor tube *DD* has two graphite or iron anodes *A* and *A′* from either of which current may flow to the mercury cathode *C* and thence through the storage battery *J* (which is being charged) to the point *p* which is at a potential midway between the potentials of the terminals *t* and *t′* of the supply transformer. The coils *E* and *F* are the two coils of a transformer.

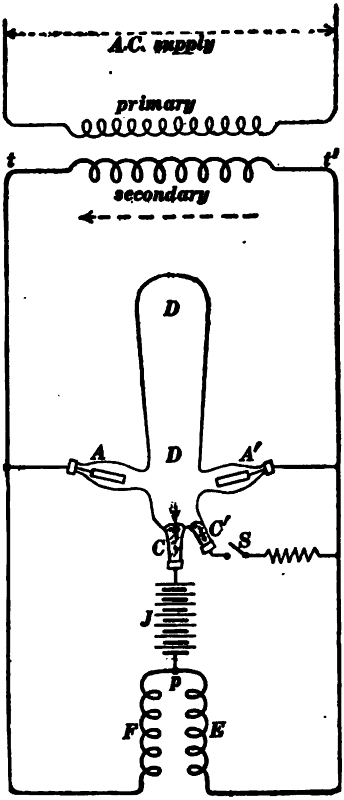

Fig. 265.

Consider the instant when the supply voltage is in the direction of the dotted arrow, *t* being at a high potential and *t′* being at a low potential. Current enters the rectifier at *A*, flows to *C*, and thence through the storage battery J to the point p. The potential of this point p is midway between the potentials of the terminals t and t', and the current divides at p, half of it flows down-hill, as it were, through E to t' delivering energy to E as a primary coil. This energy is transferred to the coil F by transformer action and the other half of the current is pumped up-hill, as it were, to the terminal t.

When the supply voltage is in the opposite direction, the current enters at A' and half of it flows down-hill through the coil F delivering energy which is transferred to E, thus pumping the other half of the current up-hill to the terminal t'.

The mercury cathode C' is called the *starting cathode* and it is used as follows: The switch S is closed and the tube is gently rocked so as to bring the mercury in the pools C and C' momentarily into contact. Alternating current then starts to flow between C' and C, and the moment C becomes a cathode, current begins to enter at A and A' as above explained. The switch S is opened after the tube is in operation.

The transformer FE is constructed so as to give the effect of a fairly large inductance in the main circuit CJ, and the momentum of each pulse of direct current in this inductance keeps the current flowing during the reversal of the alternating supply voltage. The negative blast is thus permanently maintained on the mercury cathode C.

Some idea of the voltage relations of the mercury-arc rectifier may be obtained by considering a numerical example as follows. Let us assume that the supply voltage (across the terminals tt' in Fig. 265) is harmonic. Let its voltmeter value be E. The voltmeter value of the alternating voltage A to p or A' to p is $E/2$, and the average value during each half-cycle of the voltage A to p or A' to p is $\sqrt{2}E/\pi$. But there is approximately 15 volts drop from A to C or from A' to C, so that the average value of the uni-directional voltage between C and p is $(\sqrt{2}E/\pi) - 15$ volts.

185. The synchronous converter or rotary converter.—Figure 266 represents a 2-pole direct-current generator having a ring-wound armature, and two collector rings are shown connected to opposite points a and b of the armature winding. The electromotive force between the two collector rings is an alternating electromotive force, and taking alternating current from these collector rings the machine becomes an alternating-current generator. Such a machine might be engine driven and deliver direct-current from its direct-current brushes, or alternating-current from its alternating-current brushes, or both; it might be driven as a direct-current motor and deliver mechanical power from its pulley, or alternating current from its alternating-current brushes, or both; or it might be driven as a synchronous alternating-current motor and deliver mechanical power from its pulley, or direct current from its direct-current brushes, or both.

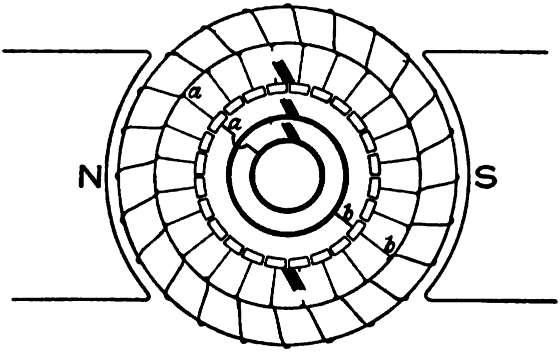

Fig. 266.

This machine is called a *synchronous converter* or *rotary converter*, and its most important use is to deliver direct current when it is driven as a synchronous alternating-current motor. This use is sometimes called the *straight* use of the machine to distinguish it from what is called the *inverted* use where the machine delivers alternating current when driven as a direct-current motor.

The arrangement shown in Fig. 266 would be called a *two-ring* or *single-phase* converter. The *three-ring* or *three-phase* converter has three collector rings connected to the armature winding at three equidistant points (in a 2-pole machine), the *four-ring* or *two-phase* converter has four collector rings connected to the armature winding at four equidistant points (in a 2-pole machine), and so on.

The n collector rings of a multipolar n-ring converter are connected as follows: Ring No. 1 is connected to all the armature turns which for a given position of the armature lie midway under the north poles of the multipolar field magnet. Let l be the distance between adjacent connection points of ring No. 1; then ring No. 2 is connected at points at a distance l/n forwards from the connection points of ring No. 1, ring No. 3 is connected at points at a distance $2l/n$ forwards from the connection points of ring No. 1, and so on.* Thus Fig. 267 shows a four-pole two-ring converter with its commutator and four direct-current brushes and with two collector rings and two alternating-current brushes.

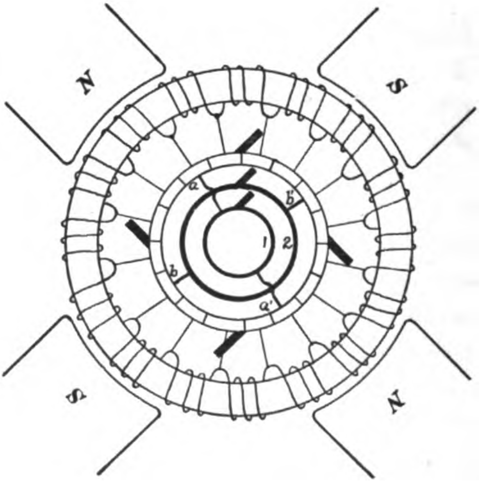

Fig. 267.

186. The internal actions of a synchronous converter.—In its relation to the alternating-current supply a synchronous converter is like a synchronous motor, and in its relation to the direct-current receiving circuit a synchronous converter is like a direct-current generator; but in regard to (*a*) the actual motor and generator actions in the machine, (*b*) the magnetizing action of the armature current, and (*c*) the heating effect of the armature current, the machine is essentially unlike a synchronous motor and essentially unlike a direct-current generator.

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* This statement applies primarily to the simple ring-wound armature.

The details of motor and generator action are of little practical importance and they are described in the following paragraphs of this article, but the magnetizing action and heating effects of the armature current are important, inasmuch as they determine the power rating of the machine.

Motor and generator action in the synchronous converter.[*]—Let a and b, Fig. 268, be the direct-current brushes of a synchronous converter. Let the negative brush b be chosen as the zero or reference point of potential so that the electromotive force between b and any other given point may be spoken of as the potential at the given point.

Let the half-circle represent one side of the armature, and let r be the position at a given instant of the point of attachment of one of the collector rings. At the given instant a definite current i is entering the armature at r from the alternator, which is driving the converter; this current is entering at a potential which is less than the potential of the brush a, a portion B of the current i flows down hill (*against* the induced electromotive force in the windings, which electromotive force is represented by the fine line arrow) to brush b and delivers power to the portion rb of the armature in the form of motor action, and the remainder A of the current i flows up hill (*with* the induced electromotive force) to brush a and receives power from the portion ra of the armature in the form of generator action. That is to say, at each instant certain portions of the armature of a synchronous converter have motor action and develop torque in the direction of

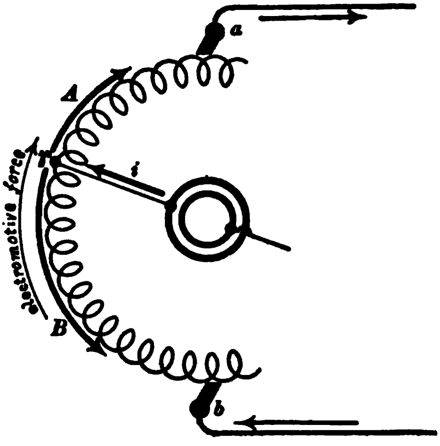

Fig. 268.

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[*] See a paper on "The energy transformations in the synchronous converter," by W. S. Franklin, *Trans. Am. Inst. Elec. Eng.*, Vol. 22, pages 17–33, 1903.

the motion of the armature, and other portions of the armature have generator action and develop torque tending to oppose the motion of the armature. In the two-ring converter the motor action at times greatly exceeds the generator action, at other times the generator action greatly exceeds the motor action, and the armature is alternately accelerated and retarded. This must be so because the single-phase alternating current delivers power to a two-ring converter in pulses. In a polyphase converter, however, the motor action is at all times very nearly equal to the generator action, being just enough in excess to supply the mechanical and magnetic friction losses in the converter armature.

187. The starting of the synchronous converter.—The synchronous converter is a synchronous motor in its relation to the alternating current supply, and it is started as any synchronous motor would be. Usually the starting is accomplished by induction motor action as explained in Art. 182. The alternating currents in the armature, however, produce a very considerable alternating magnetic flux through the field coils at starting, the field winding of a synchronous converter usually consists of a great number of turns of fine wire (the field winding is what is called a shunt winding), and therefore electromotive forces may be induced in the field winding large enough to break down the insulation. To obviate this effect of high voltage a group of switches is usually arranged to break the circuit of the field winding at many places during the starting period, the field circuit being closed and connected to the direct-current brushes of the machine when it reaches full speed.

In many cases direct current is available from a rotary converter already in operation, and in such a case a second converter may be started as a direct-current shunt motor and brought into synchronism and into step by manipulating its shunt field rheostat; but whenever this method of starting is used arrangements are also made for induction motor starting.

188. The synchronous converter in operation.—Not only in the matter of starting but also in certain matters of operation the synchronous converter is like a synchronous motor which delivers mechanical power. Thus the clock diagrams of Art. 177 apply to the synchronous converter, and the discussion of hunting oscillations in Art. 179 is applicable to the synchronous converter. The syncnronous converter, however, will stand an excessive overload (direct-current output) without dropping out of synchronism. In this respect the converter is unlike the synchronous motor (delivering mechanical power) as described in Arts. 176–178, and this unlikeness is due to the fact that the armature reaction in the converter is very different from the armature reaction in a mechanically loaded synchronous motor as explained in the next article. In fact the clock diagrams of Art. 177 are based on idealized conditions in which armature reaction is largely ignored.

189. Armature reaction of the synchronous motor. The need of commutating poles.—The kind of field distortion which is shown in Figs. 191 or 192 is always associated with heavy torque action exerted on the armature by the field magnet, and this kind of field distortion does not exist in the synchronous converter because the torque which is exerted on the armature is very small (it is just sufficient to supply the stray power losses as explained in Art. 125). Therefore the field intensity under the pole tips is not weakened in a synchronous converter as in a direct-current generator as described in connection with Figs. 193 and 194; and therefore commutation is naturally better in a synchronous converter than in a direct-current generator according to Art. 135.

There are however two conditions which tend to annul and more than annul the naturally good commutation of a synchronous converter, namely, (*a*) The speed is usually very high, especially in the case of a synchronous converter adapted to a 60-cycle alternating current supply, and consequently the time is very short for the reversal of current in an armature section while it is short-circuited under a brush; and (*b*) The direct-current output of a given machine as a synchronous converter may be nearly twice as great (without overheating) as the output of the same machine operating as an engine-driven direct-current generator. Therefore commutating poles are now generally used on synchronous converters, especially on 60-cycle synchronous converters.

190. Electromotive force relations of the synchronous converter.—There is fairly definite relation between the value of the electromotive force between the direct-current brushes of a synchronous converter and the effective value of the alternating electromotive force between collector rings. The following discussion of these electromotive force relations is based on the assumption that the alternating electromotive force of the machine is harmonic.*

Relation between the voltage E_0 between the direct-current brushes and the effective value E_2 of the voltage between the collector rings of a two-ring converter. When the two commutator bars to which the collector rings are connected (in Fig. 266 or Fig. 267) are under the direct-current brushes the varying voltage between the rings is at its maximum value, $\sqrt{2}E_2$ [see equation (67)], and this maximum value of the alternating voltage is evidently equal to E_0. Therefore

$$E_2 = \frac{E_0}{\sqrt{2}} \qquad \text{(i)}$$

Relation between the effective value of the voltage E_2 of a two-ring converter and the effective value of the voltage E_n between adjacent collector rings of an n-ring converter. By adjacent collector rings is meant collector rings whose connections to the commutator bars or armature winding are adjacent. Consider a given armature conductor of a two-pole machine. The electromotive force induced in this conductor is an alternating electromotive force and we may without great error assume it to be a harmonic alternating electromotive force so that it may be represented by a vector in a clock diagram. Evidently the same value of alternating electromotive force is induced in every armature conductor, and therefore these electromotive forces are all represented by clock-diagram vectors of the same length; but these electromotive forces differ in phase from each other, in fact the alternating electromotive forces which are induced in the various armature conductors which lie on one side of the armature between the connection points of the two rings in Fig. 266 are evenly distributed through a phase range of 180° as indicated by the equal-length vectors in Fig. 269*a*. Now all these armature conductors are in series with

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* The RI drops of voltage always produce non-harmonic voltage effects which are here neglected.

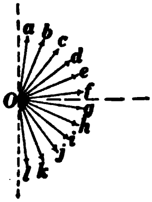

Fig. 269*a*.

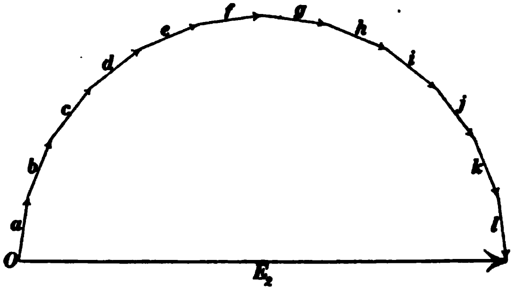

Fig. 269*b*.

each other between the collector rings and therefore the clock-diagram vector, E_2 in Fig. 269*b*, which represents the alternating voltage E_2 is the vector sum of all the vectors in Fig. 269*a*. Adjacent rings of a 3-ring converter would be connected at points 120° apart in Fig. 266, the adjacent rings of a 4-ring converter would be connected at points 90° apart in Fig. 266, and adjacent rings of a 6-ring converter would be connected at points 60° apart in Fig. 266. Therefore the relation between E_2, E_3, E_4 and E_6 is as shown in Fig. 270, in which the semicircle represents the polygon $a \cdots l$ of Fig. 269*b*. The line E_2 is the diameter (chord corresponding to half of a circumference), E_3 is a chord corresponding to 1/3 of a circumference, E_4 is a chord corresponding to 1/4 of a circumference, and E_6 is a chord corresponding to 1/6 of a circumference. Using the relation $E_2 = E_0/\sqrt{2}$ as given by equation (i) above, and making use of the geometrical relations as shown in Fig. 270 we get

$$E_2 = \frac{E_0}{\sqrt{2}} = 0.707 E_0 \tag{84}$$

$$E_3 = \frac{\sqrt{3}E_0}{2\sqrt{2}} = 0.612E_0 \tag{85}$$

$$E_4 = \frac{E_0}{2} = 0.500E_0 \tag{86}$$

$$E_6 = \frac{E_0}{2\sqrt{2}} = 0.354E_0 \tag{87}$$

where E_0 is the voltage between the direct-current brushes.

Remark 1. The alternating electromotive force of a 4-ring converter is nearly always given as the electromotive force between *opposite* collector rings (rings which are connected at opposite points in Fig. 266), and this is identically the same as E_2 as given above. Also the alternating electromotive force of a six-ring converter is often given as the electromotive force E_2 between *opposite* collector rings. The two-ring converter is seldom or never used in practice. The six-ring converter is most extensively used for reasons explained in Art. 192.

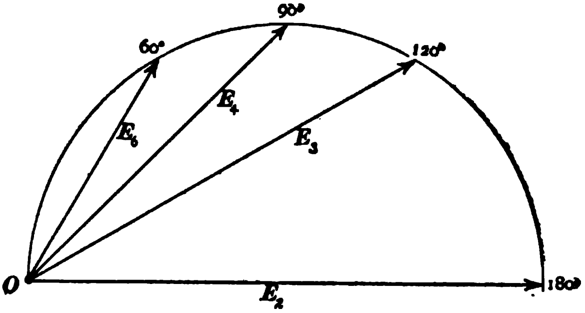

Fig. 270.

Remark 2. All of the above discussion applies primarily to the 2-pole machine, but the results are entirely general and they apply to multipolar machines.

Actual voltage relations of the synchronous converter. The voltage relations given by equations (84) to (87) are ideal because they are based on simplifying assumptions which are only approximately true. The accompanying table gives the values of E_n/E_0 as determined by actual tests of moderately large

3-ring, 4-ring and 6-ring converters, where E_0 is the voltage between the direct-current brushes, and E_n is the voltage (voltmeter value, of course) between adjacent rings of an n-ring converter. In the following table, however, the alternating electromotive forces of the 4-ring and 6-ring machines are specified by giving the value of E_2 in each case as above explained.

VOLTAGE RATIOS OF CONVERTERS* BY TEST.

Three-ring converter

	at zero load	**at full load straight**	**at full load inverted**	**ideal value**
$\frac{E_3}{E_0}$	= 0.610	0.620	0.600	0.612

Four-ring and six-ring converters

	at zero load	**at full load straight**	**at full load inverted**	**ideal value**
$\frac{E_2}{E_0}$	= 0.715	0.725	0.705	0.707

Example. Consider a converter which gives 550 volts between its direct-current brushes at zero load, therefore $E_2 = 0.715 E_0 = 393.3$ volts, and at full load straight we have $E_0 = 393.3$ volts $\div$ 0.725 = 542 volts if the alternating supply voltage does not change.

This example shows that the voltage regulation of a synchronous converter is very good; the eight-volts' rise from 542 volts at full load to 550 volts at zero load is only 1.48 per cent. of full-load voltage, or in other words the percentage voltage regulation is 1.48 per cent.

191. Voltage control of the synchronous converter.—It is evident from the above discussion that the voltage between the direct-current brushes of a synchronous converter cannot be perceptibly altered if the alternating supply voltages are kept at constant values. If the field current of a synchronous converter is reduced, the machine takes *lagging* wattless current† from the alternating-current supply, the effect of which in the armature is to *help* the field flux, *and make up almost completely for the decrease of field current.* If the field current of the machine is increased, the machine takes *leading* wattless current from the supply mains as explained in Art. 181, and this leading wattless current as it flows through the armature *opposes* the field flux *and counteracts almost completely the increase of field current.*

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* These results refer to converters of about 500 kilowatt rating, with the direct-current brushes in the neutral axis *ab* Fig. 190, and with pole faces which cover about 3/4 of the armature surface.

† The statements here made are worded to apply to the single-phase converter.

This inflexibility of the synchronous converter may, however, be to some extent overcome by connecting an inductance in each pair of armature leads (in each phase). Then the *lagging* wattless current produced by a decrease of field excitation as above stated gives a *drop of voltage* in the inductance (see Art. 157*b*) and thereby the voltage (alternating) across the terminals of each phase of the machine is lowered and the voltage across the direct-current brushes is lowered accordingly. Or, the *leading* wattless current produced by an increase of field excitation as above stated gives a *rise of voltage* in the inductance (see Art. 157*c*) and thereby the voltage (alternating) across the terminals of each phase of the machine is increased, and the voltage across the direct-current brushes is increased accordingly.

The synchronous converter is generally provided with compound field windings (shunt and series), and there is a very considerable increase of field excitation due to the series winding when the direct-current output is heavy. In order that this increase of field excitation may be effective in its influence on the voltage between the direct-current brushes, outside inductances must be connected in the alternating-current leads as above explained.

An ideal method for controlling the voltage between the direct-current terminals of the synchronous converter would be to shift the direct-current brushes; by so doing the voltage between the direct-current terminals could be varied from its full value to zero independently of any change of the voltage between the alternating-current terminals of the machine. This method of voltage control is, however, impracticable because of the sparking which is produced at the direct-current brushes when they are shifted from the neutral axis. To overcome this difficulty, Mr. J. L. Woodbridge has proposed the *split-pole converter*[*] in which each field pole is split into two (or three) parts.

192. Synchronous converter ratings.—The use of commutating poles eliminates sparking at the commutator of a synchronous converter to such an extent that the permissible heating of the converter armature, alone, sets the limit to direct-current output. A machine which can deliver 1000 amperes of direct current without overheating when operating as an engine driven direct-current generator, could deliver without overheating only 850 amperes of direct current as a 2-ring or single-phase converter; but it could deliver 1320 amperes of direct current as a 3-ring converter, 1620 amperes of direct current as a 4-ring converter, or 1920 amperes of direct current as a 6-ring converter without overheating.[†] This great capacity of the 6-ring converter has led to its general use. The six-ring converter is usually operated from three-phase supply mains as explained in Art. 206.

PROBLEMS.

183. Find the voltmeter value of the alternating voltage across the supply terminals tt' in Fig. 265 in order that the average value of the uni-directional voltage across the receiving circuit J may be 110 volts. Ans. 278 volts.

Note. The average value during half a cycle of a harmonic electromotive force is $2\sqrt{2}E/\pi$, where E is the voltmeter value of the harmonic alternating electromotive force.

184. Find the ideal values of E_2, E_3, E_4, and E_6 (voltmeter values, of course) of a two-ring, three-ring, four-ring and six-ring converter when the voltage between the direct-current brushes is 110 volts with these brushes in the neutral axis.

185. From the table in Art. 190 find the zero-load and full-load values of E_2 and E_3 when a moderately large converter is used to convert direct-current supplied at 110 volts to alternating current, the direct-current brushes being in the neutral axis, and the pole faces being broad enough to cover 3/4 of the armature surface. Ans. Zero-load values $E_2 = 78.6$ volts, $E_3 = 67.1$ volts; full-load values $E_2 = 77.5$ volts, $E_3 = 66$ volts.

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[*] See a paper by Comfort A. Adams, *Proceedings of the American Institute of Electrical Engineers*, June, 1908, pages 899–925.

[†] These results are derived theoretically and they are fully confirmed by tests. It is not, however, worth while to give the derivation here.

186. Make a connection diagram showing a 4-ring converter taking currents from 4-wire, 2-phase supply mains with an inductance connected in each receiving circuit so as to permit the voltage E_0 between the direct-current brushes to increase or decrease when the field excitation of the machine is increased or decreased.

Note. Each phase of the 2-phase supply is connected to opposite collector rings A 3-wire 2-phase supply cannot deliver currents to a 4-ring converter.

187. A direct-current 110-volt generator can deliver an output of 50 kilowatts without overheating when driven by an engine. What output could it deliver as a 2-ring converter, as a 3-ring converter, as a 4-ring converter and as a 6-ring converter without overheating?

CHAPTER XVI.

SIMPLE THEORY OF THE TRANSFORMER. TRANSFORMER CONNECTIONS.

193. The ideal transformer.—The construction of the alternating-current transformer is described in Art. 43 and the great practical importance of the transformer is explained in Art. 44. The action of a transformer would be very simple (*a*) if the coils of wire had zero resistance, (*b*) if there were no magnetic leakage, that is, if all of the magnetic flux which links with one coil also linked with the other coil, and (*c*) if the magnetic reluctance of the iron core were zero. A transformer satisfying these conditions would be called an *ideal transformer.* In an actual transformer, however, the coils always have more or less resistance; some of the magnetic flux, the so-called leakage flux, links with one coil and does not link with the other; and a certain amount of magnetomotive force (ampere-turns) is necessary to force the magnetic flux through the transformer core. *The effects of these things on the action of a well-designed commercial transformer are small, and therefore the very simple theory of the ideal transformer is sufficiently exact for most practical purposes.* All operating engineers in dealing with transformers, make use of the very simple theory of the ideal transformer, whereas designing engineers find it necessary to take account of the effects of coil resistance, magnetic leakage, and core reluctance.

194. Voltage and current relations of the ideal transformer.*—Let N' and N'' be the numbers of turns of wire in the primary and secondary coils, respectively, of the transformer, and let b and e'' be the values *at any instant* of the electromotive forces induced in the primary and secondary coils, respectively.

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* The action of the ideal transformer is brought out clearly in problems 63 and 64 on page 76.

Then $b/e'' = N'/N''$ according to equation (17) of Art. 39. But the value of the supply voltage e' at each instant must be equal and opposite* to b. Therefore we have

$$\frac{e'}{e''} = -\frac{N'}{N''} \tag{88}$$

It is more important, however, to know the relation between the voltmeter values of e' and e''. Let a be written for N'/N''. Then $e' = -ae''$ or $e'^2 = a^2e''^2$ or $\overline{\text{average } e'^2} = a^2 \times \overline{\text{average } e''^2}$ or $\sqrt{\text{average } e'^2} = a\sqrt{\text{average } e''^2}$. But $E' = \sqrt{\text{average } e'^2}$ and $E'' = \sqrt{\text{average } e''^2}$, so that

$$\frac{E'}{E''} = \frac{N'}{N''} \tag{89}$$

where E' and E'' are the voltmeter values of primary and secondary voltage respectively.

Let i' and i'' be the values at any instant of the currents in the primary and secondary coils respectively. Then if the combined magnetizing action of i' and i'' is to be zero† we must have:

$$N'i' + N''i'' = 0,$$

or

$$\frac{i'}{i''} = -\frac{N''}{N'} \tag{90}$$

and by an argument similar to the above we get

$$\frac{I'}{I''} = \frac{N''}{N'} \tag{91}$$

where I' and I'' are the ammeter values of primary and secondary currents respectively.

Example. A transformer has a 10 : 1 ratio of transformation $(= N'/N'')$. It takes 10 amperes from 1100-volt supply mains and delivers 100 amperes to 110-volt service mains.

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* In the ideal transformer in which coil resistances are negligibly small.

† And it would have to be zero in an ideal transformer. See problems 63 and 64.

The clock diagram of the ideal transformer. If the supply voltage e' is harmonic, then, according to equation (88), the secondary voltage e'' is also harmonic and exactly opposite to e' in phase. Let us suppose that this harmonic secondary voltage e'' delivers a harmonic current i'' to the receiving circuit according to Arts. 151–153; then, according to equation (90), the primary current i' must be harmonic and exactly opposite to i'' in phase.

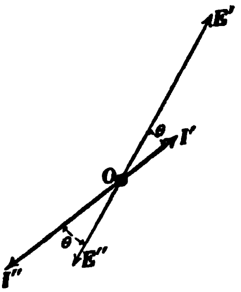

Fig. 271.

For $N'/N'' = 2$.

Let the clock-diagram vector E', Fig. 271, represent the given primary supply voltage. Then E'' is exactly opposite to E' and the value of E'' is determined by equation (89). The value of the secondary current I'' and the phase angle are determined from equations (73), (74) and (75) which give

$$I'' = \frac{E''}{\sqrt{R^2 + X^2}}$$

and

$$\tan\theta = \frac{X}{R}$$

where R is the resistance and X is the reactance of the secondary receiving circuit. Finally the primary current I' is exactly opposite to I'' in phase and its value is determined by equation (91).

195. Relation between core flux and primary voltage.—Let us assume that the varying value, ϕ, of the core flux of a transformer is given by the equation

$$\phi = \Phi \sin \omega t \qquad (a)$$

where ϕ is the value of the flux at any instant t, and Φ is the maximum value of the flux. [Compare this equation with equations (64) and (65).] The rate of change of ϕ is found by differentiating equation (a). Thus we get

$$\frac{d\phi}{dt} = \omega\Phi \cos \omega t \qquad (b)$$

but, according to equation (17) of Art. 39, $-N'\frac{d\phi}{dt}$ is the electromotive force in abvolts, or $-N'\dfrac{d\phi}{dt} \div 10^8$ is the electromotive force in volts induced in the primary coil. Therefore, ignoring algebraic signs, we have

$$e' = \frac{\omega N'\Phi}{10^8}\cdot\cos\omega t \tag{c}$$

Therefore the maximum value, $\mathbf{E}'$, of the primary voltage is $\omega N'\Phi \div 10^8$ or the voltmeter value, E', of the primary voltage is $\mathbf{E} \div \sqrt{2}$ so that

$$E' = \frac{\omega N'\Phi}{\sqrt{2}\times 10^8}$$

or, solving for maximum core flux Φ, we get

$$\Phi = \frac{\sqrt{2}E'}{\omega N'}\times 10^8 \tag{92}$$

Example. Let $E' = 1100$ volts at a frequency of 60 cycles per second ($\omega = 2\pi \times 60$), and let N' be 900 turns of wire. Then the maximum core flux Φ is 459,000 lines. The usual limit of magnetic flux density in a transformer core is from 4000 to 6000 lines of flux per square centimeter of section, and therefore the sectional area of the transformer core would have to be about 100 square centimeters to carry a maximum of 459,000 lines.

196. The magnetizing current of a transformer and the power lost in the transformer core.—In an actual transformer the primary current consists of two fairly distinct parts, namely, a part M (ammeter value) which flows when the secondary current is zero (transformer at zero load). This part of the primary current is called the *magnetizing current*, and the power which it takes from the supply mains is lost in the iron core as eddy current and hysteresis losses (all except the power lost in heating the primary coil, and this is quite small because M is small and the resistance of the primary coil is small).

When the secondary coil delivers current I'' the primary coil takes *additional current* I', called the *load current* to distinguish it from the magnetizing current, and this additional current I' is related to the secondary current I'' exactly in accordance with equation (91).

The power taken from the supply mains by the magnetizing current, and which is lost in the transformer core as stated, is very nearly independent of the transformer load, that is to say this loss of power takes place steadily as long as the primary coil of the transformer is connected to the source of alternating-current supply and whether the transformer is loaded or not.

Copper loss in a transformer. The power lost in heating the transformer windings is equal to $R'I'^2 + R''I''^2$, where R' is the resistance of the primary coil and R'' is the resistance of the secondary coil, and this so-called *copper loss* evidently varies with the transformer load (with the value of I'').

The total loss of power in a well designed transformer at full load ranges from about 1 per cent. of the transformed power for a large transformer to about 3 per cent. or more of the transformed power for a small transformer.

197. Transformer ratings.—A transformer is always designed for a particular frequency and value of primary voltage, and for a certain ratio of primary to secondary voltage; and the permissible current output of a transformer is determined by the ultimate allowable rise of temperature of the transformer under load. Thus a given transformer might be rated as a 60-cycle, 1100–110-volt, 10–100-ampere transformer. It is customary, however, to specify the current rating indirectly by giving the power in kilowatts which would be delivered by the transformer at full load to a unity power factor receiving circuit.

The ultimate rise of temperature of a given transformer at full load depends upon the arrangements for cooling; and in the assignment of a particular current rating to a transformer it is understood that certain cooling arrangements will be used. Thus small transformers are usually enclosed in sealed metal cases filled with mineral oil, and the cases are freely exposed to the surrounding air. Large transformers, on the other hand, are cooled by a draft of air forced through specially designed ducts in the core and coil structure, or by the circulation of oil through these ducts, and through or around a series of pipes where the oil itself is cooled by a stream of water or by a draft of air. If the cooling arrangement should for any reason become inoperative, the transformer must be taken out of service and kept out of service until the cooling arrangement is in proper operating condition,

198. Voltage regulation of a transformer.—The secondary terminal voltage of a transformer usually increases from full load to no load, and the increase is generally less than two or three per cent. of full-load secondary voltage. In other words the voltage regulation of a well designed transformer is generally less than two or three per cent.

The decrease of secondary voltage with increase of load is due to coil resistances and to magnetic leakage (see Art. 193). When the secondary receiving circuit is non-inductive (unity power factor) the decrease of secondary terminal voltage with increase of load is due almost wholly to coil resistances, and when the secondary receiving circuit is highly inductive (taking lagging current) the decrease of secondary terminal voltage is due almost wholly to magnetic leakage.

199. The constant-current transformer.—When a number of arc lamps (or other receiving units) are operated in series, individual lamps must be short-circuited to cut them out of service, and in order that the lamps may be independent of each other,

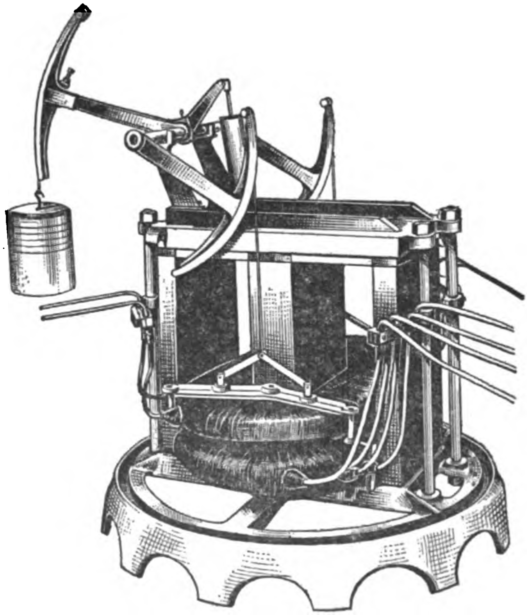

Fig. 272.

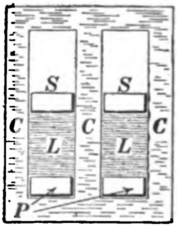

Fig. 273.

the circuit must be supplied with a constant current. When alternating current is used to operate arc lamps in series, the *constant-current transformer* is used. This transformer takes current from a constant-voltage source and delivers current of constant ammeter value to the arc-lamp circuit. Figure 272 shows a constant-current transformer of the General Electric Company. The essential features of this transformer are shown in Fig. 273, in which CCC is the laminated iron core, P is the stationary primary coil, and SS is the secondary coil which is suspended from a counterpoised lever system so as to move up and down with great ease. When lamps are short-circuited, the slight increase* of current causes an increased repulsion between P and S, the coil S moves upwards, and the increase of leakage flux across the regions LL lessens the useful flux through the secondary coil SS, which reduces the voltage induced in SS, thus counteracting the tendency of the current to decrease.

200. Simple transformer connections. Connections in parallel.—When transformers are used to supply current at constant voltage (constant voltmeter value, of course) to groups of lamps or motors, stepping down from high-voltage street mains, the primary coil of each step-down transformer is connected directly across the high-voltage street mains, and each transformer secondary delivers current to a separate group of lamps or motors. In this arrangement the primary coils of all the transformers are in parallel with each other across the street supply mains, and this arrangement is called the scheme of parallel connection of transformers.†

201. Transformers with divided coils.—Alternating-current street mains usually supply current at 1100 or 2200 volts, and the widely accepted standards for lamps and motors are 110 volts and 220 volts. Therefore small and medium sized transformers for general service are usually made with two-part primary coils and with two-part secondary coils, so that the two parts of the primary coil may be connected in series with each other for 2200-volt supply or in parallel with each other for 1100-volt supply, and so that the two parts of the secondary coil may be connected in series with each other to deliver current to 220-volt lamps or motors or in parallel with each other to deliver current to 110-volt lamps or motors.

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* An initial increase of current must be produced in order to start the moving coil upwards, but the lever system may be so designed as to keep the equilibrium value of the current almost exactly constant with decrease of resistance of receiving circuit, or even to cause the equilibrium value of the current to decrease with decrease of resistance of receiving circuit.

† The primary coils of several transformers may be connected in series with each other, but this arrangement is seldom used in practice.

In connecting the two parts of a secondary coil of a transformer in parallel with each other the utmost care must be taken to connect the parts so that one part does not act as a short-circuit for the other, and in connecting the two parts of the primary coil of a transformer in series with each other the utmost care must be taken to be sure that the two parts work together in opposing the primary impressed voltage. Failure in either case to make the proper connections may lead to serious damage to the transformer when it is connected to the mains. Transformers for general service have terminal boards on top of the transformer inside of the containing case, and the terminals of the transformer coils are arranged on this board in a way to facilitate right connections and hinder wrong connections,

202. The autotransformer.*—Consider the supply mains which

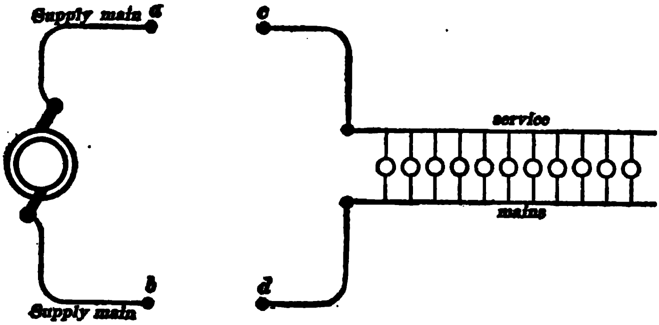

Fig. 274.

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* It is misleading to speak of *the* autotransformer inasmuch as a so-called auto-

bring current from an alternator, and the service mains which deliver current to a customer, as shown in Fig. 274. There are three methods for establishing communication from *ab* to *cd*, as follows: (*a*) By conductive connections, that is, by connecting wires from *a* to *c* and from *b* to *d* in Fig. 274, (*b*) by transformer connections, that is, by connecting the primary of a transformer to *ab* and the secondary to *cd*, and (*c*) by a combination of conductive and transformer connections. Any transformer which is connected up for this combination method is called an *autotransformer*.

The action of the autotransformer may be brought out most clearly by considering a numerical example. Let the coils *A* and *B*, Fig. 275, represent the fine-wire coil and the coarse-wire coil respectively of an ordinary 10 to 1 transformer. The fine-wire coil is connected across 100-volt supply mains, and of course an electromotive force of 10 volts is induced in the coarse-wire coil. One terminal *c* of the receiving circuit is connected directly to one of the supply mains as shown, and *the other terminal b is connected through the coil BB to the supply main a.* This connection may be made so that the 10 volts in *BB* is *added* to the supply voltage thus giving 110 volts across *bc*, or it may be made so that the 10 volts in *B* is *subtracted* from the supply voltage thus giving 90 volts across *bc*. In the first case we have

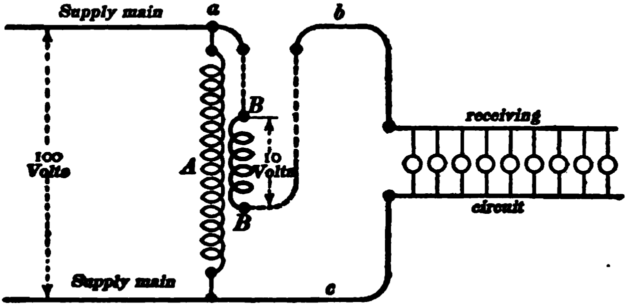

Fig. 275.

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transformer is exactly the same thing as an ordinary transformer, and it acts in the same way. The name autotransformer refers to a scheme of connections.

auto-step-up transformation, and in the other case we have auto-step-down transformation. In the first case, the *additive connections*, the coil A acts as a primary, receiving power from the supply mains; and the coil BB acts as a secondary, delivering power to the receiving circuit. In the second case, the *subtractive connections*, the coil BB acts as a primary, receiving power from the current which is forced through it in opposition to its voltage; and the coil A acts as a secondary, delivering power back to the supply mains. In both cases the power which is actually transformed from A to B or from B to A is equal to 10 volts multiplied by the current I which is delivered to the receiving circuit, whereas the total amount of power delivered to the receiving circuit is $110 \times I$ in the first case, and $90 \times I$, in

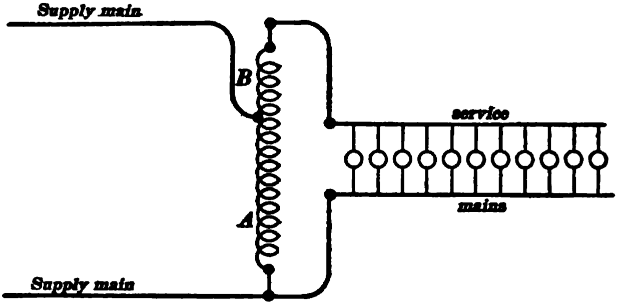

Fig. 276.

Auto-step-up transformation.

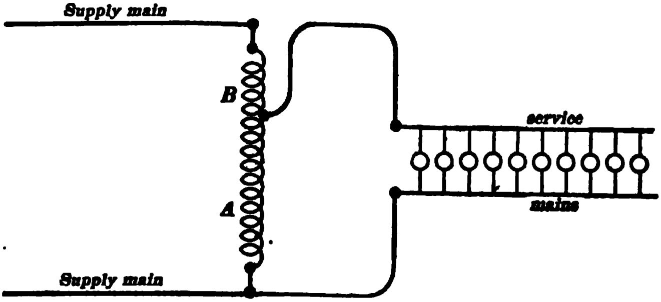

Fig. 277.

Auto-step-down transformation.

the second case. Therefore a 10 to 1 transformer rated as a 1-kilowatt transformer would suffice for the delivery of 11 kilowatts to the receiving circuit in the first case, or 9 kilowatts in the second case. This illustrates the most important feature of the autotransformer when the supply voltage is to be only slightly raised or lowered, namely, that a small transformer suffices for the delivery of a large amount of power, and of course the losses of power are less than would be involved in a large transformer used in the ordinary way.

The additive and subtractive autotransformer connections are most suggestively shown in Figs. 276 and 277 respectively. In each of these figures the two coils *A* and *B* are related to each other as parts of one continuous winding, and the voltage of the whole winding (*A* and *B* together) is to the voltage of any given part of it; as the whole number of turns is to the number of turns in the given part.

ARRANGEMENTS OF TRANSFORMERS IN POLYPHASE SYSTEMS.

203. Step-up and step-down transformation without changing the number of phases.—Step-up and step-down transformation in a polyphase system is usually accomplished by using a separate and independent transformer for each phase.* Thus Fig. 278

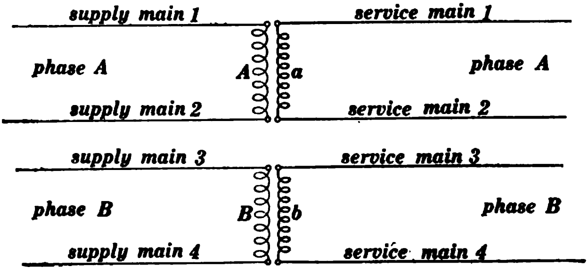

Fig. 278.

Two-phase step-down transformation.

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* In some cases the electrically distinct transformers have a connected core structure; but best practice favors entirely separate transformers for the various phases, because of greater reliability.

shows two transformers *Aa* and *Bb* connected for two-phase step-down transformation, and Fig. 279 shows three transformers *Aa*, *Bb* and *Cc* as usually arranged for three-wire, three-phase step-down transformation. Connections for step-up transformation are exactly the same as connections for step-down transformation, the only difference is that in the one case the energy comes from the low-voltage side, and in the other case the energy comes from the high-voltage side.

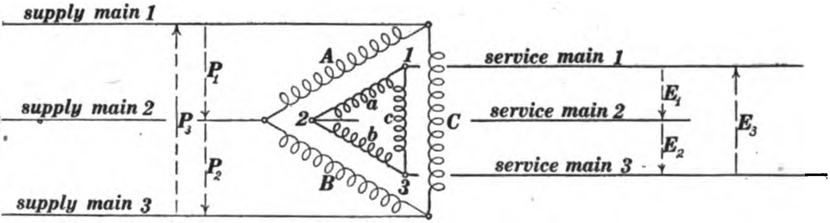

Fig. 279.

The ΔΔ-connection for three phase step-down transformation. Service mains 1, 2 and 3 are connected to the points 1, 2 and 3 respectively.

When three transformers are used for three-wire three-phase transformation the primary coils may be Δ-connected or Y-connected to the supply mains, and the secondary coils may in either case be Δ-connected or Y-connected to the service mains. Therefore four arrangements are possible, namely, the ΔΔ-connection, the ΔY-connection, the YΔ-connection, and the YY-connection. The ΔΔ-connection, as shown in Fig. 279, is nearly always used in practice because the system remains operative when one transformer is disconnected (because of a burn-out, let us say). Thus two transformers connected as shown in Fig. 280 give proper three-phase step-down transformation, but this arrangement is not desirable for steady service for the following reason: Three similar 100-kilowatt transformers would have a combined capacity of 300 kilowatts in Fig. 279; but two of these transformers would have a combined capacity of only 173 kilowatts in Fig. 280. This matter is discussed in the following paragraph as an example of the use of the clock-diagram in the study of transformer connections.

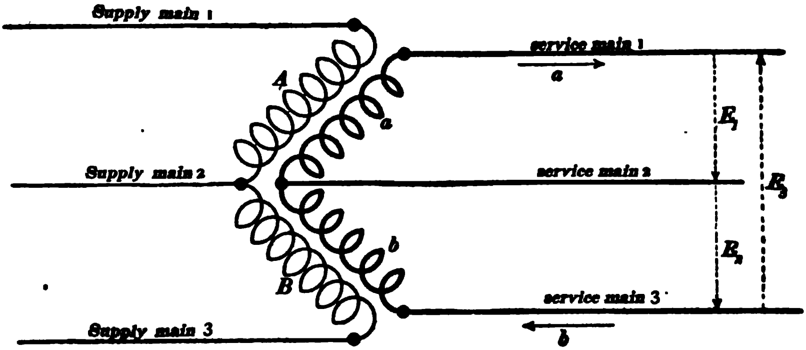

Fig. 280.

Three-phase step-down transformation. The "open-Δ" arrangement using two transformers.

Figure 279 shows three transformers properly connected for three-phase transformation. Let the three clock-diagram vectors P_1, P_2 and P_3 in Fig. 281 represent the voltages of the three-phase supply, P_1 being voltage across supply mains 1 and 2 which is transformed to E_1 across service mains 1 and 2, P_2 being voltage across supply mains 2 and 3 which is transformed to E_2 across service mains 2 and 3, and P_3 being voltage across supply mains 3 and 1 which is transformed to E_3 across service mains 3 and 1 as indicated in Fig. 279. According to Fig. 271 it is proper to consider E_1, E_2 and E_3 as being exactly opposite in phase to P_1, P_2 and P_3 respectively as shown in Fig. 281.

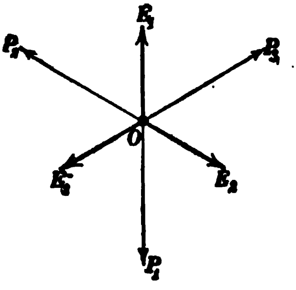

Fig. 281.

To show that E_1, E_2 *and* E_3 *of Fig. 280 are proper three-phase voltages as represented by the clock-diagram vectors* E_1, E_2 *and* E_3 *in Fig. 281.* It is evident that transformer Aa in Fig. 280 transforms from P_1 to E_1 and that transformer Bb transforms from P_2 to E_2 so that the clock-diagram vectors E_1 and E_2 in Fig. 281 represent voltages E_1 and E_2 of Fig. 280. Now the dotted arrows in Fig. 280 represent the chosen positive directions, so that the value e_3 at any instant is equal to $-e_1 - e_2$ and therefore the voltage e_3 must be represented by the clock-diagram vector E_3 which is equal and opposite to the vector sum of clock-diagram vectors E_1 and E_2.

To show that power rating of the two transformers in Fig. 280 is 173/300 of the power rating of the three similar transformers in Fig. 279. Let the arrows

E_1, E_2 and E_3 in Fig. 280 represent chosen positive directions and let i_1, i_2 and i_3 be the instantaneous values of the currents in the respective receiving circuits.

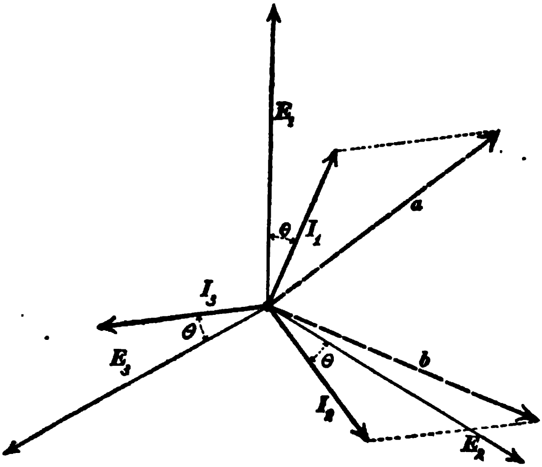

Fig. 282.

Then

$$\text{instantaneous value of current } a = i_1 - i_3$$

and

$$\text{instantaneous value of current } b = i_2 - i_3$$

and the corresponding vector equations are

$$\text{vector } a \text{ in Fig. 282} = I_1 - I_3$$

and

$$\text{vector } b \text{ in Fig. 282} = I_2 - I_3$$

It is evident therefore that the angle between I_1 and a is 30° and that the angle between I_2 and b is 30°.

Let E_1 be the effective value of the voltage induced in the coil a in Fig. 280 and let a be the effective value of the current delivered by this coil. Then $E_1 a \cos(\theta + 30°)$ is the power delivered by transformer secondary a in Fig. 280; and a similar expression, namely, $E_2 b \cos(\theta - 30°)$ gives the power delivered by transformer secondary b in Fig. 280.* In these two expressions for power let us use E for the common numerical value of E_1 and E_2, and let us use a for the common numerical value of a and b (lengths of lines in Fig. 282 may be thought of as representing effective values), then the total power delivered by both transformer secondaries in Fig. 280 is

$$P = Ea\cos(\theta + 30°) + Ea\cos(\theta - 30°)$$

which is easily reduced to

$$P = 2 \times 0.866 Ea \cos\theta$$

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* See equation (71) Art. 149.

But $Ea \cos \theta$ is the full-load output of one transformer in Fig. 279 where a is the full-load current. Therefore the full-load output of the two transformers in Fig. 280 is 2×0.866 or 1.732 times the full-load output of one transformer.

It is to be noted that the current a is 30° behind E_1 and the current b is 30° ahead of E_2 in phase even when the receiving circuits have unity power factor ($\theta = 0$). This out-of-phase relationship of electromotive forces and currents in the transformers Aa and Bb is due to the non-symmetrical arrangement in Fig. 280, and it is called *de-phasing action*.

204. **Phase transformation.**—Transformers in polyphase systems are often arranged so as to change the number of phases and at the same time to produce step-up or step-down transformation. Thus at one of the large power stations at Niagara Falls, two-phase alternators are used, and the step-up transformers which supply the Buffalo transmission line are arranged to change from two-phase to three-phase so as to take advantage of the fact that three-phase transmission requires less copper than two-phase transmission, for the same voltage between wires. Also when it is desired to operate, from an n-phase supply, a synchronous converter having m collector rings, the step-down transformers must be arranged to change the number of independent phases.

It is impossible by means of transformers to convert a single-phase supply into a polyphase supply, because a transformer cannot convert the pulsating power delivered by a single-phase alternator into the steady flow of power required in the polyphase system. A transformer (that is, an ideal transformer) delivers energy at each instant at the same rate that it receives energy.

The principle involved in the transformation of phases may be best brought out by considering the transformation from two phases to any number of phases, say, three; although any arrangement of transformers which will transform from two phases to n phases will serve to transform from n phases to two phases.

Let A and B, Fig. 283, represent the two electromotive forces of a two-phase supply, and let E_1 be any given electromotive force which it is desired to produce. The component of

E_1 parallel to A is $E_1 \sin \beta$, and the component of E_1 parallel to B is $E_1 \cos \beta$; and *to produce the desired electromotive force* E_1 *it is only necessary to produce the components of* E_1 *in the respective secondaries of two transformers, of which the primaries are supplied from* A *and* B *respectively, and to connect these secondary coils in series.* Thus Fig. 284 shows two separate transformers with their primary coils connected to phase A and phase B, respectively, of a two-phase supply, the secondary coil a has a number of turns sufficient to produce the electromotive force $E_1 \sin \beta$, the secondary coil b has a number of turns sufficient to produce the electromotive force $E_1 \cos \beta$, and the two coils a and b connected in series give the required electromotive force E_1. Similarly, the electromotive forces E_2 and E_3, Fig. 283, may be produced each by a pair of properly proportioned and properly connected secondary coils. Therefore, *the two-phase, three-phase transformer consists of two transformers with their primary coils supplied from the two-phase system, and having, in general, six secondary coils which are con-*

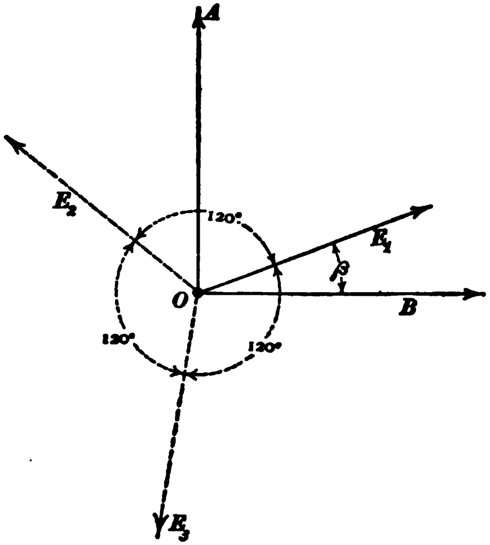

Fig. 283.

nected in pairs in series, each pair supplying one of the three-phase voltages. Such an arrangement can be used to change from two-phase to three-phase or to change from three-phase to two-phase.

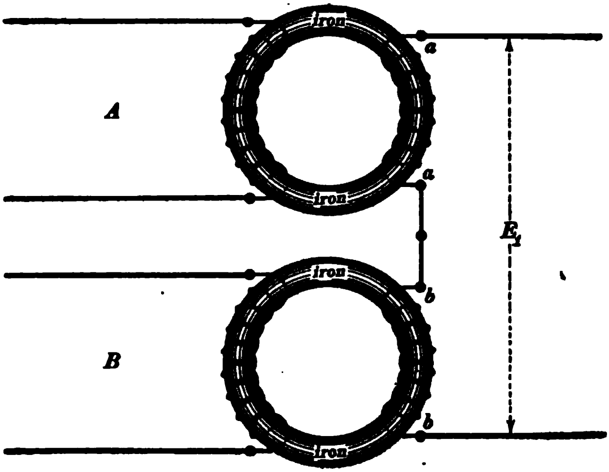

Fig. 284.

205. The Scott transformer connections.—The generalized scheme of transformer connections for two-phase to three-phase transformation may be greatly simplified and the simplest arrangement is due to Charles F. Scott. To understand the Scott connections, however, it is necessary to consider an even more highly generalized scheme of connections than that described above as follows:

Consider three units *a*, *b* and *c* of any kind (these units may be alternators or combinations of transformer secondaries in which alternating electromotive forces are produced), and let these units *a*, *b* and *c* be Y-connected to three mains 1, 2 and 3 as indicated in Fig. 285. Let the electromotive forces in the three units *a*, *b* and *c* be represented by the three clock-diagram vectors *a*, *b* and *c* in Fig. 286. Then, if the vectors *a*, *b* and *c* terminate at the apices of an equilateral triangle *pqr*, the three units in Fig. 285 will produce symmetrical three-phase voltages between the three wires 1, 2 and 3 in Fig. 285. This is evident when we consider the positive directions as indicated by the arrows in Fig. 285 showing that $E_1 = a - b$,

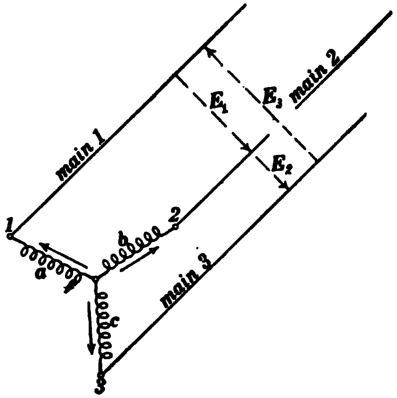

Fig. 285.

Arrows represent positive directions.

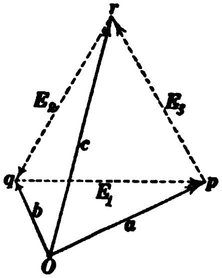

Fig. 286.

$E_2 = b - c$ and $E_3 = c - a$, as shown in Fig. 286. To give two-phase to three-phase transformation each of the units a, b and c in Fig. 285 would consist of two properly proportioned and properly connected secondary coils one on each transformer of Fig. 284.

This highly generalized scheme becomes greatly simplified in the Scott connections. The point O in Fig. 286 is located at the middle point of pq as shown in Fig. 287, and the clock-diagram vectors A and B which represent the voltages of the

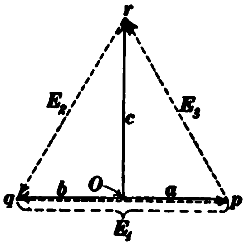

Fig. 287.

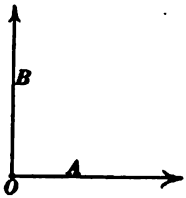

Fig. 288.

two-phase supply are parallel to *Or* and *qp* respectively as shown. *The voltage c is then produced by a single secondary coil on a transformer supplied from phase B, and the voltages a and b are produced by a pair of secondary coils* on a transformer supplied from *phase A*, and the three units *a*, *b* and *c* are Y-connected to the three-phase mains as indicated in the connection diagram Fig. 289. The two coils *a* and *b* are related to each other as the two halves of a continuous winding, a connection tap of lead being brought out from its middle point.

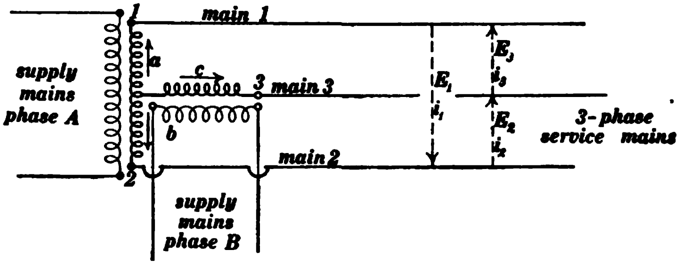

Fig. 289.

All arrows represent positive directions.

De-phasing action in the Scott scheme of connections. The Scott connections are lacking in symmetry with reference to the three-phase mains 1, 2 and 3 in Fig. 289, and this lack of symmetry produces out-of-phase relationships between voltages and currents in the two secondary coils *a* and *b* as follows: The clock-diagram vectors *a*, *b* and *c* in Fig. 290*a* represent the electromotive forces in coils *a*, *b* and *c* in Fig. 289, and the vectors E_1, E_2 and E_3 in Fig. 290*a* repre-

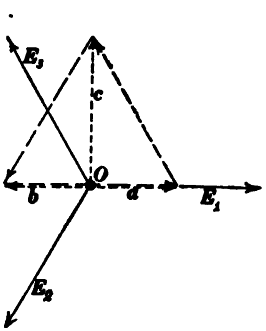

Fig. 290*a*.

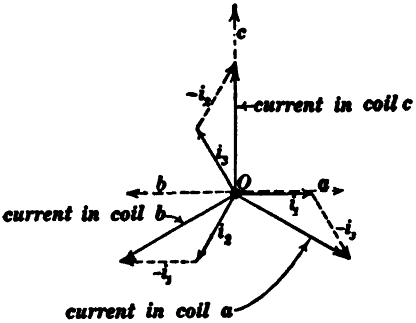

Fig. 290*b*.

Vectors *a*, *b* and *c* the same as *a*, *b* and in Fig. 290*a*.

sent the three-phase voltages as indicated in Fig. 289. Imagine three similar Δ-connected receiving circuits taking equal currents i_1, i_2 and i_3 as indicated in Fig. 289 and as represented by the clock-diagram vectors i_1, i_2 and i_3 in Fig. 290*b*; these currents are *shown* in phase with E_1, E_2 and E_3 respectively, but if the receiving circuits are inductive all of the current vectors in Fig. 290*b* would be displaced clock-wise through the angle of phase-lag θ. The current in coil *a* is evidently equal to $i_1 - i_3$, the current in coil *b* is evidently equal to $i_2 - i_1$, and the current in coil *c* is evidently equal to $i_3 - i_2$, according to the arrows in Fig. 289. Therefore the current in coil *a* is 30° behind voltage *a* in phase, the current in coil *b* is 30° ahead of voltage *b* in phase, and the current in coil *c* is in phase with voltage *c*—when angle of phase-lag θ is zero. It is interesting to note that the primary current of phase *B* in Fig. 289 is in phase with the primary voltage of phase *B* when the angle θ above mentioned is zero.

206. Transformer connections for synchronous converters.

(*a*) *The three-ring converter* is supplied from a three-wire three-phase system, the three service mains in Fig. 279 being connected to the three collector rings. Interchange of any two connections reverses the direction of running of the machine.

(*b*) *The four-ring converter* is supplied from a four-wire two-phase system. If the two phases are transmitted over a three-wire line the step-down transformers must deliver current to four service mains. The wires of phase *A* are connected to two opposite* collector rings, and the wires of phase *B* are connected to the other two rings. Reversal of connections of either phase reverses direction of running.

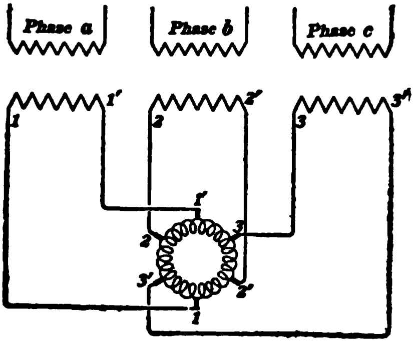

Fig. 291.

(*c*) *The six-ring converter* must be supplied from a six-wire three-phase system; the primaries of the transformers may be, and usually are supplied from a three-wire three-phase line, but the secondaries of the transformers must deliver current to six service mains as shown in Fig. 291. Each phase is connected* to a pair of "opposite" collector rings, the collector rings are indicated in Fig. 291 by their points of connection to a two-pole armature winding. One pair of service wires being connected, the connection of the second pair determines the direction of running, and the third pair must be connected so as to obviate short-circuit condition.

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* Opposite collector rings are those which are connected to opposite points of a two-pole armature winding.

PROBLEMS.

188. The primary coil of a transformer has 100 turns of wire and its resistance is negligible. The magnetic reluctance of the transformer core is assumed to be constant and its value such that 1,000,000 lines of flux are produced by 200 ampere-turns. The primary coil is connected at a given instant to a 100-volt storage battery. (*a*) Find the current in the primary coil 0.02 second after the given instant, the secondary coil being open, (*b*) find the total current in the primary coil 0.02 second after the given instant, the secondary coil (of 50 turns) being connected to a non-inductive receiving circuit of 5 ohms resistance. Ans. (*a*) 4 amperes. (*b*) 9 amperes.

Note. The transformer coils are both assumed to have zero resistance and magnetic leakage is assumed to be zero. In case (*a*) the primary current is what is required to produce the necessary flux, the necessary flux being the value to which the flux rises in 0.02 second in order to induce 100 volts in the primary coil. Find the electromotive force induced in the secondary coil when 100 volts is induced in the primary. Find the secondary current produced by this electromotive force (there is no question of inductance anywhere, not even in the secondary coil because the mutual action of primary and secondary coils of an ideal transformer annuls all inductance), find the primary current corresponding to this secondary current, and add this primary current to that found under (*a*).

189. The secondary coil of the transformer of problem 188 is connected to a circuit of which the inductance is 0.125 henry and of which the resistance is negligible. Find the current in the primary coil 0.02 second after the connecting of the 100-volt battery. Ans. 8 amperes.

190. An ideal transformer has 1300 turns of wire in its primary coil and 130 turns of wire in its secondary coil. The primary coil is connected to 1100-volt supply mains, and the secondary coil delivers current to a circuit having a resistance of 0.46 ohms and a reactance of 0.35 ohm. Find the resistance and reactance of a simple receiving circuit which if connected directly to the high-voltage supply mains would take the same current and the same power as the transformer and its connected secondary circuit. Ans. resistance 46 ohms; reactance 35 ohms.

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* Another scheme of connections is possible, but the scheme shown in Fig. 291 is nearly always used.

191. An ideal transformer, $N'/N'' = 10$, has its fine-wire coil connected to 1100-volt supply mains through a non-inductive resistance of 200 ohms, and its coarse wire coil (its secondary) connected to a non-inductive receiving circuit of which the resistance is 3 ohms. Find (*a*) The current in the primary circuit, (*b*) The current in the secondary circuit, (*c*) The voltage across the terminals of the primary coil, and (*d*) The voltage across the terminals of the secondary coil. Ans. (*a*) 2.2 amperes; (*b*) 22 amperes; (*c*) 660 volts; (*d*) 66 volts.

Note. Let a, b, c and d represent the desired quantities then

$$a/b = 1/10 \tag{i}$$

$$c/d = 10 \tag{ii}$$

$$d = 3b \tag{iii}$$

$$1100 = 200a + c \tag{iv}$$

There is no question of out-of-phase relationships because there is no inductance anywhere. See note to problem 188.

192. An ideal transformer $N'/N'' = 20$ has its fine-wire coil connected to 1100-volt supply mains through a non-inductive resistance of 150 ohms, and its coarse wire coil connected to a non-inductive receiving circuit of which the resistance is 1 ohm. Find a, b, c and d as in the previous problem.

193. The transformer specified in problem 191 is connected as specified except that the secondary receiving circuit has a negligible resistance and a reactance of 3 ohms. Find (*a*) The primary current, (*b*) The secondary current, (*c*) The primary

terminal voltage, and (*d*) The secondary terminal voltage. Ans. (*a*) 3,05 amperes; (*b*) 30.5 amperes; (*c*) 915 volts; (*d*) 91.5 volts.

194. A bundle of iron wires of which the total sectional area is 12.6 square centimeters is to be magnetized by alternating current from 133-cycle 110-volt mains so that the maximum flux density in the core may be 4,500 lines per square centimeter. (*a*) How many turns of wire are required? (*b*) How many turns of wire would be required at 60 cycles per second, other things remaining the same? Ans. (*a*) 328 turns, (*b*) 728 turns.

Note. Resistance of winding is considered negligible.

195. A transformer rated at 1100–110 volts, 60 cycles, 10–100 amperes is found by test to take a magnetizing current of 0.21 ampere (ammeter value) in its fine wire winding, and the power delivery represented by the magnetizing current is 165 watts as measured by a wattmeter. The resistance of the fine wire coil is 0.86 ohm and the resistance of the coarse wire coil is 0.0081 ohm. Find (*a*) The core loss in watts, (*b*) The efficiency of the transformer for 1/4 load, 1/2 load, 3/4 load and full-load, secondary receiving circuit having unity power factor in each case. Ans. (*a*) 164.96 watts; (*b*) Efficiency at full load is 97 per cent.

Note. The core loss is so nearly equal to 165 watts that the difference is entirely negligible—smaller in fact than the unavoidable error in the wattmeter measurement. The theoretical correction is however applied, giving 164.96 watts so that the rational aspects of the matter may be called to mind.

196. Find the all-day efficiency of the transformer of problem 195 on the following schedule: 12 hours at zero load, 3 hours at 1/4 load, 3 hours at 1/2 load, 3 hours at 3/4 load, and 3 hours at full load. Ans. 93.5 per cent.

Note. The all-day efficiency is obtained by dividing the total energy in watt-hours delivered by the transformer by the total energy delivered to the transformer during 24 hours. Of course the transformer is connected to the live supply wires continuously.

197. A 10-kilowatt 1,100 : 110 volt transformer is connected as an autotransformer to step up from 1,100 volts to 1,210 volts. (*a*) Find the amount of power that can be delivered to a unity power factor receiving circuit at 1,210 volts without exceeding the rating of the transformer; (*b*) find the current in each coil of

the transformer; and (*c*) specify the amount of power delivered to or delivered by each coil. Ans. (*a*) 110 kilowatts; (*b*) current in fine wire coil 9.09 amperes, current in coarse wire coil 90.9 amperes; (*c*) 10 kilowatts delivered to fine wire coil, 10 kilowatts delivered by coarse wire coil.

198. A 10-kilowatt 1,100 : 110-volt transformer is connected as an autotransformer to step down from 1,100 volts to 990 volts. Find (*a*) the amount of power that can be delivered to a non-inductive receiving circuit at 990 volts without exceeding the rating of the transformer; (*b*) find the current in each coil of the transformer; and (*c*) specify the amount of power delivered to or by each coil. Ans. (*a*) 90 kilowatts; (*b*) current in fine wire coil 9.09 amperes, current in coarse wire coil 90.9 amperes; (*c*) 10 kilowatts delivered by fine wire coil back to supply mains and 10 kilowatts delivered to coarse wire coil by the current which flows to the service mains.

199. A transformer has two coils AB and CD arranged as shown in Fig. 292. The coil AB is connected to 11-volt mains, point A being connected to main number one, and point B being connected to main number two. Specify the exact manner in which the coil CD must be connected in order that auto-step-up transformation may be accomplished and find the value of the voltage across the service mains.

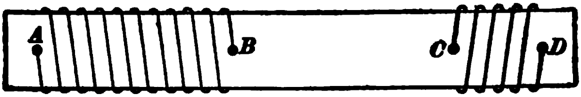

Fig. 292.

200. Specify the exact manner in which the coil CD, Fig. 292, must be connected in order that auto-step-down transformation may be accomplished, with the coil AB connected to 11-volt supply mains, and find the value of the voltage across the service mains.

201. The service mains in problem 200 deliver current to a non-inductive receiving circuit of which the resistance is 0.2

ohms. Find (*a*) the current and power delivered to the receiving circuit; (*b*) the power developed in coil *CD* (specify whether this coil gives out power or receives power); (*c*) the current flowing in coil *AB* and the power developed in coil *AB* (specify whether this coil gives out power or receives power), and (*d*) the total current delivered by the supply mains. Ans. (*a*) 30 amperes, 180 watts; (*b*) the coil *CD* receives 150 watts; (*c*) 13,64 amperes of current flow in the coil *AB*, this current flows back into the supply mains, that is to say, coil *AB* delivers 150 watts to the supply mains; (*d*) of the 30 amperes delivered to the service mains, 13.64 amperes come out of the fine wire coil and the remainder, 16.36 amperes, are delivered by the supply mains.

202. Three similar 1,000 to 100-volt transformers have their 1,000-volt coils delta-connected to three-phase 1,000-volt mains, and the secondaries are Y-connected to service mains. Give a diagram of the connections and find the electromotive force between the pairs of service mains. Ans. 173 volts.

203. The primaries of the transformers in problem 202 are Y-connected to the three-phase supply mains and the secondaries are delta-connected to the service mains. Find the electromotive force between each pair of service mains. Ans. 57.8 volts.

204. The primary coils of two transformers have each 560 turns of wire and they are connected to two-phase mains, the electromotive force of each phase being 800 volts. Calculate the turns of wire in each of two secondary coils (one on each transformer) so that these coils when connected in series may give an electromotive force of 400 volts, 30° ahead of one of the two-phase electromotive forces. Ans. 140 turns, 242.5 turns.

205. Scott connections are to be arranged to transform from 1,000 volts two-phase to 110 volts three-phase. Allowing 0.7 volt per turn of wire on each transformer, find (*a*) Number of turns in each primary coil, (*b*) Number of turns in each secondary coil as shown in Fig. 289.

206. A three-ring synchronous converter is to take alternating current from a 11,000-volt three-wire three-phase line and deliver direct-current at 500 volts. Step-down transformation is to be accomplished by three transformers with the double-delta connection. What is the transformation ratio of each transformer? Ans. The value of the ratio for ideal conditions would be 35.95 to 1.

CHAPTER XVII.

THE INDUCTION MOTOR.

207. The induction motor.—The polyphase induction motor which is here described and the single-phase induction motor which is described later are both similar to the direct-current shunt motor in their running characteristics, that is they run at practically constant speed at all loads up to a very considerable overload; but the direct-current shunt motor can easily give two or three times normal full-load torque at starting, whereas the polyphase induction motor rarely gives more than 1.5 times normal full-load torque at starting and in many cases much less, and the single-phase induction cannot start as such but must operate either as a polyphase induction motor or as a commutator motor at starting.

It can start [illegible] [illegible]

The polyphase induction motor consists of a primary member to the windings of which polyphase currents are supplied, and a secondary member with short-circuited windings. The primary member is generally stationary and it is called the *stator*; the secondary member generally rotates and it is called the *rotor*.

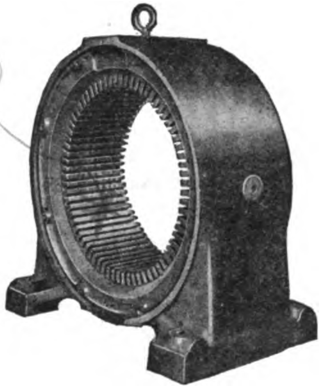

Fig. 293.

The structure of the induction motor is shown in Figs. 293, 294 and 295. Figure 293 shows a cast iron frame or housing which holds the sheet iron stampings which constitute the stator core. This stator core presents a large number of slots on its inward face, and in these slots the stator winding is placed. *This stator winding is identical to the armature winding of a two-*

phase or three-phase alternator armature as described in Arts. 160 and 161. Figure 294 shows the stator complete with its windings, and *squirrel-cage type* of rotor is shown complete in Fig. 295. This type of rotor is made by laying insulated copper bars in slots on the outer face of the laminated rotor core and connecting the projecting ends of these bars to two copper rings one at each end of the rotor core. The *wire wound rotor* has a

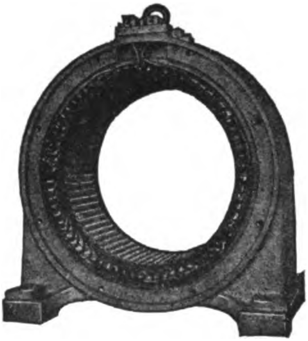

Fig. 294.

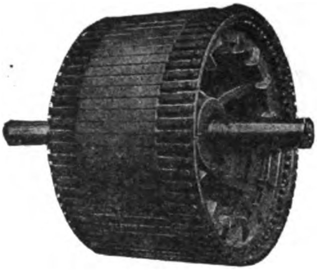

Fig. 295.

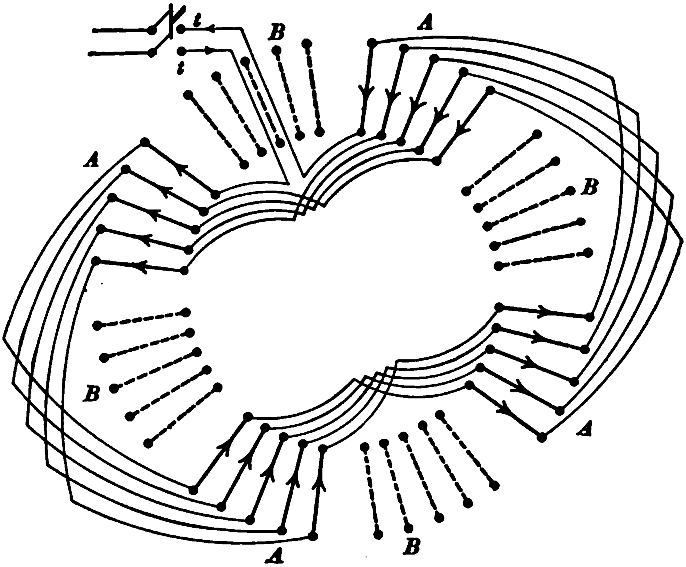

Fig. 296.

winding exactly like a polyphase alternator armature and usually in the wire wound rotor these windings are connected to three or four collector rings exactly as in the three-phase or two-phase alternator armature.

As an example, Fig. 296 shows a two-phase four-pole stator winding. The radial lines represent the conductors which lie in the stator slots, the outside fine curved lines show the winding connections at one end of stator, and the fine curved lines near the middle show the winding connections at the other end of stator. The figure shows complete connections of one winding only, the *A* winding, and it shows this winding connected through its supply switch to the supply mains. The *B* winding is exactly like the *A* winding and it is connected to the other phase of the two-phase supply mains.

208. The action of the induction motor.—The action of the induction motor may be described in two steps, namely, (*a*) the production of rotating magnetism by the stator windings, and (*b*) the action of this rotating magnetism on the short-circuited rotor.

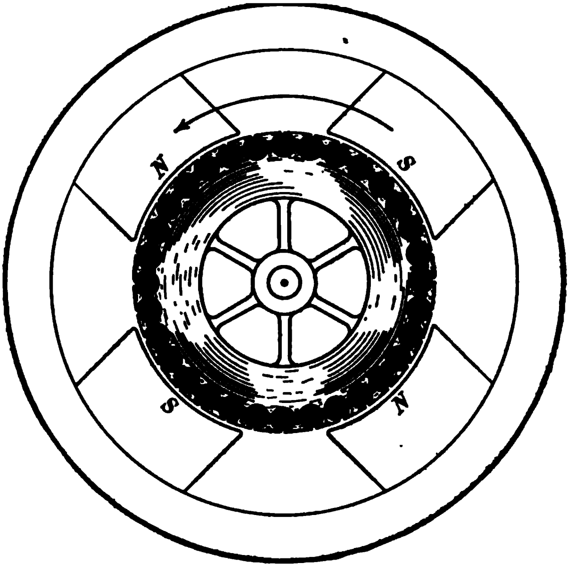

Fig. 297.

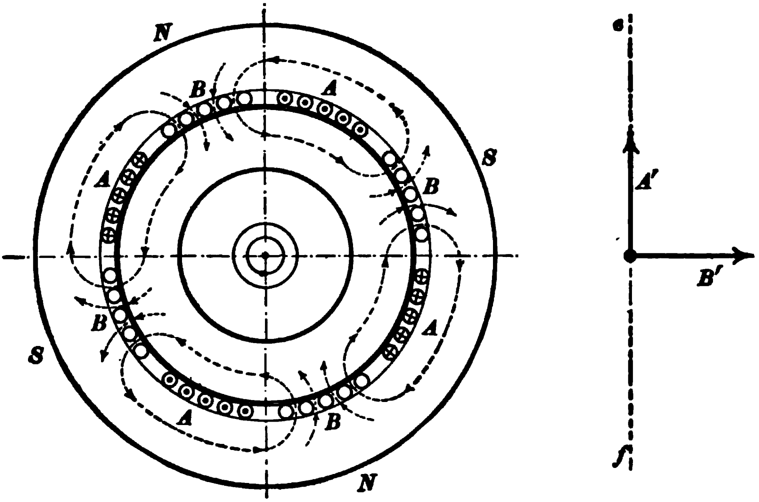

Fig. 298*a*.

Action (*b*) is very simple and may be stated as follows: Imagine a squirrel-cage rotor to be placed between the poles of a mechanically rotated direct-current-excited field magnet as indicated in

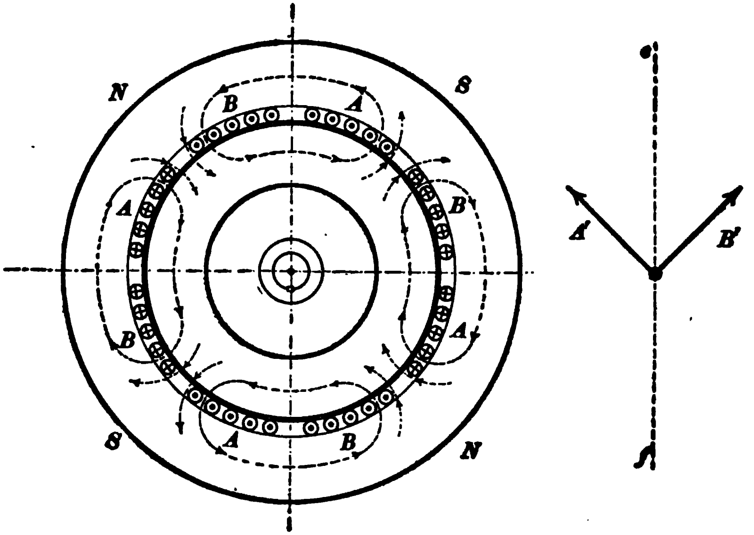

Fig. 298*b*.

Fig. 297. The action of this revolving magnet upon the short-circuited rotor windings is exactly equivalent to the action of the stator of an induction motor upon the rotor. The revolving magnet induces currents in the short-circuited rotor rods, and because of these currents the rotor rods are pushed sidewise by the magnetic flux which emanates from the revolving magnet poles.

The production of rotating magnetism by the stator windings.—Figures 298 represent an end view of a two-phase induction motor with a stator winding like that which is represented in Fig. 296 (four-pole winding). The rotor slots and conductors are omitted for the sake of clearness, and the bands of stator conductors are shown slightly separated from each other so that they may be easily distinguished. Before proceeding to the discussion of the action of the stator windings it is necessary to consider the effect of current in a band of conductors between two masses of iron. This effect is shown in Figs. 299*a* and 299*b*. The small circles with dots represent conductors carrying up-flowing currents, and the small circles with crosses represent conductors with down-flowing currents.* *The action of the currents in these bands*

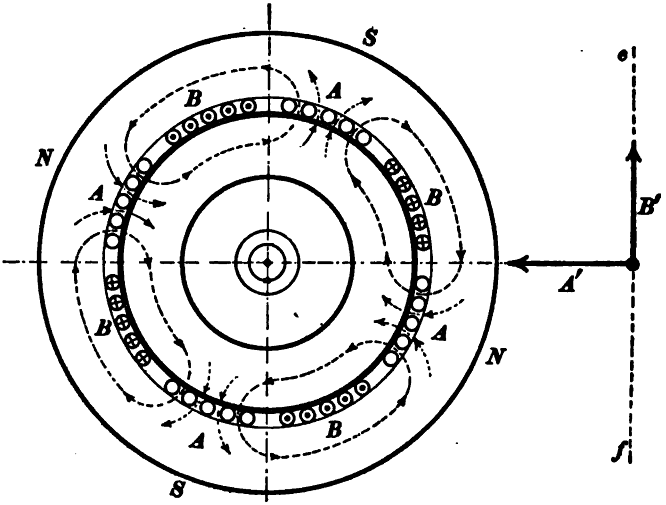

Fig. 298*c*.

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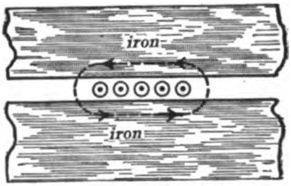
* Plane of paper supposed to be horizontal.

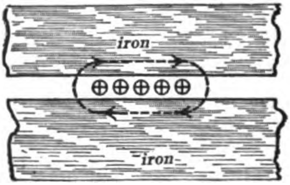
of conductors is to produce magnetic flux along the dotted lines in the directions indicated by the arrows.

The lines A' and B' in Figs. 298 are clock-diagram lines to be thought of as rotating in a counter-clockwise direction; and their projections on the fixed line ef represent at each instant the values of the currents which flow in the A bands and B bands of conductors respectively.



up flowing currents

Fig. 299*a*.



down flowing currents

Fig. 299*b*.

(*a*) At the instant represented in Fig. 298*a* the current in the A bands is at its maximum value, the current in the B bands is zero, and the magnetic flux due to the stator windings is at this instant represented by the dotted lines in Fig. 298*a*. Where the flux passes from stator iron into rotor iron, the stator iron has north magnetic polarity; and where the flux passes from rotor iron into stator iron, the stator iron has south magnetic polarity. The locations of these polar* areas at the given instant are indicated by the letters $NSNS$.

(*b*) When the lines A' and B' have rotated one eighth of a revolution (one eighth of a cycle) as shown in Fig. 298*b*, the current in the A bands has decreased and the current in the B bands has increased, the two currents have become equal in value, and the magnetic flux due to the stator windings has shifted as indicated by the dotted lines in Fig. 298*b*.

(*c*) When the lines A' and B' have rotated one quarter of a revolution (one quarter of a cycle) as shown in Fig. 298*c*, the current in the A bands has decreased to zero, the current in the

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* These letters are shown on the *outside* of the stator iron because there is no room to show them at $BBBB$.

B bands has increased to its maximum value, and the magnetic flux due to the stator windings has shifted still farther as indicated by the dotted lines in Fig. 298*c*.

As a matter of fact, the magnetic flux due to the stator windings shifts *continuously* forwards, and by comparing Fig. 298*a* with Fig. 298*c*, it may be seen that the forward shift of the polar regions during one quarter of a cycle is equal to one half the distance from N to S in Fig. 298*a*. In a whole cycle, therefore, the forward shift of the polar regions is two times the distance from N to S, which is equal to the distance from N to N. That is to say, the stator magnetism makes a complete revolution in $p/2$ cycles where p is the number of polar regions on the stator iron. Therefore if the alternating currents supplied to the stator windings have a frequency of f cycles per second, we have

$$n = \frac{2f}{p} \tag{93}$$

in which n is the speed of the stator magnetism in revolutions per second. This speed $2f/p$ is usually spoken of as *synchronous speed*, and the rotor always runs at a speed slightly less than synchronous speed because at synchronous speed the rotor rods would move along with the stator magnetism so that no electromotive force would be induced in the rotor rods, no current would flow in the rotor rods, no side force would be exerted on the rotor rods by the stator magnetism, and therefore there would be no torque action to keep the rotor turning.*

204. The torque-speed relations of the polyphase induction motor.—In Art. 126 on the speed characteristics of the direct-current shunt motor, the relation between torque and speed is expressed indirectly in terms of armature current and speed for the sake of simplicity and because for a given motor with given field excitation the torque is very nearly proportional to the armature current. In the induction motor, however, it is better to consider the speed characteristics directly in terms of torque and speed.

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* As a matter of fact some torque does exist at synchronous speed of rotor because of hysteresis loss and some other effects.

The relation between speed and torque *for different values of rotor resistance* is shown by the different curves in Fig. 300. The values of rotor speed n' are expressed as fractions of synchronous speed $n(=2f/p)$. When an induction motor is in full operation its rotor speed n' is very nearly equal to synchronous speed n, and with increasing load (increasing torque) the speed n' falls off only a very little up to full rated load. With increasing overload the torque rises still further to a sharp maximum as shown by curve 1, and then decreases more and more with decreasing rotor speed n' as indicated. The motor is said to be at the *break-down point* when it has been overloaded and its torque increased to the maximum point. Any further overload brings the motor to a full stop.

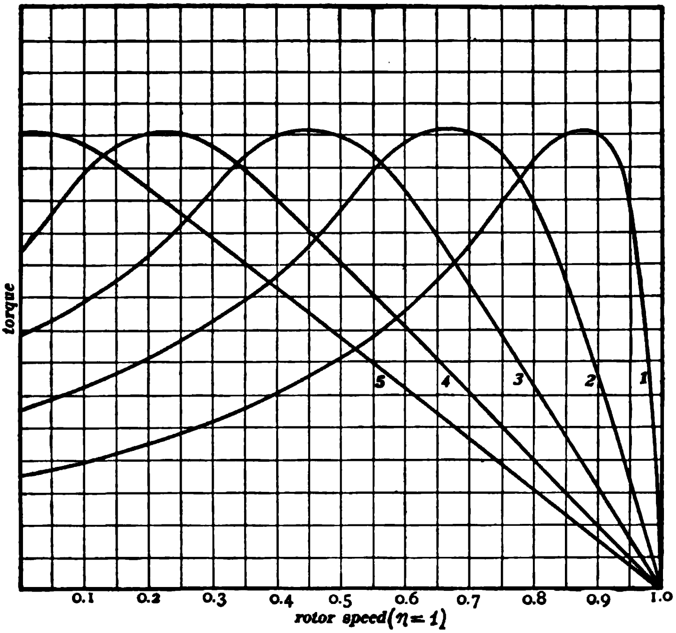

Fig. 300.

The speed at which the maximum torque occurs is lower and lower the higher the rotor resistance. Thus the curves 1, 2, 3, 4 and 5 in Fig. 300 correspond to increasing values of rotor resistance obtained, for example, by connecting outside resistance to the collector rings of a wire wound rotor. The curves 1, 2, 3, 4 and 5 show increasing values of torque at zero speed (starting torque), and in curve 5 the maximum torque occurs at zero speed.

Whenever the use of an induction motor requires it to exert a large starting torque provision must be made for having its rotor resistance large at starting, and this large rotor resistance must be cut down to the lowest possible value to bring the running speed of the motor up nearly to synchronous speed. This high running speed is desirable first because it means high efficiency and second because it means very little change of speed with load as may be seen from curve 1 in Fig. 300.

The equation expressing the relation between torque and speed of a polyphase induction motor can be easily established as follows in so far as the general shape of the curves in Fig. 300 and their dependence on rotor resistance are concerned, the constants in the equations being left undetermined. For the purpose of this derivation let us consider the action of the polyphase induction motor as being resolved into the following elements, namely, (*a*) The stator windings produce a *certain invariable amount of magnetic flux* Φ coming out of the stator iron into rotor iron under the polar areas NN in Figs. 298, and going back from rotor iron into stator iron under the polar areas SS in Figs. 298, and the stator windings cause this flux distribution to rotate at speed $n = 2f/p$; (*b*) This rotating flux, or the part of it which penetrates into the rotor, induces electromotive forces in the rotor rods because the rotor runs at a lower speed n'; (*c*) These induced electromotive forces produce currents in the rotor rods; and (*d*) The flux which penetrates the rotor exerts sidewise forces on the rotor rods thus developing torque. Let us represent this state of affairs as in Fig. 301 which shows a small portion of Fig. 297.

Let Φ be the value of the flux mentioned under (*a*) above. Part of this flux is diverted along the gap space without entering the rotor. Let Φ' be the flux which *does* enter the rotor; then $\Phi - \Phi'$ is the flux that is diverted, and this diverted or leakage flux is proportional to the rotor current I or equal to kI. That is

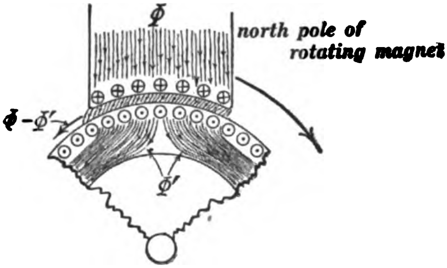

Fig. 301.

$$\Phi - \Phi' = kI \qquad \text{(i)}$$

where k is a constant whose value does not concern us here. For the sake of simplicity we may suppose the rotor to be wire wound (like an alternator armature) then I will be the ammeter value of the current in one of the rotor windings.

The electromotive force induced in one of the rotor windings is proportional to the flux Φ' and proportional to the relative speed $n - n'$, where n is the speed of the stator magnetism (speed of magnet in Fig. 297) and n' is the speed of the rotor; and the current I in the rotor winding is equal* to the induced electromotive force divided by the resistance R of the rotor winding including the outside resistance which is connected between each pair of rotor collector rings. Therefore

$$I = \frac{h\Phi' s}{R} \qquad \text{(ii)}$$

where h is a constant whose value does not concern us, and s is written for $n - n'$, that is

$$s = n - n' \qquad \text{(iii)}$$

The relative speed $s(= n - n')$ is usually called the *slip speed*,

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* There is no question of inductance of the rotor winding after taking the leakage flux $\Phi - \Phi'$ into account.

The torque T is proportional to $\Phi' I$, so that we may write

$$T = j\Phi' I \tag{iv}$$

where j is a constant whose value does not concern us here.

Using equations (i) and (ii) we may eliminate Φ' and I from (iv) and we get

$$T = \frac{aRs}{(R + bs)^2} \tag{94}$$

where a and b are constants. This equation expresses T as a function of $s(n - n')$, and the curves of Fig. 300 are plotted from this equation, using an increasing series of values for R.

Differentiating equation (94) and placing the differential coefficient $\frac{dT}{ds}$ equal to zero we get the condition for maximum torque, namely, $s = Rb$, which shows that maximum torque occurs at greater and greater slip s as R is made larger and larger, and it also shows that R must have a certain value to give the maximum torque at zero speed $(s = n)$.

This matter is taken up again in Chapter XIX.

210. The friction clutch analogy of the polyphase induction motor.—Let us assume for the sake of simplicity that no bearing or air friction acts on either the driving element A or the driven element B of a friction clutch, and let us suppose that the driving element A is running steadily at speed n, exerting a steady torque T on the driven element B, and turning B steadily at speed n'. Under these assumed and supposed conditions the only torque opposing the motion of A would be the *reaction* of torque T, and an outside torque equal to T would therefore have to act on A to maintain its motion. Under these conditions the power delivered to A to drive it is $2\pi nT$;* the mechanical power delivered to B is $2\pi n'T$, and, under the assumptions as to absence of bearing and air friction, all of this power is delivered by B to the outside machine that is being driven by the clutch; and the difference $2\pi nT - 2\pi n'T$

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* See footnote on page 195.

is lost as heat in the clutch because of the slip between A and B. Considering the assumed absence of bearing and air friction, the efficiency of the clutch is $2\pi n'T/2\pi nT$ or n'/n.

Let us assume for the sake of simplicity that there is no resistance loss in the stator windings nor any eddy current or hysteresis loss in the stator iron; let us also assume that the rotor runs without bearing or air friction* (we are not assuming however that the resistance of the rotor windings is zero); and let us suppose that stator magnetism rotates at speed n exerting a steady torque T on the rotor and driving the rotor steadily at speed n'. Under these assumed and supposed conditions $2\pi nT$ is the total power delivered to the stator (from the supply mains of course), $2\pi n'T$ is the mechanical power delivered to the rotor and delivered by the rotor to the outside machine that is being driven by the motor, and the difference $2\pi nT - 2\pi n'T$ is lost on account of the " slip " $n - n'$; this lost power appears as the RI^2 loss in the rotor windings. Considering the assumed absence of rotor friction and of stator losses the efficiency of the motor is $2\pi n'T/2\pi nT$ or n'/n.

211. Speed control of the induction motor.—The speed n of the stator magnetism of an induction motor is fixed by the frequency f of the alternating-current supply and by the arrangement of the stator windings (number of polar regions on stator). Therefore the zero-load speed of an induction motor, which is very nearly equal to the speed of the stator magnetism, is invariable, unless the frequency of supply or the arrangement of the stator windings is changed.

On the other hand, the rotor speed n' of an induction motor *with given load* may have any value between zero and normal full speed if the rotor resistance is increased to the proper value.

There are therefore two feasible methods for controlling the speed of an induction motor which is supplied with alternating currents at a given frequency, namely, (*a*) to arrange the stator windings so that they may be changed quickly from, say, a four-pole winding to an eight-pole winding by means of conveniently arranged switches, and (*b*) to bring out the rotor windings to collector rings, as described in Art. 207, and short-circuit the rotor windings through external adjustable resistances.

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* Eddy current and hysteresis losses in rotor core are extremely small and actually negligible.

The difficulty with the first method of speed control is that the arrangement of the stator windings becomes very complicated if more than two speeds are provided for; and the difficulties with the second method of speed control are (*a*) that the motor is very inefficient at low speeds, and (*b*) that the speed varies greatly with changes of load when the rotor resistance has been adjusted so as to give a low speed with a given load.

212. The starting of the polyphase induction motor.—(*a*) *Small induction motors* with squirrel-cage rotors are nearly always started by connecting their stator windings directly to the polyphase supply mains.

(*b*) *Large induction motors* with wire wound rotors are usually started by inserting resistance in the rotor windings, as explained in Art. 209. The advantages of using a high resistance in the rotor windings at starting are (*a*) that the starting torque is thereby increased and (*b*) the stator currents at starting are reduced in value.

(*c*) *The starting compensator* consists of two or three autotransformers (two for two-phase supply and three for three-phase supply) with switches arranged for conveniently supplying the motor with low voltage polyphase currents at starting, which voltage is increased in steps as the motor speeds up, until finally the full supply-voltage is connected to the motor. The objection to the use of the starting compensator is that the starting torque is very small when the voltage is small; indeed, the starting compensator can scarcely be used in the case of a motor which is to be started under load. The advantage of the starting compensator is that the motor takes very much smaller currents from the supply mains at starting than it would if connected directly to the supply mains.

The starting compensator is nearly always used when it is desired to start a polyphase alternator or a polyphase synchronous converter by inherent induction motor action, as explained in Art. 182.

213. Rating of the induction motor.—An induction motor is always rated to run at a specified speed and deliver a specified amount of power when supplied from a specified number of phases of alternating current supply at a specified frequency and at a specified voltage per phase. Thus one would speak, say, of an 1800-revolutions-per-minute, 1-horse-power, single-phase, 60-cycle, 110-volt induction motor; or of a 900-revolutions-per-minute, 10-horse-power, two-phase, 60-cycle, 110-volt induction motor. Single-phase motors are almost always used for powers less than one or two or three horse-power, and polyphase induction motors are nearly always used for medium amounts of power. For large amounts of power the synchronous motor is nearly always used when the service is steady; but the induction motor is used if frequent starting and stopping is necessary; indeed the induction motor must be used if moderately large starting torques are required or if steady running below full speed is desired.

The rating of an induction motor should always include the value of the starting torque, and this is usually given in per cent. of normal full-load torque. Thus a given induction might with the rotor-resistance scheme of starting be rated to give say 1.50 per cent. of normal full-load torque at starting, and another induction motor arranged to be started with a starting compensator might give only, say, 25 per cent, of normal full-load torque at starting.

Any polyphase induction motor will, after being started, continue to run and carry load as a single-phase induction motor if all supply phases but one are disconnected; but the load capacity of the motor is considerably reduced thereby.

214. The induction motor as a frequency changer.—Consider a wire-wound rotor of an induction motor with three or four collector rings. The rotor delivers three-phase or two-phase currents to the resistances which are connected to the collector rings, and the frequency of these rotor currents is $\frac{n - n'}{n} \times f$, where f is the frequency of the alternating currents supplied to the stator windings, n is the speed of the stator magnetism, and n' is the rotor speed. This is evident when we consider that the stator magnetism in sweeping past the stator conductors at speed n induces electromotive forces of frequency f in these conductors and the stator magnetism in sweeping past the rotor conductors at speed $n - n'$ induces electromotive forces in these conductors at a correspondingly lower frequency. By driving the rotor backwards the difference $n - n'$ becomes $n + n'$, numerically, and it is larger than n. For example let the rotor be driven backwards at speed $n' = n$, then the difference $n - n'$ (paying proper attention to fact that n' is negative) is $2n$, and the frequency of the rotor currents becomes $\frac{2n}{n} \times f$ or $2f$. Thus 25-cycle polyphase currents are delivered to the stator windings of a 25-cycle induction motor, the rotor of the machine is driven backwards by an auxiliary motor at a speed $n' = 1.4n$ so that $(n + n')/n = 2.4$. Then the backward driven rotor will deliver 60-cycle currents.

Let a be the total power output of the rotor in the above example, let b be the power required to drive the rotor backwards at the specified speed $(1.4n)$, and let c be the power delivered to the stator windings. A careful consideration of the relations explained in Art. 210 will show that $b/c = n'/n$ approximately; and, ignoring losses a is of course equal to $b + c$.

A polyphase induction motor used in this way is called a *frequency changer.*

215. The induction generator.—The stator of an induction motor is connected to the polyphase supply mains and the motor started in the regular way; then an outside source of power is arranged to drive the rotor forwards so that the rotor speed n' may exceed the speed n of the stator magnetism. Under these conditions the stator windings deliver power back to the alternating-current supply mains, and an induction motor used in this way is called an *induction generator*. An induction generator can be operated only when the frequency of the alternating-current supply voltages is fixed by an ordinary alternating-current generator (or it might be a synchronous motor) connected to the mains.*

The argument of Art. 210 may be used here. Let T be the torque action between rotor and stator when rotor is driven so that n' is greater than n. Then $2\pi n'T$ is the power required to drive rotor (friction neglected), $2\pi nT$ is the power delivered to stator and thence to the supply mains (stator losses neglected), and $2\pi n'T - 2\pi nT$ is lost in heating the rotor windings.

216. The single-phase induction motor.—As stated above, a

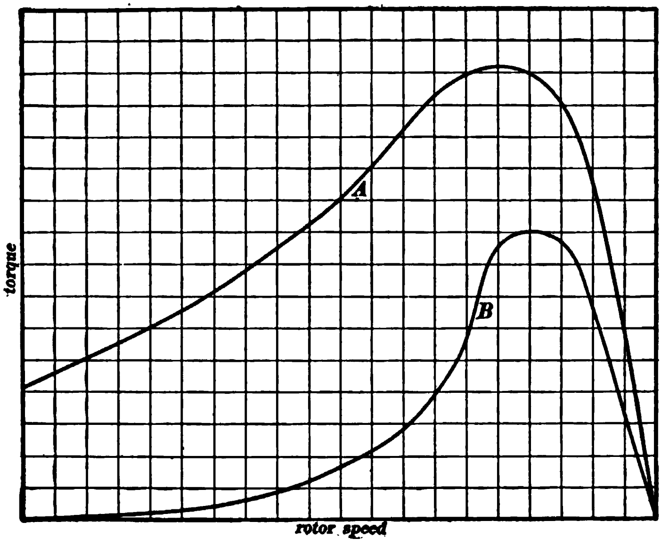

Fig. 302.

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* To be more exact, the induction generator requires a certain phase difference between its terminal voltage and its current output (each phase), and amount of this phase difference varies with the current output.

polyphase induction motor, once in operation, continues to run and carry a moderate load when all of the stator windings (phases) but one are disconnected from the supply mains. When so operated the induction motor is called a *single-phase induction motor*. The curve A, Fig. 302, shows the relation between torque and speed of a two-phase induction motor with both phases connected to two-phase supply mains, and curve B shows the relation between torque and speed of the same motor with only one phase of the stator winding connected to one phase of the supply mains. This figure shows that when driven by single-phase current (at normal voltage), the motor runs at a slightly lower speed to develop a given torque, it shows that the maximum or break-down torque occurs at a higher speed and is less in value than the break-down torque of the same motor when driven by two-phase currents, and it shows that the single-phase induction motor develops zero torque at stand-still. In fact, the single-phase motor must be started either as a polyphase induction motor or by other means as described in the next article.

Action of the single-phase induction motor. Figure 303*a* shows the essential features of a two-pole single-phase induction motor. The stator winding SS has, say, $2N$ conductors or N turns, and we will think of the rods of the squirrel-cage rotor as *at each instant* equivalent to two windings, namely, the winding A (of $2N$ conductors or N turns) with its axis parallel to the axis of the stator winding, and the winding B (of $2N$ conductors or N turns) with its axis at right angles to the axis of stator winding as shown in Fig. 303*b*.

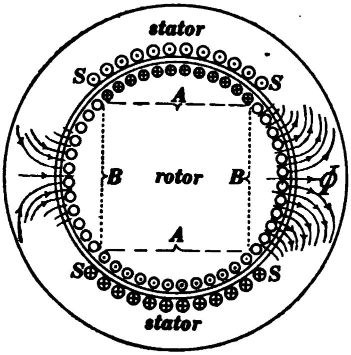

Fig. 303*a*.

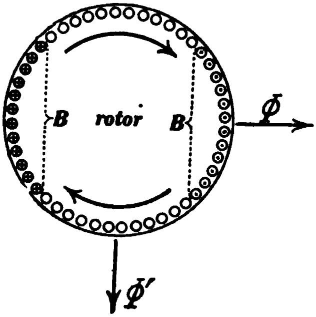

Fig. 303*b*.

For the sake of simplicity we will assume the stator winding to have no resistance, we will assume that *all* of the flux Φ encircles all of the stator conductors and all of the rotor rods of the A group, and we will assume the magnetic reluctance of iron and air gaps to be negligibly small.

The stator winding S and the rotor winding A together constitute an ordinary transformer with a pulsating flux Φ which induces equal electromotive forces in S and A; but the electromotive force induced in S must balance the supply voltage E so that it must be equal and opposite thereto. Therefore the electromotive force induced in A by the pulsating flux Φ is equal to E; *however a very moderate current only is produced in the short-circuited winding* A for the following reason: The motion of the rotor causes an induced electromotive force in winding B as it cuts the flux Φ; the *very small current*[*] thus set up in winding B produces a pulsating flux Φ' (which is omitted from Fig. 303*a* to avoid confusion); the motion of the rotor causes an induced electromotive force in winding A *as it cuts the flux* Φ'; and this electromotive force opposes the electromotive force which is induced in A *by the pulsations of the flux* Φ. Indeed at synchronous speed (which in a two-pole motor is equal to the frequency of the alternating current supply) the electromotive forces thus induced in winding A exactly balance each other, and no current is produced in this winding.

Below synchronous speed, however, the electromotive force induced in A by the cutting of Φ' is less than the electromotive force induced in A by the pulsations of Φ so that current flows in the A winding, *and because of this current the rotor rods* A *are pushed sidewise by the flux* Φ' *and the torque thus developed drives the rotor.* The pulsating flux Φ is called the "transformer field" and the flux Φ', which, of course, also pulsates, is called the "motor field" because it produces on the rotor rods the side forces which drive the motor.

Suppose the rotor is running at speed n'. Then, since the flux Φ is the same at all speeds, the electromotive force induced in the B winding by the cutting of Φ is n'/n of its value at synchronous speed n; the flux Φ' is reduced to n'/n of its value at synchronous speed;[†] and this *reduced* value of Φ' is cut at *reduced* speed by the A winding so that the electromotive force induced in the A winding is $\frac{n'}{n} \times \frac{n'}{n}$ of its value E at synchronous speed. Therefore the net electromotive force in the A winding is $E - \left(\frac{n'}{n}\right)^2 E$, and the current in the A winding is $I = \left[E - \left(\frac{n'}{n}\right)^2 E\right] + R$ where R is the resistance of the A winding. But the torque is proportional to $\Phi' I$, or to $\frac{n'}{n} \times \Phi I$, or to $E\Phi\left[1 - \left(\frac{n'}{n}\right)^2\right]\frac{n'}{n} \div R$, or, since E and Φ are constant, we have

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[*] Indefinitely small if magnetic reluctance is negligible.

[†] An argument of this kind would be much easier to follow if every statement would be accepted not as self-evident but as something which in the writer's opinion may be easily thought out by the reader. Let the reader always think vividly of physical things and physical actions. The pulsating flux Φ' induces in winding B an electromotive force which balances the electromotive force produced in winding B by the cutting of flux Φ, magnetic reluctance being negligible. This is true at all rotor speeds.

$$T = k\left[\frac{n'}{n} - \left(\frac{n'}{n}\right)^2\right] \div R$$

This is the equation to the curve B in Fig. 302. This torque-speed relation is altered somewhat by magnetic leakage which is here assumed to be zero.

217. The two-phase induction motor as a phase-splitter.—When a two-phase induction motor is driven by one phase A, the idle stator winding B has induced in it an electromotive force which is in quadrature with (90° ahead of or 90° behind) the single-phase supply voltage, and a very considerable current (and power) may be taken from winding B, thus *the single-phase supply mains and stator winding B together constitute a two-phase supply.* The getting of a two-phase supply from single-phase mains is called *phase splitting*, and any arrangement therefor is called a *phase splitter.*

218. Starting of the single-phase induction motor.—Two schemes are in general use for starting single-phase induction motors, namely, *split-phase starting* and *repulsion motor starting.* The former is here described and the latter is described in Art. 225.

Split-phase starting.—The single-phase motor may have a two-phase stator winding and be started as a two-phase motor. The auxiliary stator winding (phase No. 2) is usually made of finer wire than the main stator winding (phase No. 1) because its use is only momentary.

In the operation of the large locomotive induction motors of the Norfolk and Western Railway, a two-phase induction motor is installed in the locomotive cab and run continuously from the single-phase trolley supply as a phase splitter for starting and for low-speed operation of the induction motors which drive the locomotive.

The numerous and widely scattered small single-phase induction motors in a city employ split-phase starting only when a very small starting torque is required; the necessarily cheap phase splitting device is very crude. The two stator windings are connected in parallel to the single-phase supply mains with considerable resistance in circuit with the auxiliary stator winding (phase No. 2). The currents in the two stator windings are thus slightly out of phase with each other and a slight starting torque is produced.

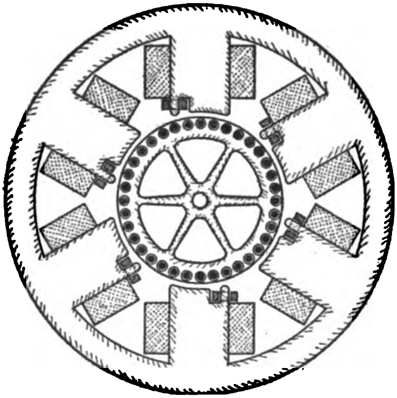

Fig. 304.

In very small single-phase fan motors the split-phase effect is sometimes produced by *shading coils*. Thus Fig. 304 shows a single-phase induction motor with a squirrel-cage rotor, and a small part of each pole face is surrounded by a short-circuited coil called a *shading coil*.

Repulsion motor starting.—Whenever considerable starting torque is required a single-phase induction motor is arranged to start as a repulsion motor, as explained in Art. 225.

PROBLEMS.

207. What are the zero-load speeds of 2-pole, 4-pole, 6-pole and 8-pole polyphase induction motors (*a*) For 60 cycles per second, and (*b*) For 25 cycles per second?

208. The synchronous speed of an induction motor is 900 revolutions per minute. The motor developes its maximum torque at 740 revolutions per minute when the rotor resistance is R. Find the value of the rotor resistance to give maximum torque at starting. Ans. $5.625 \times R$.

209. An 8-pole, 60-cycle polyphase induction motor runs at a speed of 875 revolutions per minute and developes a certain torque T. Under these conditions the rotor resistance is R. Find the value of rotor resistance which would permit the motor to develope the same torque at a speed of 500 revolutions per minute. Ans. $16R$.

Note. In using equation (94) for the solution of this problem the answer comes out (independently of b) as one of the roots of a quadratic equation in which the ratio of the two resistances is the quantity which is to be evaluated. The result may be arrived at directly by considering that the slip is increased 16 times so that *if the rotor flux* Φ' *is the same* 16 times as much electromotive force would be induced in the rotor windings and therefore the rotor current would be the same if the resistance is 16 times as great. But with same rotor current the assumption of same value of Φ' is correct; and therefore with same I and same Φ' the torque will be the same.

210. A 4-pole 60-cycle 2-phase 220-volt induction motor takes 260 watts from the two-phase supply mains when it is running without load, this power being measured by a wattmeter. The motor takes 5420 watts from the supply mains and runs at a speed of 1692 revolutions per minute at full load. The resistance of each stator winding is 0.3 ohm, the zero-load current in each phase is 1.15 amperes, and the full-load current in each phase is 14.9 amperes. Find (*a*) Power factor at zero load, (*b*) Power factor at full load, (*c*) Total power losses at full load, (*d*) Power output at full load, and (*e*) Efficiency at full load. Ans. (*a*) 0.514; (*b*) 0.827; (*c*) 701.7 watts; (*d*) 4718.3 watts or 6.32 horse-power; (*e*) 87 per cent.

Note. The RI^2 losses in stator at zero load should be subtracted from zero-load intake to get core and friction losses; these losses are nearly independent of load. The rotor RI^2 losses at full load are s/n of the total power transferred to the rotor according to Art. 210. A portion, only, of core and friction loss is transferred to rotor—let this portion be estimated at one half, therefore $\{5420 -$ stator RI^2 loss $-$ half of core and friction loss$\}$ is transferred to the rotor and from this we can calculate rotor RI^2 loss at full load.

211. A polyphase induction motor running at given slip developes a certain amount of torque T. What amount of torque would be developed at same slip if the supply voltages were halved, frequency remaining unchanged. Ans. $T/4$.

Note. Using the notation of Art. 209 the argument is as follows: With supply voltage halved Φ would be halved according to Art. 195. Let us assume that Φ' would also be halved, then voltages induced in rotor would be halved (speed remains the same) and rotor current would be halved. But with Φ reduced to half the magnetizing action of stator winding is halved and the halved rotor current causes the same fraction of Φ to leak around the rotor as before. Therefore the assumption that Φ' is halved is justified. Therefore Φ' and I being both halved

the torque will be 1/4 as great inasmuch as the torque is proportional to the product $\Phi' I$.

212. A 220-volt two-phase 60-cycle induction motor with squirrel-cage rotor has a starting torque equal to 25 per cent. of its maximum or break-down torque. What would the starting torque of this motor be if a starting compensator were used to reduce the voltages to 1/4 normal value at starting?

213. The rotor of a 4-pole 25-cycle 2-phase induction motor is driven backwards at a speed of 1200 revolutions per minute, and two-phase currents are taken from the rotor collector rings. What is the frequency of the currents delivered by the rotor, and if 100 kilowatts of power is taken from the rotor slip rings how much power would be required to drive the rotor, ignoring losses. Ans. 65 cycles per second, 61.5 kilowatts.

CHAPTER XVIII.

THE SINGLE-PHASE COMMUTATOR MOTOR.

219. **The single-phase series motor.**—The series dynamo, as described in Arts. 18 and 113, when provided with laminated iron in its field structure and designed so as to reduce sparking at the brushes, can be driven by alternating-current. When so driven it is called the *single-phase series motor*. It has the same speed characteristics* as when driven by direct current, and it is used extensively for electric traction and for operating elevators and cranes where frequent starting under heavy load and more or less steady running at a fraction of full speed are necessary.

The repulsion motor is most easily explained as a modification of the single-phase series motor. It is not used to any extent in practice except in connection with repulsion-motor starting of the single-phase induction motor (see Art. 225).

The following theoretical discussion is of comparatively little direct importance to the operating engineer; it is given here merely as an example of theory.

Behavior of a direct-current motor when the supply voltage is suddenly reversed.—Any direct-current motor will continue to run in an unchanged direction when the supply voltage is reversed, because the reversal of the supply voltage reverses both the armature current and the field current. Therefore any direct-current motor might be operated by a repeatedly reversed supply voltage, that is, by an alternating supply voltage, if it were not for certain effects which are produced at the moment of voltage reversal.

In the case of a shunt motor a reversal of the supply voltage would be followed by a slow dying away of the previously existing field current and a slow growth of a reversed field current, because of the very large inductance of the field winding; whereas the current in the armature would quickly drop to zero and an excessive momentary flow of reversed current would take place. This difference in the behavior of the field current and armature current of a shunt motor when the supply voltage is suddenly reversed would show itself as a very considerable phase difference between field current (field magnetism) and armature current if the attempt were made to operate a shunt motor from alternating-current supply mains, and the motor would develop but little torque. The shunt motor cannot be operated satisfactorily from alternating-current supply mains.

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* See Art. 223 for fuller statement.

In the case of a series motor a reversal of the supply voltage would not be followed by an excessive flow of current, because the armature and field windings are in series; and there can be no phase difference between armature current and field magnetism of a series motor which is operated from alternating-current supply mains.

Eddy currents.—To avoid excessive eddy-current losses and to permit of quick reversal of field magnetism the field iron must be laminated.

Sparking.—The most serious difficulty that is encountered when the attempt is made to operate an ordinary series motor by alternating current is the excessive sparking at the brushes. Whenever a brush touches two commutator bars, an armature coil or section is short-circuited; at the moment of this short-circuit the armature coil surrounds a field pole* as shown by the heavy line in Fig. 305, so that the armature coil is related to the field winding as the short-circuited secondary coil of a transformer of which the primary coil is the field winding. Therefore an excessive short-circuit current is produced through the armature coil and through the tip of the brush; and the result is an increased heating of the armature coils, commutator, and brushes, and an excessive sparking at the brush tips as the commutator bars pass from under the brushes.

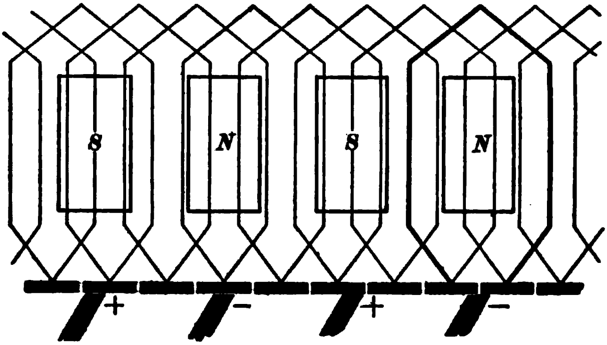

Fig. 305.

Low power factor.—A less serious difficulty in the operation of a series motor by alternating current is that the inductance of the field and armature windings cause the current to lag behind the supply voltage in phase, so that the power factor of the series motor when driven by alternating current is always considerably less than unity.

The single-phase series motor.—The ordinary series motor when designed so as to operate satisfactorily with alternating current is called the ***single-phase commutator motor***, or the ***single-phase series motor***.

220. The clock diagram of the single-phase series motor.—Let the line *OI*,

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* This particular statement applies to the lap drum winding, but the effect referred to exists in every variety of armature winding.

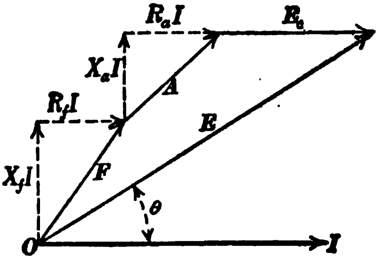

Fig. 306.

Fig. 306, represent the current flowing through the field winding and armature of a series motor, let X_f be the reactance of the field winding, X_a the reactance of the armature winding, R_f the resistance of the field winding, and R_a the resistance of the armature winding. The components of the electromotive force F which is required to force the current I through the field winding are X_fI and R_fI, as shown in Fig. 306, and the components of the electromotive force A which is required to overcome the reactance and resistance of the armature winding are X_aI and R_aI as shown.

The effect of the current I in flowing through the field winding is to produce an alternating flux Φ through the armature. This flux is in phase with the current and it induces in the armature windings an electromotive force E_c which is equal to $\Phi Z'n$.* Since Φ is an alternating flux which is in phase with the field current I, it is evident that E_c (equals $\Phi Z'n$) is an alternating electromotive force of the same frequency and in phase with I. Therefore since E_c is a counter electromotive force, the portion of the supply voltage which overcomes it is equal to it and is represented by the line E_c in Fig. 306, and the total supply voltage is represented by the line OE which is equal to the vector sum of F, A and E_c.

The power lost in the field winding is R_fI^2, the power lost in the armature is R_aI^2, and the power which is used to force the current against the counter electromotive force E_c, namely, E_cI, appears as mechanical power.

Design of single-phase series motor to give large power factor.—It is evident from Fig. 306 that a large power factor (small value of the angle θ) depends upon a large value of E_c in comparison with X_fI and X_aI; of course R_fI and R_aI should both be as small as possible because they represent energy losses in the windings.

(*a*) In the first place the reactance X_a of the armature winding may be reduced nearly to zero by the use of the neutralizing winding as explained in the following article. A neutralizing winding is therefore an important feature of the single-phase series motor.

(*b*) In the second place, it is to be noted that the voltage X_fI which is lost in the field winding is proportional to the product of the number of field turns and the maximum value of the alternating field flux Φ, exactly as in the case of the primary coil of a transformer,† and we may assume that we have a motor in which a specified maximum flux is to be produced, then to make the reactance voltage X_fI a minimum, the machine must be designed so that the desired flux may be produced by the least possible number of field turns. That is to say, the reluctance of the magnetic circuit must be small, or in other words the gap space between the pole faces and the armature core must be short and the iron

* See Art. 115.

† See Art. 195.

of the field magnet and armature core must be operated at a low degree of magnetic saturation. Of course there is no clearly defined value of magnetic saturation which gives the best results, and all that can be said in regard to this second point is that low degree of magnetic saturation and short air gaps are important features of the single-phase series motor.

(*c*) In the third place the value of E_c at a given speed may be made fairly large, even though the value of Φ be rather small, by increasing the number of armature conductors. Therefore the use of a large amount of armature copper is an important feature of the single-phase series motor.

(*d*) In the fourth place, suppose that we have a single-phase series motor running at given speed with *given (effective) ammeter value of current* and given value (effective or maximum) of field flux. Under these conditions, the counter electromotive force is definite in value (effective), the mechanical power E_cI is definite, and a reduction of the frequency of the supply voltage would involve a proportional reduction of the reactance voltages X_fI and X_aI without affecting the value of E_c; that is, a reduction of frequency would increase the power factor of the motor. Therefore it is important to use a low frequency alternating current for a single-phase series motor. A low frequency is also desirable in order to reduce the tendency to spark at the brushes.

221. The neutralizing of armature inductance.—A winding of wire on an iron core has considerable inductance, that is, a current in the winding magnetizes the iron core and causes a considerable amount of magnetic flux through the core. The inductance of such a winding may be annulled by anything which counteracts the magnetizing action of the winding on the iron core. Thus, Fig. 307 shows a layer of wire *WW* wound on an iron rod, the magnetic flux due to the current in the winding is indicated by fine lines; and Fig. 308 shows a second layer of wire *aa* wound over *WW* and so connected in series with *WW* that the current flows in opposite directions around the rod in *aa* and in *WW*, as indicated by the dots and crosses. The magnetizing action of the winding *WW* is neutralized by the opposite magnetizing action of the auxiliary winding *aa*, the only magnetic flux produced is the very small amount of flux between *W* and *a*, and therefore the inductance of *WW* and *aa* combined is nearly zero.

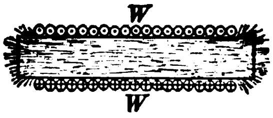

Fig. 307.

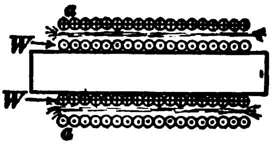

Fig. 308.

If an alternating current is made to flow through the winding *WW*, Fig. 308, and if the auxiliary winding *aa* is short-circuited, then the magnetizing action of the current in *WW* will be very nearly neutralized by the current which is induced in the short-circuited winding *aa*; in this case the number of turns in the auxiliary coil *aa* need not be the same as the number of turns of wire in the winding *WW*.

The auxiliary winding *aa*, Fig. 308, which is used to annul the inductance of the given winding WW is called a ***neutralizing*** or ***compensating winding.*** When the auxiliary winding is connected in series with the given winding, we have what is called ***forced neutralization*** or ***forced compensation,*** and when the auxiliary winding is short-circuited we have what is called ***inductive*** or ***transformer neutralization.*** Forced compensation can be used both for direct current and for alternating current, whereas transformer compensation can be used only for alternating current.

The magnetizing action of the armature current of a direct-current motor or of a single-phase series motor produces magnetic flux, as shown by the dotted lines in Fig. 309,* the inductance of the armature winding is due to this flux, and this inductance may be annulled by a neutralizing winding embedded in the pole faces, as shown in Fig. 310. This neutralizing winding is connected so that the current flows in one direction in all of the conductors which lie under north poles and in the opposite direction in all of the conductors which lie under south poles as indicated by the dots and crosses in Fig. 310. In the single-phase series motor this neutralizing winding may be connected in series with the armature (forced compensation), or it may be short-circuited (transformer compensation).

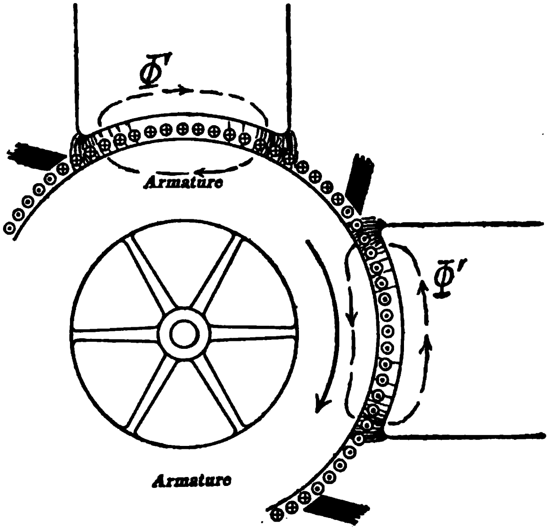

Fig. 309.

222. Prevention of sparking in the single-phase series motor.—A number of undesirable effects are included under the general term of sparking, as follows: (*a*) The effects that are present in the direct-current dynamo† and which are due to the necessity of a sudden reversal of the current in a given armature section as the terminal bars (commutator bars) of the section pass under a brush; and

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* See Arts. 133 and 134.

† See Arts. 133 to 136.

(*b*) the effects of the excessive current produced in each short-circuited section by the electromotive force which is caused by the pulsations of the field flux. The effects (*a*) need not be considered because they are less serious than effects (*b*), and because anything which obviates the great difficulties due to effects (*b*) will likewise obviate the lesser difficulties due to effects (*a*).

The effects of excessive short-circuit currents due to pulsations of the field flux are, to heat the armature winding, to heat the commutator, to heat the brushes, and to roughen the commutator by sparking, and these short-circuit currents represent a considerable loss of power. The cure for these undesirable effects is, of course, to prevent excessive short-circuit currents and to shorten their duration as much as possible.

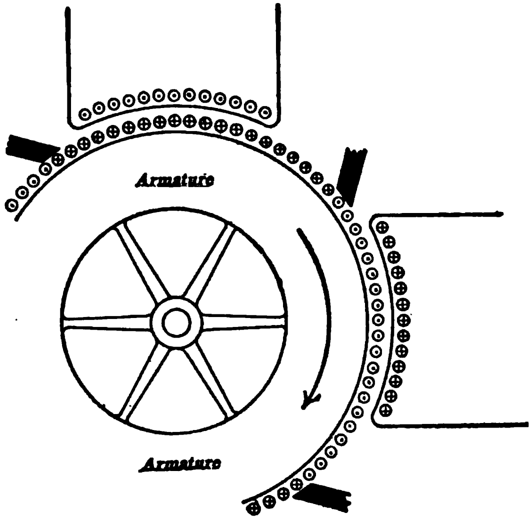

Fig. 310.

The electromotive force induced in a short-circuited armature section by the pulsating field flux is proportional to the number of turns of wire in the section, proportional to the maximum value reached by the pulsating field flux, and proportional to the frequency of the pulsating flux. Therefore the short-circuited current may be reduced by reducing the number of turns of wire per armature section (the resistance of the short circuit not being reduced in proportion to the turns because of the relatively large resistance at the brush contacts), by reducing the maximum value of the pulsating flux, and by reducing the frequency. It is essential, therefore, in the design of the single-phase series motor to provide for few turns of wire per armature section (many commutator bars), to provide for small field flux per pole (many field poles), and to use alternating current of low frequency.

The duration of a short circuit of an armature section by a brush is proportional to the thickness of the brush, and therefore it is desirable to use thin brushes.*

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* This statement refers to the simplex armature winding.

Special devices for preventing sparking.—The foregoing discussion refers to those general features of design which tend to reduce short-circuit currents in the armature sections of single-phase series motors. It does not seem to be possible, however, to produce a single-phase series motor which will operate satisfactorily without special devices for preventing excessive short-circuit currents. The simplest device for this purpose is the insertion of a moderate amount of resistance in each commutator lead, and a number of special inductive devices have been proposed, of which, perhaps, the simplest is the balanced choke-coil arrangement of Professor S. S. Seyfert.

The resistance leads.—The coils *bcdef*, Fig. 311, represent the sections of an armature winding, and *rrrr* represent resistances inserted in the commutator leads. The figure shows the armature section *d* short-circuited by a brush, and the heavy arrow represents the electromotive force induced in the section *d* by the pulsating field flux. The short-circuit current must flow through two of the resistances *r* in series, as shown by the short arrows, whereas the useful current which enters or leaves the armature through the brush must flow through two of the resistances *r* in parallel. When the brush touches but one commutator bar there is no short circuit, and the whole of the useful current must flow through one of the resistances *r*. The resistances *r* are entirely idle except at the moments when the bars to which they are connected pass under a brush.

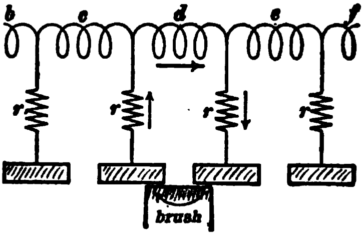

Fig. 311.

Resistance leads are objectionable because they involve a loss of power, and because of the difficulty of finding room for them in a compact armature structure. While a motor is in actual operation the resistance leads come into service in rapid succession, and the heat is distributed among all of the leads; at starting, however, there is a possibility of the motor being stuck fast, so that a given resistance lead may be in circuit for an indefinite time. In view of this possibility it is necessary to make the resistance leads of large current-carrying capacity, and to place the leads so that they can be easily repaired in case of their being burned out. The resistance leads are usually made of heavy strips of German silver thoroughly insulated and placed in the bottoms of the armature slots.

223. Characteristics of the single-phase series motor.—The single-phase series motor is like the direct-current series motor in its speed and torque characteristics. The series motor requires a very low voltage at starting, and the voltage across the motor terminals must be increased as the speed increases. When direct current is used, this variation of motor voltage can be accomplished only by the use of rheostat control, as explained in Art. 130, but when alternating current is used, a number of taps can be brought out from the secondary coil of the step-down transformer so that the low voltage required at starting can be taken directly from a few secondary turns, and the voltage can be increased at will by bringing a larger and larger number of secondary turns into action as the speed increases. Rheostat control is therefore not necessary when alternating current is used and the loss of power in rheostats is obviated. Furthermore, the series parallel system of direct-current series motor control, the object of which is to reduce the necessary range of rheostat control of voltage, is unnecessary when alternating current is used.

224. The repulsion motor.—The repulsion motor consists of a primary or stator member which is identical to the stator of the single-phase induction motor, and a secondary or rotor member which is like an ordinary direct-current armature with a commutator and brushes. The primary member is supplied with single-phase alternating current, and the secondary member is short-circuited by connecting its brushes directly together. The essential features of the repulsion motor are shown in Fig. 312, which represents a two-pole model. The stator winding produces

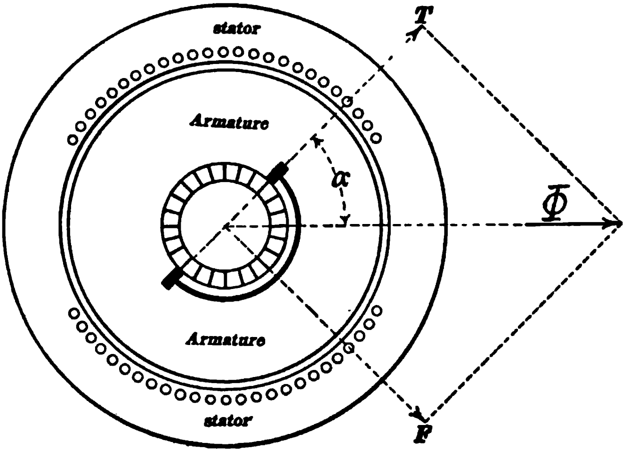

Fig. 312.

through the armature a pulsating flux in the direction of the arrow Φ, and, in order to describe the action of the motor, this flux may be resolved into two component parts, one of which T is parallel to the line of the brushes, and the other of which F is perpendicular to T. The stator winding may also be considered in two parts, namely, the part t which produces the flux T and the part f which produces the flux F, as indicated in Fig. 313.

The repulsion motor may be shown to be fundamentally equivalent to the simple series motor in its action and in its torque-speed characteristics as follows: The stator winding t, Fig. 313, produces the pulsating flux T and acts like the primary of a transformer of which the secondary is the short-circuited armature winding. Let us assume that this transformer action is ideal (zero reluctance for flux T) and let us assume that the ratio of transformation is 1 : 1. Then the armature current is identical in value to and in phase[*] with the current in the coil t, *just as if the armature were included in the primary circuit.*

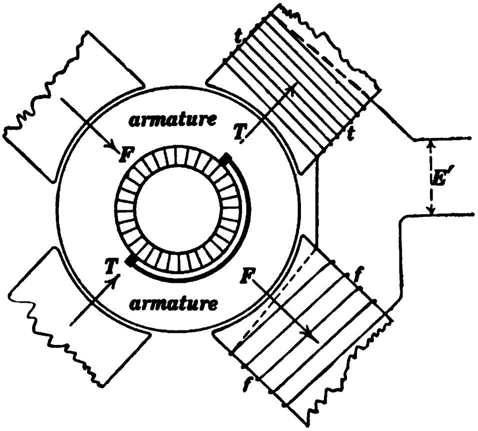

Fig. 313.

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[*] The secondary current of a transformer is usually thought of as exactly opposite in phase to the primary current, but whether it be considered as *opposite to* or *in phase with* the primary current depends on a mere convention as to the choice of the direction in the secondary coil which is to be considered as the positive direction.

The stator winding f produces the pulsating flux F. The armature is on open circuit with respect to the electromotive forces induced by the pulsations of this flux, but the electromotive forces produced by the *cutting* of F by the rotating armature act through or around the short circuit formed by the brushes, and these electromotive forces are reproduced by transformer action as *back electromotive forces*, in the stator winding t, *just as if the armature itself were included in the primary circuit.*

The simple series motor is different from the repulsion motor in that the transformer flux T does not exist in the simple series motor, and this transformer flux has an interesting effect on the rotating armature conductors as follows: The armature sections which are short-circuited under the brushes have electromotive forces induced in them by the pulsation of the field flux F exactly as in the simple series motor but these short-circuited sections are *cutting* the transformer flux T at the time of their short circuit and the electromotive force due to this cutting is opposed to the electromotive force produced by the pulsation of the field flux F. In fact, the electromotive forces due to cutting of the transformer flux T are equal and opposite to the electromotive forces due to pulsation of the field flux F at synchronous speed, and therefore the repulsion motor has no great tendency to spark at synchronous speed.

The repulsion motor has not come into practical use to any extent because the series motor is on the whole more satisfactory. However repulsion-motor action is employed extensively for starting the single-phase induction motor.

225. Repulsion-motor starting of the single-phase induction motor.—The single-phase induction motor as arranged for repulsion-motor starting has a rotor exactly like a direct-current-dynamo armature with a commutator and short-circuited brushes as indicated in Fig. 312, and the stator winding is as indicated in Figs. 303*a* and 312. At starting the machine is a repulsion motor, and at nearly full speed a centrifugal device pushes a metal ring over the commutator thus short-circuiting the commutator bars and converting the wire-wound armature into the equivalent of a squirrel-cage rotor. The centrifugal device also usually lifts the brushes off the commutator.

PROBLEMS.

214. A 5-horse-power, 4-pole, 110-volt, 25-cycle, single-phase series motor has a full-load current of 50 amperes. The resistance of the field winding is 0.072 ohm and the combined resistance of armature and compensating windings is 0.1760 ohm. With field winding open and armature standing still, a current of 50 amperes is produced through the armature and compensating windings by 12.88 volts at rated frequency. With the armature and compensating windings open, a current of 50 amperes is produced through the field winding by 48.67 volts at rated frequency. Calculate the following quantities at full load: (*a*) Field reactance, (*b*) Reactance of armature and compensating windings combined, (*c*) Wattless component of supply voltage, (*d*) Power component of supply voltage, (*e*) Power factor of motor at full load. Ans. (*a*) 0.9708 ohm; (*b*) 0.188 ohm; (*c*) 57.94 volts; (*d*) 93.5 volts; (*e*) 0.85.

215. Given a 2-pole, 25-cycle, single-phase series motor of which the full-load current rating is 25 amperes. The resistance of the field winding alone is 0.15 ohm, and the resistance of armature and neutralizing windings together is 0.25 ohm. With the armature stationary the full-load current is produced through the motor by 70 volts at rated frequency. With the field winding open the full-load current is produced through the armature and neutralizing windings by 20.73 volts at rated frequency. What is the total reactance of the motor, what is the reactance of the field winding alone, and what is the reactance of the armature and neutralizing windings together? Ans. (*a*) 2.77 ohms; (*b*) 1.979 ohms; (*c*) 0.791 ohm.

216. The motor of problem 215 is driven by an auxiliary motor at a speed of 1,500 revolutions per minute with its field

Note. In solving this problem assume that core losses are zero.

excited by 20 amperes of direct current, and the steady electromotive force across the armature terminals is observed to be 60 volts.

The machine is to be driven as a single-phase motor from a 25-cycle source at a speed of 1,500 revolutions per minute taking full-load current. Find: (*a*) The terminal voltage that is required, and (*b*) the power factor of the motor. Ans. (*a*) 109.6 volts; (*b*) 77.5 per cent.

Note. In this and the following problems assume that the reluctance of the magnetic circuit of the motor is invariable so that the useful flux may be considered to be at each instant proportional to the field current.

217. The motor of problems 215 and 216 has 240 conductors upon its armature. Find the maximum value of the alternating armature flux when the motor is running as specified in problem 216, and find the number of field turns. Ans. (*a*) 1,770,000 lines, (*b*) 55 turns.

Note. In this problem assume the field flux and armature flux to be equal, or, in other words, assume magnetic leakage to be zero.

The maximum value of the alternating armature flux corresponds to the maximum value of the alternating current in the field winding as explained in the note to problem 216. The reactance voltage X_fI is induced in the field winding by the alternating field flux exactly as in the coil of a transformer so that equation (92) of Art. 195 gives the relation between maximum field flux, field turns, induced voltage and frequency.

218. The motor of problems 215–217 is driven from a 15-cycle supply at a speed of 1,200 revolutions per minute taking 25 amperes of current. What is the terminal voltage of the motor and what is its power factor? Ans. (*a*) terminal voltage 81.4, (*b*) $\cos\theta = 0.86$.

219. The motor of problems 215–217 is driven from a 60-cycle supply at a speed of 1,800 revolutions per minute taking 25 amperes of current. What is the terminal voltage of the motor and what is its power factor? Ans. (*a*) terminal voltage 192.5, (*b*) $\cos\theta = 0.52$.

220. The motor of problems 215–217 has 60 commutator bars, that is, a section of the armature winding consists of 4 conductors (two turns). The motor operates at full-load current at 25 cycles per second frequency. What is the effective value of the voltage which is induced in the sections of the armature winding as they are successively short-circuited by a brush? Ans. 3.93 volts.

221. With the motor operating as specified in problem 220, calculate the resistance required in each resistance lead to give a short-circuit current of 62½ amperes effective; each armature section consists of 36 inches of 102 mil copper wire, and the brush tip introduces a resistance into the short circuit such that the *RI* drop through the brush is 2.0 volts. Ans. 0.014 ohm.

Note. The answer to this problem is here calculated on the assumption that the armature section is non-inductive. This assumption is not exactly true. The tendency of the current in the armature section to produce flux through the main magnetic circuit of the motor does not give to the armature section a reactance which has to be overcome by the electromotive force induced by the actual given flux through the magnetic circuit of the motor. The only thing which does give a reactance to the armature section in this problem is the leakage flux which circles around the bundles of wires in the armature slots.

CHAPTER XIX.

GENERAL THEORY OF THE TRANSFORMER AND OF THE INDUCTION MOTOR.

226. The ideal transformer and the actual transformer.—The needs of the operating engineer are met almost completely by the very simple theory of the transformer in which the effects of coil resistances, core reluctance (magnetizing current) and magnetic leakage are ignored. In this chapter these effects are considered, and the theory of the transformer is developed to the point where it may be applied to the induction motor for the calculation of performance curves.

The effects of core reluctance (magnetizing current), coil resistances and magnetic leakage are small and their mutual influence on each other is extremely small; it is permissible therefore to treat each effect by itself, that is, as if the transformer were ideal except for the single effect under consideration.

227. The simple-circuit equivalent of an ideal transformer and its secondary receiving circuit.—It is a great help in the development of the theory of the transformer to reduce a transformer and its secondary receiving circuit to an equivalent simple circuit as indicated in Figs. 314 and 315. It is easy to show that the simple circuit in Fig. 315 takes from the supply mains the same current and the same power as the primary coil takes in Fig. 314, or, in other words, the simple circuit in Fig. 315 is equivalent to the whole arrangement in Fig. 314.

228. The magnetizing current.*—When the secondary coil of a transformer is open, a certain amount of current M flows through the primary coil and produces the amount of pulsating core flux which is required to induce sufficient back electromotive force in the primary coil to balance or nearly balance the supply voltage. When the secondary coil is connected to a receiving circuit, a certain amount of current I'' flows through the secondary coil and the receiving circuit, and *additional current* I' flows through the primary coil. The magnetizing action of I'' is exactly balanced or annulled by the opposite magnetizing action of I', that is, $N'I' + N''I'' = 0$, and the primary current M produces, as before, the pulsating core flux which is required to induce sufficient back electromotive force in the primary coil to balance or nearly balance the supply voltage. The current M is called the *magnetizing current*, and the current I' is called the *load current*.

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* See Art. 196.

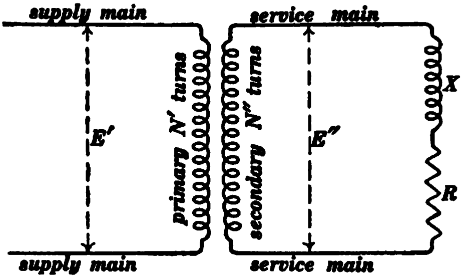

Fig. 314.

Ideal transformer with its secondary receiving circuit of resistance R and reactance X.

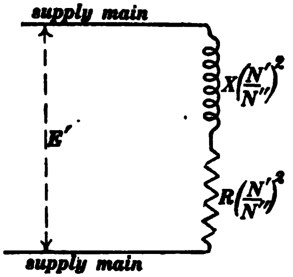
supply main
E'
$X\left(\frac{N'}{N''}\right)^2$
$R\left(\frac{N'}{N''}\right)^2$
supply main

Fig. 315.

Simple circuit equivalent to Fig. 314.

The magnetizing current is not harmonic, but it is treated as if it were harmonic in all transformer calculations into which it enters. Let E' be the measured voltmeter value of the supply voltage, M the measured ammeter value of the magnetizing current, and P_m the power delivered to the transformer when the secondary coil is open as measured by a wattmeter. Then, thinking of M as a harmonic current lagging $m°$ behind* E'

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* The wattmeter does not distinguish between a lagging or a leading current and it is impossible to tell from equation (i) whether m is positive (an angle of lag) or negative (an angle of lead). The magnetizing current does in fact lag behind E'.

in phase, we have

$$P_m = EM \cos m \tag{i}$$

from which the phase angle m may be calculated.

The clock diagram, Fig. 316, shows the relation between I'', I' and M, the magnetizing current M being thought of as harmonic. The secondary current I'' and its phase angle θ depend upon the resistance and reactance of the secondary receiving circuit, the load current I' is directly opposite to I'' and equal to $\left(\frac{N''}{N'}\right) I''$, and the total primary current is represented by the vector sum of M and I' as indicated.

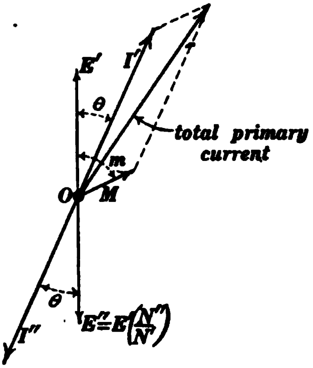

Fig. 316.

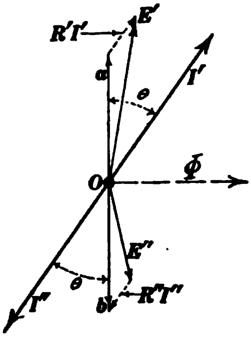

Fig. 319.

Figure 317 shows a circuit of resistance r_1 and reactance x_1 connected in parallel with the primary coil of an ideal transformer so as to make the ideal transformer equivalent to an actual transformer insofar as magnetizing current is concerned; the values of r_1 and x_1 being chosen so that the circuit r_1x_1 may take current M and power P_m from the supply mains.

Figure 318 shows a combination of simple circuits which is equivalent to the whole arrangement in Fig. 317.

229. Effect of coil resistances.—The resistances R' and R'' of the primary and secondary coils of a transformer cause a certain loss of power as explained in Art. 196, but we are here interested more particularly in the influence of coil resistances on the phase relations and values of the primary and secondary

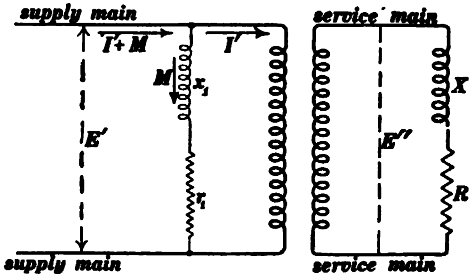

Fig. 317.

Ideal transformer arranged to be equivalent to a transformer taking magnetizing current.

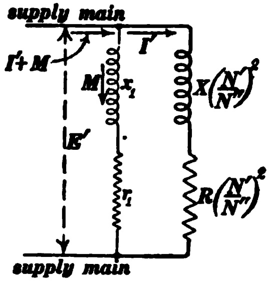
supply main
I'+M
I'
M
x_1
E'
r_1
$X\left(\frac{N'}{N''}\right)^2$
$R\left(\frac{N'}{N''}\right)^2$
supply main

Fig. 318.

Combination of simple circuits which is equivalent to Fig. 317.

voltages. The clock diagram, Fig. 319, shows this effect greatly exaggerated. The line $O\Phi$ represents the harmonically varying flux in the transformer core; the line Oa represents the useful part of E' (the part which balances the electromotive force which is induced in the primary coil by the pulsating core flux); and the line Ob represents the total electromotive force induced in the secondary coil (magnetic leakage assumed to be zero). The ratio Oa/Ob is equal to N'/N''. The line OI'' represents the secondary current, the power factor of the secondary receiving circuit being equal to cos θ, where θ is the angle shown in the figure; and the line OI' represents the primary current (magnetizing current assumed to be zero).

The primary voltage E' is the vector sum of Oa and $R'I'$ as shown, $R'I'$ being, of course, parallel to I'. That is to say, part of E' is used to overcome $R'I'$ and part of E' is used to overcome the back electromotive force as above explained.

The line Ob represents the total electromotive force induced in the secondary coil, as stated; part of Ob is used to overcome $R''I''$ (which is, of course, parallel to OI'') and the remainder is the secondary terminal voltage E'', that is, Ob is the vector sum of E'' and $R''I''$ as shown.

When the receiving circuit has unity power factor (angle θ equal to zero), then $R'I'$ and $R''I''$ are parallel to Oa and Ob respectively. In this case (for given values of I' and I'') the useful part a of the primary voltage is much less than E' in value, and E'' is much less than $b\left(=\frac{N''}{N'}\cdot a\right)$, so that E'' is considerably smaller than its ideal value $\frac{N''}{N'}\cdot E'$.

When, however, the angle θ is nearly 90°, *plus* or *minus*, the useful part a of the primary voltage is very nearly equal to E', and the secondary terminal voltage E'' is very nearly equal to $b\left(=\frac{N''}{N'}\cdot a\right)$, so that the value of E'' is very nearly equal to its ideal value $\frac{N''}{N'}\cdot E'$.

Therefore coil resistances do not appreciably affect the value of secondary terminal voltage when the angle θ in Fig. 319 is nearly 90°.

Figure 320 shows a combination of simple circuits which is equivalent to a transformer with core reluctance (taking magnetizing current) and of which the coil resistances are R' and R'' respectively.

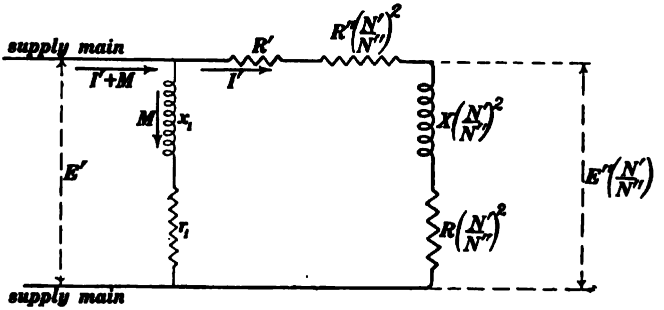

Fig. 320.

Combination of simple circuits which is equivalent to a transformer having core reluctance and coil resistance delivering current to a secondary receiving circuit of resistance R and reactance X.

230. Effects of magnetic leakage.—*Magnetic leakage is equivalent to a certain inductance* P *or reactance* ωP *connected in series with the primary coil of a transformer.* Figure 321 is a sectional view of a transformer, *pp* is the primary coil and *ss* is the secondary coil; and these coils are shown widely separated with the leakage flux *aa* $(=\varphi)$ passing between them.* The total flux through the primary coil may be considered in two parts, namely, (1) The part Φ which passes through both coils and produces ideal transformer action (this flux is represented by the clock-diagram vector $O\Phi$ in Fig. 319), and (2) The part φ which crosses over between the coils as the leakage flux *aa*, and does not pass through the secondary coil.

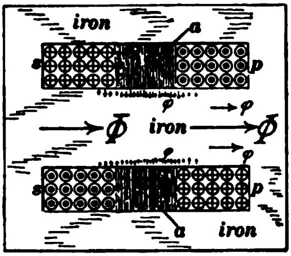

Fig. 321.

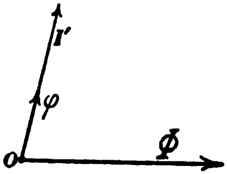

Fig. 322.

The primary and secondary currents i' and i'' work together to produce the leakage flux *aa* as may be seen by considering the dots and crosses which show the directions of i' and i'' at a given instant. Now i' and i'' rise and fall in value together, and therefore the leakage flux *aa* $(=\varphi)$ rises and falls in value with the primary current, or, in other words, the leakage flux is in phase with i' and it is proportional to i'. The two fluxes Φ and φ (which are, of course alternating fluxes) are represented by the clock-diagram vectors Φ and φ in Fig. 322.

The flux φ, being proportional to i', is equivalent to a certain inductance P with the primary current i' flowing through it, and such that $Pi' = \varphi N'$ at each instant as explained in Art. 48.

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* A portion of the leakage flux crosses from iron to iron through the coils them selves, but it is not necessary for the purpose of the present to distinguish between the flux *aa* and that which crosses through the coils.

The short-circuit test. When the secondary coil of a transformer is short-circuited, the primary current I which is produced by primary voltage E is equal to $E/\omega P$, coil resistances being negligible, and therefore the value of ωP may be calculated from the measured values of E and I. Indeed, allowance may be made in this calculation for coil resistances by using the equation

$$I = E\sqrt{R^2 + (\omega P)^2}, \text{ where } R = R' + \left(\frac{N'}{N''}\right)^2 R''.$$

Many writers on alternating current theory represent the effects of magnetic leakage of transformers and induction motors as equivalent to a certain reactance in the primary circuit together with a certain reactance in the secondary circuit; but when the effects of magnetizing current are ignored (and all writers do ignore them because to take these effects into account in the discussion of magnetic leakage leads to complications which do not seem to be justified by the requirements of practice) it is only possible to separate the effects of magnetic leakage into two such reactances arbitrarily.

The clock diagrams of Figs. 323 and 324 show the effects of magnetic leakage (greatly exaggerated). In Fig. 323 the secondary current I'' is behind E'' in phase, and in Fig. 324 the secondary current I'' is ahead of E'' in phase. The clock-

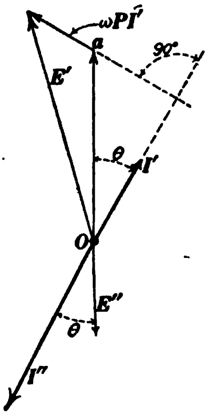

Fig. 323.

$\omega PI'$ is 90° ahead of I'.

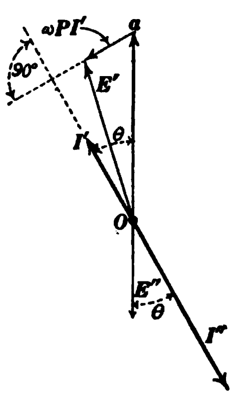

Fig. 324.

$\omega PI'$ is 90° ahead of I'.

diagram vector $\omega PI'$ is in each case that part of the primary voltage which is used to overcome the leakage reactance ωP, and it is 90° ahead of I' in phase according to Arts. 151–153.

In Fig. 323 the useful part a of the primary voltage is less than E' in value, and the effect of magnetic leakage is to make the secondary terminal voltage E'' $\left(=\frac{N''}{N'}\cdot a\right)$ less than its ideal value $\frac{N''}{N'}\cdot E'$.

In Fig. 324 the useful part a of the primary voltage is greater than E' in value, and the effect of magnetic leakage is to make the secondary terminal voltage E'' $\left(=\frac{N''}{N'}\cdot a\right)$ greater than its ideal value $\frac{N''}{N'}\cdot E'$.

When I'' is in phase with E'', the angle θ in Figs. 323 and 324 is zero, the vector $\omega PI'$ is at right angles to Oa, and the useful part a of the primary voltage is sensibly equal in value to E', so that the secondary terminal voltage E'' $\left(=\frac{N''}{N'}\cdot a\right)$ is sensibly equal to its ideal value $\frac{N''}{N'}\cdot E'$. Therefore magnetic leakage does not perceptibly affect the voltage regulation of a transformer when the secondary receiving circuit has unity power factor (angle θ in Figs. 323 and 324 equal to zero).

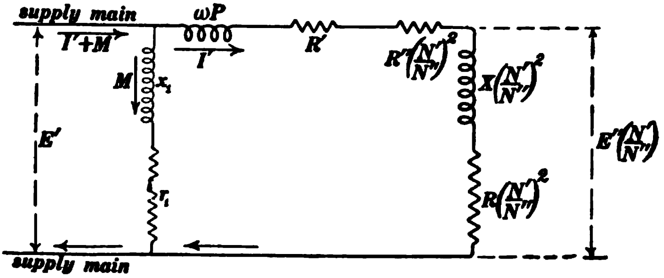

Fig. 325.

Complete simple-circuit equivalent of an actual transformer delivering current to a secondary receiving circuit of resistance R and reactance X.

Figure 325* shows a combination of simple circuits which is equivalent to an actual transformer [having core reluctance (magnetizing current), coil resistance, and magnetic leakage] delivering current to a secondary receiving circuit of resistance R and reactance X.

231. Calculation of transformer regulation.—Given a transformer, N' and N'' being known, R' and R'' being measured, and ωP being determined by "short-circuit" test as described in the previous article, it is required to calculate the secondary terminal voltage when the secondary coil delivers current to a receiving circuit of resistance R and reactance X, primary supply voltage being given. It is evident from Fig. 325 that the secondary terminal voltage is not affected by the circuit r_1x_1 which represents the effect of core reluctance.

Solution.—The zero-load value of secondary terminal voltage, E_0'', is equal to $\frac{N''}{N'}\cdot E'$. To find the full-load value, E_1'', let us suppose that R and X are known. Calculate I' by dividing E' by the impedance of the main circuit of Fig. 325, remembering that the total resistance is $R' + R''(N'/N'')^2 + R(N'/N'')^2$, and that the total reactance is $\omega P + X(N'/N'')^2$. Having thus found the value of I', calculate the voltage across the resistance $R(N'/N'')^2$, and calculate the voltage across the reactance $X(N'/N'')^2$. Then $E_1''(N'/N'')$ is equal to the square root of the sum of the squares of the resistance and reactance voltages thus calculated, whence E_1'' may be found. The percentage regulation of the transformer is then equal to $100(E_0'' - E_1'')/E_1''$.

232. Referring of rotor currents and rotor electromotive forces TO THE STATOR in an induction motor.—Consider a two-pole polyphase induction motor with the phases of its stator winding connected to 60-cycle polyphase supply mains. Then the current in each stator phase is 60-cycle current, and the stator magnetism rotates at a speed of 60 revolutions per second. Suppose the rotor speed to be, say, 57 revolutions per second. Then the speed of the rotor relative to the stator magnetism is three revolutions per second, and the frequency of the electromotive forces induced in rotor conductors is 3 cycles per second, which is also the frequency of the currents in the rotor conductors. *If the action of the induction motor is to be represented by a clock diagram it is necessary to develope a point of view such that the rotor electromotive forces and currents may be thought of as having the same frequency as the supply voltages and currents.*

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* The facts are more accurately represented by connecting the circuit r_1x_1 behind ωP and R', thus to some extent making allowance for the mutual influences of core reluctance, magnetic leakage and coil resistance.

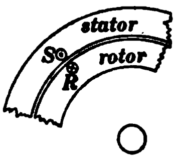

Fig. 326.

The current i' in any *given stator conductor* S, Fig. 326, has a frequency, say, of 60 cycles per second. *Consider he series of values* i'' *of current in* **successive rotor rods** *as they pass under* S. The two sets of current values i' and i'' so defined rise and fall in value together and they are always in opposite directions as indicated by the dot and cross in Fig. 326, that is to say, i' and i'' have exactly the same frequency and they are opposite in phase.

Throughout the following discussion when we speak of the current in a rotor rod, let it be understood that we refer to the series of instantaneous values of current in the successive rotor rods as they pass a given stator conductor, and when we speak of the current in a phase of a wire-wound rotor let it be understood that we refer to the series of instantaneous* values of current in the successive bands of rotor conductors as they pass a given band of stator conductors (the stator winding and the rotor winding are both like Fig. 296).

Also in speaking of the electromotive force induced in a rotor rod or in a phase of a wire-wound rotor, let it be understood that we refer to the series of values of electromotive force induced in the successive rotor rods or in the successive bands of rotor conductors as they pass a given stator conductor or band of stator conductors.

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* This statement, in the very simple form in which it is made, involves the assumption that stator and rotor are both wound for many phases instead of only 2 or 3.

We will represent by I'' the square-root-of-average-square of the current values i'' as above defined, and this value I'' is also equal to the ammeter value of the current in a *particular* rotor rod or rotor phase, and therefore the RI^2 loss in a particular rotor phase is equal to $R''I''^2$, where R'' is the resistance of a rotor winding or phase and I'' is the above mentioned current value.

233. Action of the polyphase induction motor.—*The effect of core reluctance* in an induction motor is to cause the flow of a certain magnetizing current M in each phase of the stator winding exactly as in the transformer. This magnetizing current is not harmonic but it is usually treated as if it were. Its value may be measured by placing an ammeter in each phase of the stator winding and driving the rotor at synchronous speed; at the same time each supply voltage E' and the power P_m delivered to each phase of the stator winding are measured. We thus get, in general, three slightly different values of M, three slightly different values of E' and three slightly different values of P_m. Taking the average in each case we calculate m from the equation $P_m = E'M \cos m$ where m is the phase difference between E' and M, as shown in Fig. 329.

At zero motor load, when the rotor runs at approximately the same speed as the stator magnetism (synchronous speed), the rotor conductors do not cut flux, no electromotive forces are induced in the rotor conductors, and no currents flow in the rotor conductors. Under these conditions the magnetizing current M, alone, flows in each stator winding and produces the rotating stator magnetism (which induces in each stator winding enough back electromotive force to balance or very nearly balance the supply voltage E').

When the motor is loaded the rotor speed is slightly reduced, the rotor conductors cut flux, electromotive forces are induced in the rotor conductors, and a certain current I'' flows in each rotor rod or band of rotor conductors; and *additional current* I' flows in each stator phase. The magnetizing action of I'' is balanced by the opposite magnetizing action of I', exactly as in the transformer. If the number of stator conductors is the same as the number of rotor conductors, then the currents I'' and I' in the two conductors R and S, Fig. 326, are always equal and opposite.

The effect of magnetic leakage in an induction motor, as in a transformer, is equivalent to an inductance P or a reactance ωP connected in series with each stator winding. This effect and also the effects of the resistances of stator and rotor windings are described in connection with the clock diagrams Figs. 327, 328 and 329.

Throughout the following discussion the rotor is supposed to be wire wound and to have same number of conductors and same number of phases as the stator.

Figure 327 is a clock diagram showing the action of an induction motor without core reluctance (without magnetizing

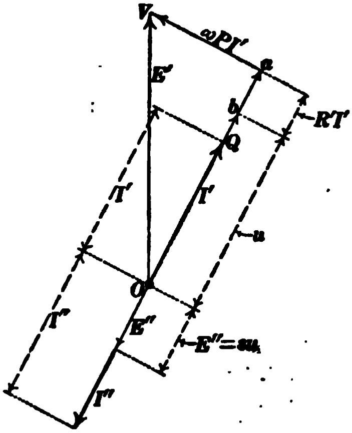

Fig. 327.

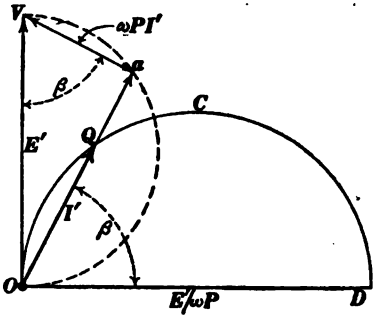

Fig. 328.

Representing one of the phases of an induction motor without core reluctance (without magnetizing current).

current). Let us think of the rotating stator magnetism as consisting of two parts* Φ and φ such that Φ as it rotates cuts all of the stator conductors and all of the rotor conductors, whereas φ as it rotates cuts the stator conductors only.

The useful part of E' is the part Ob $(= u)$ which overcomes the back electromotive force (equal and opposite to Ob) which is induced in each stator winding by the rotating flux Φ, the part ba overcomes the resistance R' of each stator winding, and the part $\omega PI'$ overcomes the electromotive force induced in each stator winding by the rotating leakage flux φ, ($\omega PI'$ is 90° ahead of I' in phase); and E' is, of course, the vector sum of u, $R'I'$ and $\omega PI'$ as shown.

The rotating flux Φ cuts the stator conductors at speed n and induces the electromotive force Ob or u in each stator winding; the same flux cuts the rotor conductors at less speed, $n - n'$, and induces correspondingly less electromotive force in each rotor winding. Therefore the electromotive force induced in each rotor winding is $E' = \frac{n - n'}{n} \cdot u = su$, where s (the so-called *slip-speed*) is written for $\frac{n - n'}{n}$.

The electromotive force E'' produces a current $I'' = E''/R''$†

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* This is the point of view which was partially developed in Art. 209; here, however, a little analysis will show that the center of the polar areas of Φ are 90° behind the center of the polar areas of ϕ in a two-pole motor, the rotating flux Φ is produced by the magnetizing currents M and the rotating flux ϕ is produced by the load currents I' in the stator, but ϕ is prevented from entering the rotor by the opposing magnetizing action of the rotor currents I''. Let it be understood, however, that *analysis* involves something besides algebra.

† This brings up again the question which has been met before but not very fully answered—What about the inductance or reactance of the rotor windings? and how can the rotor windings be considered as non-inductive circuits? The answer is that an electromotive induced in a circuit by *all the magnetic flux that actually exists* cannot be partly used to overcome an electromotive force induced by flux due to the current in the circuit; but if one considers the changing flux due, for example, to a moving magnet near a coil, one discovers later that there is another cause acting to produce changing flux, namely, the changing current in the coil; and, as an afterthought, as it were, one makes allowance for this newly discovered flux.

in each rotor winding, and current I' then flows through each stator winding, as above explained.

Figure 328 shows the triangle OaV of Fig. 327, and, since Oa and aV are at right angles to each other, it is evident that the locus of the point a is the semi-circle of which the diameter is OV or E'. *Also the locus of the point* Q *is the semi-circle* OCD *whose diameter* OD *is at right angles to* E' *and equal* (*in ampere value*) *to* $E'/\omega P$. This is evident from the following considerations: (1) The two angles β are equal, (2) The ratio: Va divided by OQ is equal to $\omega PI'$ divided by I', and this ratio is therefore constant and equal to ωP, (3) The vector OQ being proportional to Va and the angles β being equal, the locus of Q must be similar to the locus of a, a semi-circle, and (4) Because of similitude the ratio: OD divided by OQ must be equal to the ratio OV divided by Va; but OQ is I', OV is E', and Va is $\omega PI'$, so that $OD = E'/\omega P$.

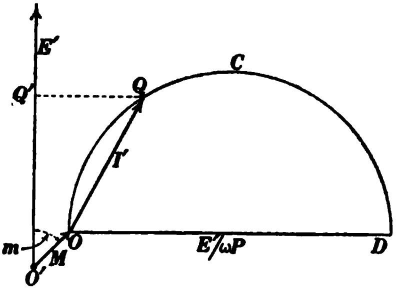

Fig. 329.

The total current in each stator winding is $M + I'$ which is represented by the line $O'Q$ in Fig. 329. Therefore, introducing the magnetizing current M, the circle diagram of Fig. 328 becomes as shown in Fig. 329.

234. Induction motor calculations.—To construct Fig. 329 one must know E', m, M and ωP; and R' and R'' must be known also for the following calculations. The value of E' is of course known or easily determined, the determination of m and M is explained in the previous article, the values of R' and R'' can be easily measured inasmuch as the rotor is understood to be wire wound with same number of conductors and same number of phases as the stator, and ωP may be determined by the following method which is exactly the same as the short-circuit transformer test as described in Art. 230:

Lock the motor, apply voltages E of normal frequency and proper phases to the stator windings so as to produce full-load current I in each stator winding. Then

$$E = I\sqrt{(R' + R'')^2 + (\omega P)^2}$$

from which ωP may be calculated.

If the rotor is wire wound with p'' phases and N'' conductors then the measured resistance of each rotor winding must be multiplied by $\left(\frac{N'}{N''}\right)^2$ and by $\frac{p''}{p'}$* to give the value of R'' to be used in the following calculations, where N' is the number of stator conductors and p' is the number of stator phases.

If the motor has a squirrel-cage rotor, the value of R'' to be used in the following calculations (as if the rotor were wire wound with same number of conductors and same number of phases as stator) may be measured as follows: Lock the motor so that it cannot run, send measured normal full-load currents I' of proper phases and at normal frequency through the stator windings, and measure the power delivered to each stator winding. Under the conditions of this test the core loss is very small, and if we neglect it we have

$$P_0 = (R' + R'')I'^2$$

where P_0 is the average value of the above measured values of power delivered to the phases of the stator, and R'' is the desired resistance of each fictitious rotor winding.

Calculations.—Choose a point Q on the circle in Fig. 329; then $E' \times O'Q'$ is the power delivered to each stator winding, and the cosine of the angle between E' and a line drawn from O' to Q is the power factor of each stator winding.

The total current in each stator winding is $O'Q$, and the square of this current multiplied by R' gives the RI^2 loss of power in each stator winding.

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* This ignores a small correction factor, the ratio of what are called the *phase constants* of the p'-phase and p''-phase windings.

The core loss of power is $E'M \cos m$ per phase.

The rotor RI^2 loss per phase is $R''I''^2$ or $R''^2I'^2$, inasmuch as $I'' = I' = OQ$.

Therefore total power supplied to motor, total loss of power in motor and total mechanical power output (friction losses being ignored) are easily calculated, and also the motor efficiency.

The slip speed s and the torque exerted on the rotor, for the chosen point Q, are calculated as follows: Knowing the value of $I'(= OQ)$, the value of $\omega PI'$ is known, and the value of

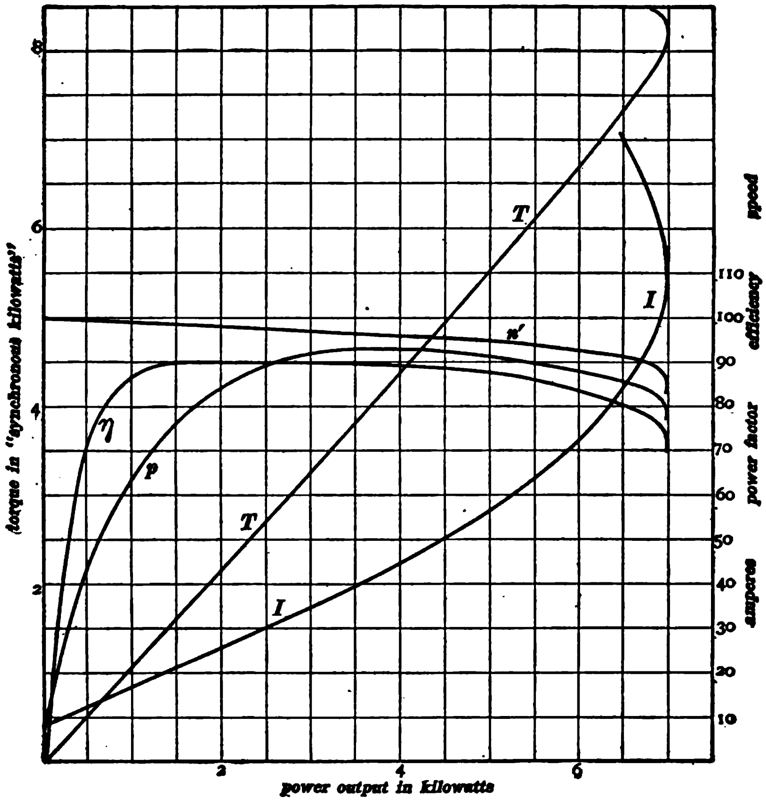

Fig. 330.

Oa, Fig. 327, may be calculated. Then the value of u is known $(= Oa - R'I')$; but $I''(= I') = E''/R''$, so that E'' is known; and therefore s may be found from the relation $E'' = su$. Knowing the value of synchronous speed n [equal to two times primary frequency divided by the known number of poles, according to equation (93)] and the value of s, the value of n' may be found, and the torque T in pound-feet may be calculated from the equation: total power output in watts $= 8.52n'T$, as explained in Art. 125.

Then choose another point Q in Fig. 329 and calculate another set of values of power intake, power factor, power output, efficiency, slip speed s, rotor speed n', and torque; and so on. In this way the data for the plotting of the curves in Fig. 330 were calculated for a certain 2-phase, 60-cycle, 6-pole, 5-horse-power induction motor. In this figure, however, each value of torque is expressed in terms of the power it would develop at synchronous speed n.

235. The single-phase induction motor.—The theory of the single-phase motor as developed in Art. 216 is exact as far as it goes, but it does not take account of core-reluctance, magnetic leakage, and stator resistance. It assumes also that the resistance of the B-part of the rotor winding, as shown in Fig. 303, is negligible, and this assumption will be retained in the following discussion, because a very small current (the magnetizing current which produces the *motor field* or *motor flux* Φ' as explained in Art. 216) flows in the B-part of the rotor winding.

Magnetic leakage in the single-phase induction motor is equivalent to an inductance P or reactance ωP connected in series with the stator winding, exactly as in the polyphase induction motor, and the value of ωP is determined by locking the motor and observing E and I exactly as in case of the polyphase motor as explained in Art. 234.

The clock diagram of Fig. 327 represents the action of the single-phase induction motor as affected by magnetic leakage and stator resistance but instead of taking E'' equal to su it must be taken equal to $s'u$, where

$$s' = 1 - \left(\frac{n'}{n}\right)^2$$

as explained in Art. 216. It is evident that the point a in Fig. 327 lies on a semi-circle of which E' is the diameter, as shown in Fig. 328, and that the point Q in Fig. 328 lies on the semi-circle of which the diameter OD is at right angles to E' and equal (in ampere value) to $E'/\omega P$, exactly as in the polyphase induction motor.

The magnetizing current M of the single-phase induction motor and its angle of lag m are determined exactly as in the polyphase motor by measuring stator current M and stator power intake P_m at zero load (synchronous speed); and if we assume that M is the same in value and in phase at all loads,* the diagram Fig. 328 becomes the circle diagram of Fig. 329.

The stator resistance R' is easily measured and the fictitious rotor resistance R'' is measured by observing the power intake P_0 of the stator when the rotor is locked and when the stator current I is about normal full-load value. Then, neglecting core loss (which is indeed quite small under the conditions of the test) we have $P_0 = (R' + R'')I^2$.

Calculations.—Choose a point Q on the circle in Fig. 329; then $E' \times O'Q'$ is the power delivered to the stator, and the cosine of the angle between E' and a line drawn from O' to Q is the power factor.

The total stator current is $O'Q$, and the square of this current multiplied by R' gives the RI^2 loss of power in the stator winding.

The core loss of power is $E'M \cos m$.

The rotor RI^2 loss of power is $R''I''^2$ or $R''I'^2$, inasmuch as $I'' = I' = OQ$.

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* In fact the magnetizing current of the single-phase induction motor decreases more rapidly with increasing motor load than do the magnetizing currents of the polyphase induction motor. But single-phase motor calculations are usually made on the assumption that M and m do not change with motor load.

Therefore total power supplied to motor, total loss of power in motor, and total mechanical power output (friction losses being ignored) are known, and also the motor efficiency.

The value of s' and the motor torque, for the chosen point Q, are calculated as follows: Knowing the value of I' $(= OQ)$, the value of $\omega PI'$ is known, and the value of Oa, Fig. 327, may be calculated. Then the value of u is known $(= Oa - R'I')$; but $I''(= I') = E''/R''$, so that E'' is known; and therefore s' may be found from the relation $E'' = s'u$. Knowing the value of synchronous speed n (equal to $2f/p$, where f is the frequency of supply voltage and p is the number of poles for which the stator winding is arranged) and the value of s', the value of n' may be found from $s' = 1 - \left(\frac{n'}{n}\right)^2$; and then the value of the torque in pound-feet can be found from the equation: power output in watts $= 8.52n'T$.

APPENDIX A

SOME ORDINARY DIFFERENTIAL EQUATIONS.*

1. Degree and order of a differential equation.—The simple equation $az + b = 0$ is said to be *linear* with respect to z because it contains no power of z higher than the first power. A differential equation of the form:

$$y + A\frac{dy}{dx} + B\frac{d^2y}{dx^2} + C = 0 \tag{1}$$

is called a *first degree* or *linear* differential equation because it contains no products of y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, and no powers of y, $\frac{dy}{dx}$, etc., higher than the first. The coefficients A, B and C in a linear differential equation may contain the independent variable x, but we shall confine our attention in this chapter chiefly to *linear differential equations with constant coefficients.*

If the first derivative,† only, appears in a differential equation, the differential equation is said to be of the *first order.* If the second derivative occurs (with or without the first derivative) the differential equation is said to be of the *second order;* and so on.

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* No branch of mathematics is of greater importance to the engineer than the theory of differential equations. This Appendix refers to what are called ordinary differential equations. The third volume of this elementary treatise on Electrical Engineering will include a discussion of some partial differential equations.

This Appendix is taken without change from Franklin. MacNutt and Charles' *Calculus*, published by the authors, South Bethlehem, Pa., price $2.00, postpaid. Every beginner in calculus should read the first chapter of the revised edition of this book. It is intelligible.

† The terms *degree* and *order* apply to partial differential equations as well as to ordinary differential equations. A function of two variables, however, has two first derivatives, three second derivatives, and so on. Therefore it is somewhat misleading to speak of *the* first derivative, or *the* second derivative in explaining what is meant by a differential equation of the first or second order.

2. Ordinary and partial differential equations.—A differential equation expressing the law of growth of a function of one independent variable is called an *ordinary differential equation*. Many examples of ordinary differential equations are given in the earlier chapters of Franklin, MacNutt and Charles' *Calculus*.

A differential equation which expresses the law of growth of a function of two or more independent variables is called a *partial differential equation*. Some examples of partial differential equations are given in Chapter V of Franklin, MacNutt and Charles' *Calculus*. See Arts. 60 and 90 in particular.

This chapter is devoted to the discussion of a few important ordinary differential equations.

3. Pure and mixed differential equations.*—A *pure differential equation* contains but one derivative (the first or any higher derivative) and does not contain the dependent variable y. Thus $dy = 6x \cdot dx$, $\frac{dy}{dx} = \sin x$, $\frac{d^2y}{dx^2} = \log x$ are pure differential equations.

A *mixed differential equation* contains more than one derivative, or it contains one or more derivatives and the dependent variable y. Thus

$$\frac{dy}{dx} = y, \qquad y + B\frac{dy}{dx} = Cx^2, \qquad B\frac{dy}{dx} + C\frac{d^2y}{dx^2} = 0$$

are mixed differential equations.

Solution of pure differential equations.—A pure differential equation can be integrated by looking up the appropriate form in a table of integrals. This is exemplified by every problem in integration. In the case of a pure differential equation of the second or third order, successive integrations are of course necessary. For example, consider the third order pure differential equation:

$$\frac{d^3y}{dx^3} = 3x^2 \tag{1}$$

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* **This and the following articles refer primarily to ordinary differential equations.**

Let q represent $\frac{d^2y}{dx^2}$, then equation (1) becomes:

$$\frac{dq}{dx} = ax^2 \tag{2}$$

This equation can be integrated, giving:

$$q = \frac{d^2y}{dx^2} = \tfrac{1}{3}ax^3 + C \tag{3}$$

Let p represent $\frac{dy}{dx}$, then equation (3) becomes:

$$\frac{dp}{dx} = \tfrac{1}{3}ax^3 + C \tag{4}$$

This equation can be integrated, giving:

$$p = \frac{dy}{dx} = \tfrac{1}{12}ax^4 + Cx + D \tag{5}$$

This equation can be integrated, giving:

$$y = \tfrac{1}{60}ax^5 + \tfrac{1}{2}Cx^2 + Dx + E \tag{6}$$

Solution of mixed differential equations. Separation of variables.—The linear differential equation (1) of Art. 1 is of course a mixed equation, and the general solution of this equation with constant coefficients is given in a subsequent article. We will here consider one or two examples of a simple transformation, called *the separation of variables*, which can sometimes be used to bring a mixed differential equation into a form which can be integrated by looking up appropriate forms in a table of integrals.

As a first example consider:

$$\frac{dy}{dx} = x^2y \tag{7}$$

This equation reduces to:

$$\frac{dy}{y} = x^2 \cdot dx \tag{8}$$

and this equation can be integrated with the help of a table of integrals, giving

$$\log y = \tfrac{1}{3}x^3 + C \tag{9}$$

As a second example consider

$$\frac{d^2y}{dx^2} = x^2\frac{dy}{dx} \tag{10}$$

Let p represent $\dfrac{dy}{dx}$ and this equation becomes

$$\frac{dp}{dx} = x^2p \tag{12}$$

and the integral of this equation, according to equation (9), is:

$$\log p = \tfrac{1}{3}x^3 + C$$

or

$$p = \frac{dy}{dx} = e^{\frac{1}{3}x^3+C} \tag{13}$$

This is a pure differential equation and its integral can be found by looking up the appropriate form in the table of integrals.

4. The general solution and particular solutions of a differential equation.—The general solution of a differential equation is an expression for y (the dependent variable) in terms of x (the independent variable) *including every possible*[*] *function which satisfies the differential equation.* Thus equation (6) of Art. 3 is the general solution of equation (1) of that article. Concerning the three constants of integration it is evident that E is the value of y when $x = 0$. Also it is evident from equation (5) of Art. 3 that D is the value of $\dfrac{dy}{dx}$ when $x = 0$, and it is evident from equation (3) that C is the value of $\dfrac{d^2y}{dx^2}$ when $x = 0$.

The *general solution* of a differential equation of the nth order contains n undetermined constants of integration.

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[*] This statement is subject to some qualification because of what is called the *singular solution.* See Johnson's *Ordinary and Partial Differential Equations,* page 43, John Wiley & Sons, New York, 1890.

When one or more of the constants of integration in the general solution of a differential equation have particular values assigned to them we have what is called a *particular solution* of the differential equation. For example, let $C = 2$, let $D = 0$ and let $E = 0$ in equation (6) of Art. 3, then the equation becomes $y = \frac{ax^5}{48} + x^2$ which is a particular solution of equation (1) of Art. 3.

5. Discussion of the first order linear differential equation with constant coefficients.—Consider the differential equation:

$$y + A\frac{dy}{dx} = 0 \tag{1}$$

The most obvious method for solving this differential equation is to "separate the variables" as explained in Art. 3. Thus equation (1) is easily reduced to:

$$A\frac{dy}{y} = -dx \tag{2}$$

and this equation can be integrated by looking up appropriate forms in a table of integrals, giving:

$$A \log y = -x + C \tag{3}$$

It is desirable, however, to solve equation (1) by the following method because the method applies to a linear differential equations of any order when the coefficients are constants.

Let

$$y = C\epsilon^{kx} \tag{4}$$

whe e C and k are constants, and ϵ is the Napierian base. Then:

$$\frac{dy}{dx} = kC\epsilon^{kx} \tag{5}$$

Substituting the values of y and $\frac{dy}{dx}$ from (4) and (5) in equation (1), we have:

$$C\epsilon^{kx} + AkC\epsilon^{kx} = 0 \tag{6}$$

whence by cancellation we have:

$$1 + Ak = 0$$

or

$$k = -\frac{1}{A} \tag{7}$$

Therefore if k in equation (4) has the value $-\frac{1}{A}$ then equation (4) satisfies equation (1); that is, equation (4) is a solution of (1) if $k = -\frac{1}{A}$; indeed equation (4) is the general solution of (1) because it contains one undetermined constant C, the constant of integration.

6. The principle of superposition.—A principle of extremely wide application in physics is the so-called *principle of superposition*. From the physical point of view a general statement of this principle is scarcely possible and therefore the following examples must suffice: (*a*) A person at A (Fig. 1) can see window No. 1 and another person at B can see window No. 2 *at the same time*. This means that two beams of light a and b can travel through the same region at the same time and not get tangled up together as it were, each beam behaving as if it were traveling through the region alone. (*b*) Two sounds can travel through the same body of air simultaneously, each sound behaving as if it were traveling through the body of air alone. (*c*) Two systems of water waves can travel over the same part of a pond simultaneously, each system behaving as if the other were not present. (*d*) Two messages* can travel over a tele-

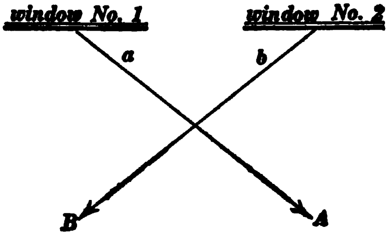

Fig. 1.

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* Indeed any number of messages can travel over a telegraph wire in either direction or in both directions simultaneously. The only limiting feature in multiplex telegraphy, when line-loss is negligible, is in the design of the sending and receiving apparatus; and the same is true in wireless telegraphy. In each of the above examples the word *two* means *two or more*.

graph wire at the same time and not get mixed up. (*e*) Two forces F and G exerted simultaneously upon an elastic structure produce an effect which is the sum of the effects which would be produced by the forces separately, provided the sum of the forces does not exceed the elastic limit of the structure; therefore each force may be thought of as producing the same effect that it would produce if it were acting alone.

All of the effects in physics which are superposable—and this includes the greater part of the effects in mechanics, heat, electricity and magnetism, light and sound, and a great many effects in chemistry—are expressible in terms of *linear differential equations with constant coefficients*, and the principle of superposition may be thought of as a property of such differential equations as follows: *If y is a function of x which satisfies a linear differential equation with constant coefficients, and if z is another function of x which satisfies the same differential equation, then $(y + z)$ is a function of x which satisfies the equation.*[*]

Proof.—Let the given linear differential equation be:

$$u + A\frac{du}{dx} + B\frac{d^2u}{dx^2} + \cdots = 0 \tag{1}$$

If a function y satisfies this equation, then:

$$y + A\frac{dy}{dx} + B\frac{d^2y}{dx^2} + \cdots = 0 \tag{2}$$

If another function z satisfies equation (1), then:

$$z + A\frac{dz}{dx} + B\frac{d^2z}{dx^2} + \cdots = 0 \tag{3}$$

Now $\dfrac{d(y+z)}{dx} = \dfrac{dy}{dx} + \dfrac{dz}{dx}$ and $\dfrac{d^2(y+z)}{dx^2} = \dfrac{d^2y}{dx^2} + \dfrac{d^2z}{dx^2}$. Therefore, adding equations (2) and (3), we get:

---

[*] This proposition is true for a partial linear differential equation also Indeed most of the superposable effects in physics are expressible in terms of partial linear differential equations with constant coefficients. Examples are given in Chapters VIII and IX of Franklin, MacNutt and Charles' *Calculus*.

$$(y+z) + A\frac{d(y+z)}{dx} + B\frac{d^2(y+z)}{dx^2} + \cdots = 0 \qquad (4)$$

But this is the same form as equation (1) which shows that the function $(y + z)$ satisfies (1).

7. Discussion of the second order linear differential equation with constant coefficients.—Consider the differential equation:

$$y + A\frac{dy}{dx} + B\frac{d^2y}{dx^2} = 0 \qquad (1)$$

It is not possible to "separate the variables" in this differential equation and therefore the second method of Art. 5 must be used. Therefore let:

$$y = C\epsilon^{kx} \qquad (2)$$

then

$$\frac{dy}{dx} = kC\epsilon^{kx} \qquad (3)$$

and

$$\frac{d^2y}{dx^2} = k^2C\epsilon^{kx} \qquad (4)$$

Substituting these values of y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in equation (1) and cancelling the common factor $C\epsilon^{kx}$, we have:

$$1 + Ak + Bk^2 = 0 \qquad (5)$$

whence

$$k = -\frac{A}{2B} \pm \sqrt{\frac{A^2}{4B^2} - 1} \qquad (6)$$

Therefore using α for one of these values of k and using β for the other value of k, we get two solutions of (1), namely:

$$w = C\epsilon^{\alpha x} \qquad (7)$$

and

$$z = D\epsilon^{\beta x} \qquad (8)$$

But according to Art. 6 the sum $(w + z)$ is also a solution.

Therefore using y for $(w + z)$, we have as a solution of (1):

$$y = C\epsilon^{\alpha x} + D\epsilon^{\beta x} \tag{9}$$

and this is the general solution of (1) because it contains the two undetermined constants C and D.

8. The starting of a boat.—At a certain instant $(t = 0)$ a constant force E begins to act on a boat, and it is desired to find an expression for the increasing velocity i of the boat. At very low speeds the backward drag or friction of the water on a boat is proportional to the velocity of the boat. *Let us assume that this proportional relation is exact,* then the frictional drag of the water on a boat is equal to Ri, where i is the velocity of the boat and R is a constant which depends on the shape and size of the boat. Therefore the net accelerating force acting on the boat is $E - Ri$. This force is equal* to the product of the mass L of the boat and the acceleration $\frac{di}{dt}$. Therefore we have:

$$L\frac{di}{dt} = E - Ri \tag{1}$$

To get this equation into the standard form of a linear differential equation, let

$$y = E - Ri \tag{2}$$

Then

$$\frac{dy}{dt} = -R\frac{di}{dt}$$

and equation (1) becomes:

$$-\frac{L}{R}\cdot\frac{dy}{dt} = y$$

or

$$y + \frac{L}{R}\cdot\frac{dy}{dt} = 0 \tag{3}$$

The general solution of this equation is given in Art. 5, but it is worth while to work it out anew, as follows: Let

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* If force is expressed in poundals (or dynes), mass in pounds (or grams), and acceleration in feet per second per second (or centimeters per second per second).

$$y = C\epsilon^{kt} \tag{4}$$

then

$$\frac{dy}{dt} = kC\epsilon^{kt} \tag{5}$$

Substituting these values of y and $\frac{dy}{dt}$ in equation (3) and cancelling out the factor $C\epsilon^{kt}$, we have:

$$1 + \frac{L}{R} \cdot k = 0$$

so that

$$k = -\frac{R}{L} \tag{6}$$

Therefore equation (4) becomes:

$$y = C\epsilon^{-\frac{R}{L} \cdot t}$$

or, substituting $E - Ri$ for y, we have:

$$E - Ri = C\epsilon^{-\frac{R}{L} \cdot t} \tag{7}$$

Now $i = 0$ when $t = 0$. Therefore placing this pair of values in equation (7) we have:

$$E = C \tag{8}$$

so that the constant of integration is determined, and equation (7) becomes:

$$E - Ri = E\epsilon^{-\frac{R}{L} \cdot t}$$

or

$$i = \frac{E}{R} - \frac{E}{R}\epsilon^{-\frac{R}{L} \cdot t} \tag{9}$$

This equation expresses the value at each instant of the increasing velocity of the boat as a function of the elapsed time t.

9. **The stopping of a boat.**—A boat is moving at velocity I when the propelling force ceases to act, and it is required to find an expression for the decreasing velocity, i, of the boat as it

gradually comes to rest. In this case the only force acting on the boat is the retarding force Ri, and in this case the acceleration $\frac{di}{dt}$ is negative so that:

$$L\frac{di}{dt} = -Ri \tag{1}$$

or

$$i + \frac{L}{R}\cdot\frac{di}{dt} = 0 \tag{2}$$

The general solution of this differential equation is:

$$i = C\epsilon^{-\frac{R}{L}\cdot t} \tag{3}$$

But $i = I$ when $t = 0$. Therefore $C = I$, and equation (3) becomes:

$$i = I\epsilon^{-\frac{R}{L}\cdot t} \tag{4}$$

Remark.—The notation used in this discussion of the starting and stopping of a boat is the standard notation used in electrical theory. This notation is used here and in the following articles

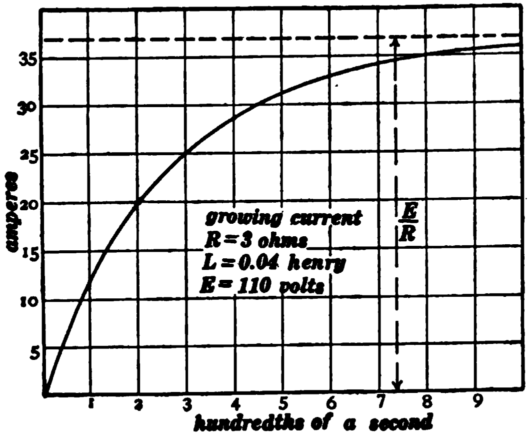

Fig. 2.

because the mechanical problems discussed in this and the following articles are strictly analogous to certain electrical problems which constitute the foundation of the theory of alternating currents. Thus the curve in Fig. 2 is a graphical representation of equation (9) of Art. 8; the ordinates of this curve represent the growing values of the current i in a circuit of resistance R and inductance L after a battery of electromotive force E is connected to the circuit. The curve in Fig. 3 is a graphical representation of equation (4) of Art. 9; the ordinates of this curve represent the decaying values of the current i when an initial current of I amperes is left to die away in a circuit of resistance R and inductance L.

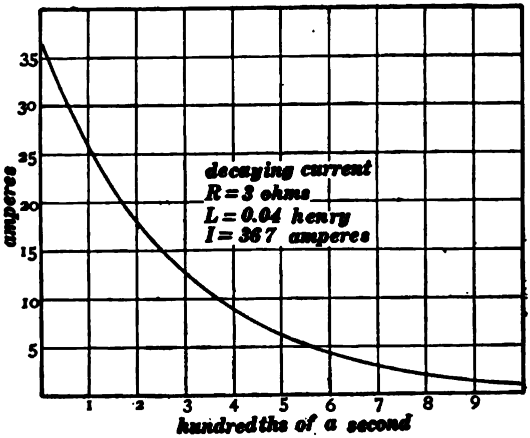

Fig. 3.

10. Undamped oscillations.—Figure 4 shows a weight suspended by a spring, and the weight stands in equilibrium in a certain position. If the weight happens to be q feet above or below its equilibrium position an unbalanced force proportional to q, and therefore equal to $\frac{1}{C}$[*] times q, acts upon the weight. This force is downwards when q is upwards, and upwards when q is downwards. Therefore the force is equal to $-\frac{1}{C}\cdot q$.

---

[*] This is simply a proportionality factor. It is written in this form so as to conform to the standard notation of electrical theory.

This force accelerates (or retards) the weight, and we have

$$L\frac{di}{dt} = -\frac{q}{C} \tag{1}$$

where L is the mass in pounds of the attached weight in Fig. 4, and $\frac{di}{dt}$ is the acceleration (it is a retardation when it is negaive) of the attached weight. But the ve'ocity i of the weight is the rate of change of its distance q from the equilibrium position. That is

$$i = \frac{dq}{dt} \tag{2}$$

so that

$$\frac{di}{dt} = \frac{d^2q}{dt^2} \tag{3}$$

Therefore equation (1) becomes

$$L\frac{d^2q}{dt^2} = -\frac{q}{C}$$

or

$$q + LC \cdot \frac{d^2q}{dt^2} = 0 \tag{4}$$

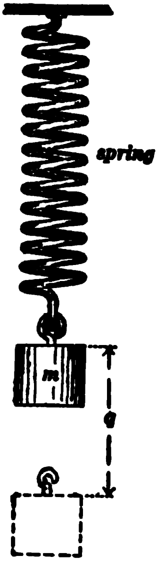

Fig. 4.

The general solution of this second order linear differential equation is given in Art. 7, but it is worth while to work out the solution anew as follows:

Let

$$q = M\epsilon^{kt} \tag{5}$$

then

$$\frac{dq}{dt} = kM\epsilon^{kt} \tag{6}$$

and

$$\frac{d^2q}{dt^2} = k^2M\epsilon^{kt} \tag{7}$$

Substituting these values of q and $\frac{d^2q}{dt^2}$ in equation (4) and cancelling the common factor $M\epsilon^{kt}$, we have:

$$1 + LCk^2 = 0 \tag{8}$$

so that

$$k = \sqrt{-\frac{1}{LC}}. \tag{9}$$

It is evident that k is imaginary because L and C are positive quantities. Therefore let us write $j\omega$ for k where j is $\sqrt{-1}$,* then:

$$\omega = \sqrt{\frac{1}{LC}} \tag{9}$$

Now of course ω may be either positive or negative, that is, k may be $+j\omega$ or $-j\omega$. Therefore

$$v = M\epsilon^{+j\omega t} \tag{10}$$

and

$$z = N\epsilon^{-j\omega t} \tag{11}$$

are two particular solutions of equation (4), where M and N are undetermined constants. Therefore, according to Art. 6, $(v + z)$ is also a solution of (4). Consequently, writing u for $(v + z)$, we have as a solution of (4):

$$u = M\epsilon^{+j\omega t} + N\epsilon^{-j\omega t} \tag{12}$$

and this is the general solution of (4) because it contains the two undetermined constants M and N.

Now from Demoivre's theorem† it is evident that u as given by equation (12) is complex (part real and part imaginary), and for the sake of generality M and N may be thought of as complex also. That is, for M and N we may write:

$$M = M' + jM'' \tag{13}$$

and

$$N = N' + jN'' \tag{14}$$

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* It is customary in electrical theory to use j for $\sqrt{-1}$ because the letter i is used for electric current.

† See Franklin, MacNutt and Charles' *Calculus*, page 162.

Now any complex equation is equivalent to two simple equations, namely the equation which expresses the equality of the real parts of the members of the complex equation, and the equation which expresses the equality of the imaginary parts of the complex equation; and this latter equation becomes real when j is cancelled from its members. Thus equation (12) may be broken up into two simple equations each of which is a solution of equation (4).

To split equation (12) up into two equations use the values for M and N from equations (13) and (14), and reduce $\epsilon^{+j\omega t}$ and $\epsilon^{-j\omega t}$ by means of Demoivre's theorem. We thus get:

$$r + js = (M' + jM'')(\cos \omega t + j \sin \omega t) + (N' + jN'')(\cos \omega t - j \sin \omega t) \qquad (15)$$

where $r + js$ has been written for q. Separating real and imaginary terms in equation (15) we get:

$$r = (M' + N') \cos \omega t - (M'' - N'') \sin \omega t \qquad (16)$$

and

$$s = (M'' + N'') \cos \omega t + (M' - N') \sin \omega t \qquad (17)$$

Now in either of these equations the coefficient of $\cos \omega t$ is *any* constant, and the coefficient of $\sin \omega t$ is *any* constant. Therefore, letting q stand for r or s, both of these equations reduce to:

$$q = G \cos \omega t + H \sin \omega t \qquad (18)$$

which is the final general solution of equation (4), G and H being undetermined constants.

In alternating current theory this equation is used in a slightly modified form as follows:

Let

$$G = - Q \sin \theta \qquad (19)$$

and

$$H = Q \cos \theta \qquad (20)$$

Then equation (18) becomes:

$$q = Q \sin (\omega t - \theta) \qquad (21)$$

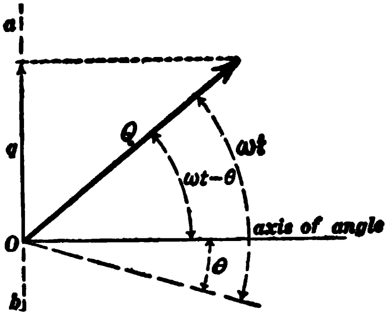

Fig. 5.

This equation is represented by Fig. 5, in which the line Q rotates at angular velocity ω radians per second, and the value of q at each instant is represented by the projection of Q on the fixed line ab.

The two integration constants Q and θ are determined by the initial conditions as follows: For example let the weight be started oscillating by pulling it upwards 2 feet ($q = 2$ feet), holding it still $\left(\frac{dq}{dt} = 0\right)$ and releasing it at the instant $t = 0$. The values of Q and θ are then determined by substituting the values $\frac{dq}{dt} = 0$ and $t = 0$ in the equation

$$\frac{dq}{dt} = \omega Q \cos (\omega t - \theta)$$

and by substituting the values $q = 2$ and $t = 0$ in equation (21).

11. **Damped oscillations.**—The weight in Fig. 4 is acted upon by the force $-\frac{q}{C}$ when it is at a distance q above or below its equilibrium position as explained in Art. 10; and, if the weight moves up and down through a resisting fluid, the backward drag due to friction will be approximately proportional to the velocity $\frac{dq}{dt}$ of the weight. *Assuming this proportional relationship to be exact,* the backward drag of friction may be written $R \cdot \frac{dq}{dt}$, and since it is always opposed to the velocity $\frac{dq}{dt}$ it must be written $-R\frac{dq}{dt}$. Therefore the total force acting on the weight is $-\frac{q}{C} - R\frac{dq}{dt}$, and this force is equal to the

product of the mass L of the weight and the acceleration $\frac{d^2q}{dt^2}$ of the weight. Therefore we have:

$$L\frac{d^2q}{dt^2} = -\frac{q}{C} - R\frac{dq}{dt}$$

or

$$q + CR\frac{dq}{dt} + LC\frac{d^2q}{dt^2} = 0 \tag{1}^*$$

The general solution of this equation may be found by the method used in Art. 10. Reduced to simple form the general solution is:

$$q = Q\epsilon^{-\alpha t}\sin(\omega t - \theta) \tag{2}$$

where Q and θ are undetermined constants of integration and where α and ω are written for the following:

$$\alpha = \frac{R}{2L} \tag{3}$$

and

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{4}$$

Equation (2) completely expresses the motion of the weight in Fig. 4 when the motion of the weight is opposed by friction as

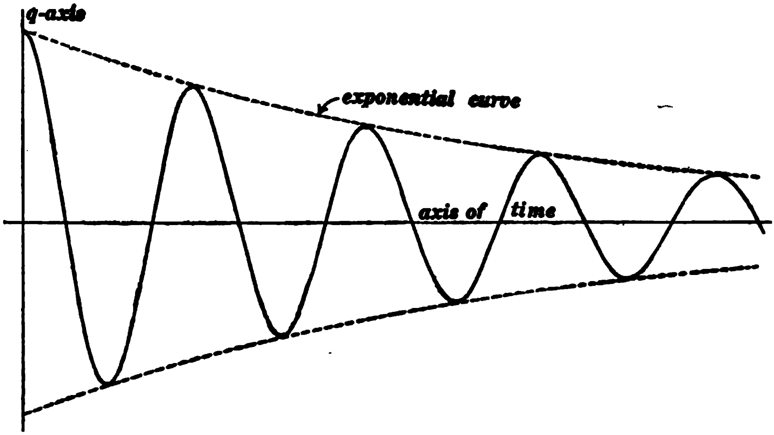

Fig. 6.

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* See equation (28) of Art. 63, Chapter IV.

above explained. Indeed equation (2) expresses a kind of harmonic motion in which the amplitude $(Q\epsilon^{-at})$ decreases continually.

The varying value of q as expressed by equation (2) is represented by the ordinates of the wavy curve in Fig. 6 for the case in which $\theta = -\frac{\pi}{2}$. The dotted curves are exponential curves of which the equations are $q = \pm Q\epsilon^{-at}$. Another graphical representation of equation (2) is shown in Fig. 7 where q is the projection on the fixed line ab of a line $Q\epsilon^{-at}$ which rotates at a constant angular velocity of ω radians per second and grows continually shorter and shorter. The end of the line $Q\epsilon^{-at}$ describes the dotted curve which is called an exponential spiral.

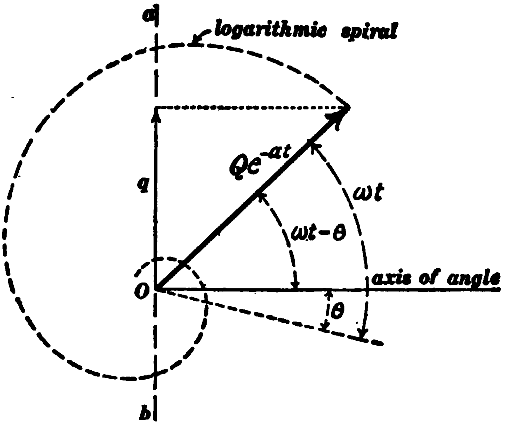

Fig. 7.

12. **Forced or maintained oscillations.**—The weight in Fig. 4 is acted upon by the force $-\frac{q}{C}$ due to the altered stretch of the spring, and by the force $-R\frac{dq}{dt}$ due to the frictional drag of the fluid in which the weight moves up and down. Let us suppose that an outside periodic force, **E** sin pt, acts upon the

weight. Then the total force acting on the weight will be $-\frac{q}{C} - R\frac{dq}{dt} + \mathbf{E}\sin pt$, and this will be equal to the product of mass times acceleration $\left(L\frac{d^2q}{dt^2}\right)$. Therefore we will have:

$$q + CR\frac{dq}{dt} + LC\frac{d^2q}{dt^2} = C\mathbf{E}\sin pt \qquad (1)$$

This is the differential equation of motion of the weight under the assumed conditions, and this identical differential equation expresses the mode of variation of the charge q in the condenser C in Fig. 8 when an alternator of which the electromotive force is $\mathbf{E}$ sin pt delivers electric current to a circuit of resistance R and inductance L, the circuit containing a condenser of capacity C. The charge q in the condenser is not so familiar a thing as the current $i\left(=\frac{dq}{dt}\right)$ which flows in the circuit, but it is convenient to use q as our variable instead of i.

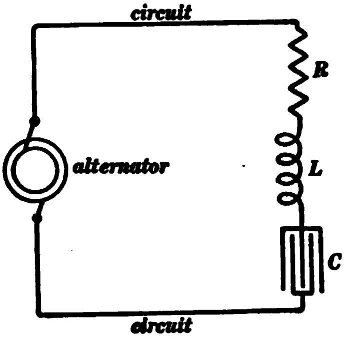

Fig. 8.

13. General solution of equation (1) of the previous article. Effects produced immediately after an alternating electromotive force is connected to a circuit.—The voltage-current relation which is discussed in Arts. 151–153 refers to the current which is *steadily maintained* in a circuit by a harmonic alternating electromotive force, whereas the following discussion refers to the more complicated state of affairs which exists for a short time after an alternating electromotive force is connected to a circuit, that is to say, for a short time after the closing of a connecting switch.

Circuit containing resistance and inductance, but no capacity (no condenser).—A connecting switch is closed at a certain instant t' and a harmonic alternating electromotive force, $\mathbf{E} \sin \omega t$, acts on a circuit of resistance R and inductance L. Part of the electromotive force, Ri, is used at each instant to overcome resistance, and part, $L\frac{di}{dt}$, is used at each instant to make the current i change. Therefore equation (1) of Art. 12 reduces to

$$L\frac{di}{dt} + Ri = \mathbf{E} \sin \omega t \tag{1}$$

If we can establish the general solution of this differential equation, a solution containing one undetermined or disposable constant, we will have the entirely general algebraic expression for the current i.

Consider the steadily maintained current

$$m = \mathbf{I} \sin (\omega t - \theta) \tag{2}$$

where*

$$\mathbf{I} = \mathbf{E}/\sqrt{R^2 + \omega^2 L^2} \tag{3}$$

and

$$\tan \theta = \omega L/R \tag*{(4)*}$$

This current m is a special solution of equation (1), and it remains to modify equation (2) so that it may still satisfy equation (1) and contain an undetermined constant.

Consider the decaying current [see equation (4) of Art. 9 of this Appendix]

$$d = D\epsilon^{-\frac{R}{L}\cdot t} \tag{5}$$

This decaying current d is a solution of the differential equation

$$L\frac{di}{dt} + Ri = 0 \tag{6}$$

as explained in Art. 9 of this Appendix.

But any solution of equation (6) added to any solution of equation (1) must give a new solution of equation (1).† Therefore adding equations (2) and (5) we get

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* See Arts. 151–153 of Chapter XI.

† This is very easily shown by the method of Art. 6 of this Appendix.

$$i = m + d = \mathrm{I} \sin (\omega t - \theta) + D\epsilon^{-\frac{R}{L}\cdot t} \qquad (7)$$

which is a solution of equation (1), and, since it contains the undetermined and disposable constant D, it is the desired general solution.

To understand the meaning of equation (7) consider Fig. 9.

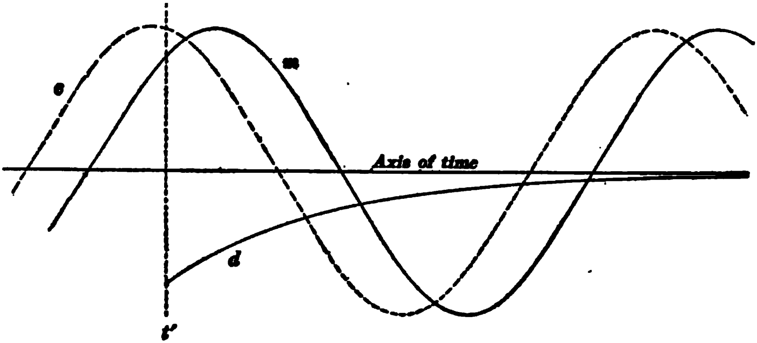

Fig. 9.

The ordinates of the curve m represent the maintained current m as expressed by equation (2), and the ordinates of the curve d represent the decaying current d as expressed by equation (5). The actual current in the circuit at the instant t' when the switch is closed must, of course, be zero, and the value of the constant D is determined by this condition. That is to say, at the instant t' the decaying current d is equal and opposite to the maintained current m.

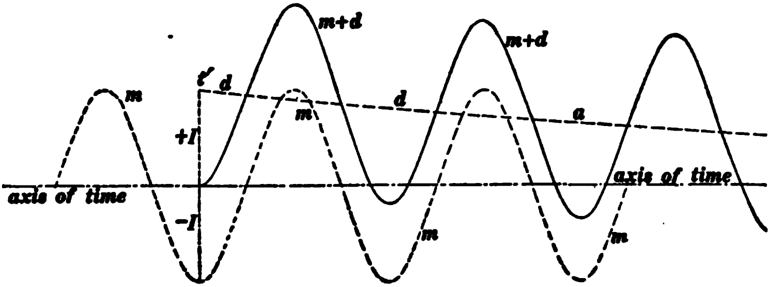

Fig. 10.

Figure 10 shows the two parts m and d, and the total current $m + d$ for the case in which the switch is closed at the instant when the maintained current m would be at its maximum negative value $\mathbf{I}$. In this case it is evident that the first pulse of actual current, $m + d$, rises to an excessively large value, a value which is nearly equal to $2\mathbf{I}$ when the current d decays slowly.

Circuit containing resistance, inductance and capacity.—The electromotive force, $\mathbf{E} \sin \omega t$, acting on such a circuit is used in part to overcome the reaction q/C of the charged condenser, in part, $R\frac{dq}{dt}$, to overcome resistance, and in part, $L\frac{d^2q}{dt^2}$, to make the current change, as explained in Art. 63 of Chapter IV. Therefore

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C} \cdot q = \mathbf{E} \sin \omega t \tag{8}$$

The maintained current is

$$m = \mathbf{I} \sin (\omega t - \theta) \tag{9}$$

where

$$\mathbf{I} = \mathbf{E} \Big/ \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{10}$$

and

$$\tan \theta = \left(\omega L - \frac{1}{\omega C}\right) \Big/ R \tag{11}$$

Equation (9) is a special solution of equation (8); and if we add to m the general solution of equation (1) of Art. 11 of this Appendix as expressed by equation (2) of Art. 11, we get the general solution of equation (8). It is not worth while to completely formulate this matter, it is sufficient to point out that the total current flowing in a circuit like Fig. 8 of this Appendix after the closing of the switch may be thought of in two parts, namely, (m) the maintained current as expressed by equations (9), (10) and (11), and (l) a decaying oscillatory current; and at the instant of closing the switch $m + l = 0$.

Excessive rush of magnetizing current through the primary coil of a transformer after the closing of a switch.—If we assume the core reluctance of a transformer to be constant so that core flux may be thought of as strictly proportional to the current in the primary coil (secondary coil on open circuit), then the primary coil would have a definite inductance L, and the flow of current through the primary coil after closing the supply switch would be as explained above and as represented in Figs. 9 and 10. The magnetic saturation of the transformer core plays an important part, however, and an extreme rush of magnetizing current takes place if the switch is closed at the instant when the supply voltage is passing through zero. Let $\frac{d\Phi}{dt}$ be the rate of increase of the core flux, and let N' be the number of turns of wire in the primary coil. Then $-N'\frac{d\Phi}{dt}$ is the back electromotive force induced in the primary coil and the instantaneous value of the primary supply voltage, e', is

$$e' = +N'\frac{d\Phi}{dt} \tag{12}$$

It is evident from this equation that Φ continues to increase while e' is positive. Let F be the total increase of Φ during half a cycle while e' is passing through a set of positive values. When a transformer is in steady operation this increase F ranges from a certain negative value of Φ to an equal positive value, that is, Φ will change from $-\frac{1}{2}F$ to $+\frac{1}{2}F$; but if the supply voltage is connected to the primary coil at the beginning of a set of positive values of e', then Φ must rise from zero to F, or, indeed, if the transformer core should happen to have a residual flux $+A$, the rise would have to be from A to $F+A$. These large values of flux, F or $F+A$, may approach magnetic saturation of the core, and to push the core flux up to these large values the magnetizing current would have to rise to an excessively large value. The above discussion ignores the effect of the resistance of the primary coil.

PROBLEMS.

1. Solve the differential equation $x^3 \dfrac{d^3y}{dx^3} = 2.$

Ans. $y = \log x + Cx^2 + Dx + E.$

2. Solve the differential equation $\dfrac{d^2y}{dx^2} + \dfrac{1}{x}\dfrac{dy}{dx} = 0.$

Ans. $C \log x = y + D.$

3. Solve the linear differential equation $\dfrac{d^2y}{dx^2} - 4y = 0.$

Ans. $y = C\epsilon^{2x} + D\epsilon^{-2x}.$

4. Solve the linear differential equation

$$\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 0$$

Ans. $y = C\epsilon^{2x} + D\epsilon^{3x} + E.$

5. The field winding of a shunt dynamo has 2,500 turns of wire and a current of 0.3 abampere produces a field flux of 1,500,000 maxwells or lines. Ignoring variability of magnetic reluctance of the iron field and armature parts find the inductance of the field winding (see Art. 48) and find the time required for the field current to rise to 9/10 of its final steady value when the field winding is connected to 110-volt supply mains, the resistance of the winding being 36.7 ohms.

Ans. 125×10^8 abhenrys; 0.784 second.

APPENDIX B.

SOME EXAMPLES SHOWING THE USE OF CHARACTERISTIC CURVES.

14. Relation between the volt-ampere curve and the magnetization curve of a shunt generator.—The terminal voltage E_x of a shunt generator may be measured for various values of current output I at given speed, and the results plotted as a curve as shown by the full-line curve *cc* in Fig. 11. This curve is called the *volt-ampere characteristic* of the generator; indeed it is called the *external volt-ampere characteristic* because it shows the relation between the external or terminal voltage E_x and the actual current I in the external receiving circuit. The *total volt-ampere characteristic* of the shunt generator is the curve showing the relation between the total electromotive force E induced in the armature and the total current I_a in the armature, at given speed.

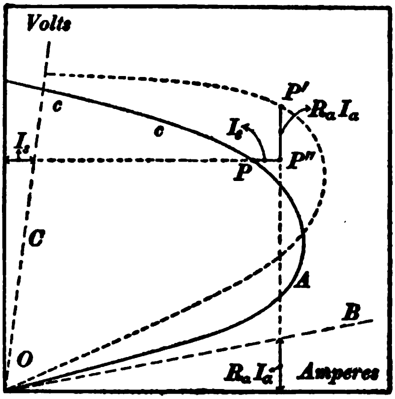

Fig. 11.

The total volt-ampere characteristic may be derived from the external volt-ampere characteristic as follows: The shunt field current I_s is equal to E_x/R_s, where E_x is the electromotive force between the terminals of the shunt field winding and R_s is the resistance of the winding. Therefore the curve showing the relation between E_x and I_s is a straight line OC, Fig. 11. Choose a point P on the external characteristic, then the point P'' is the corresponding point on the E_x, I_a curve. The electromotive force lost in the armature is equal to R_aI_a, where R_a is the resistance of the armature including brush contacts and brushes, therefore the point P' is on the total characteristic curve (the E, I_a curve).

The curve showing for given speed the relation between shunt field current and terminal voltage at zero load is called the *magnetization curve* of the shunt generator. Thus the curve OA, Fig. 12, is the magnetization curve of a shunt generator, the shunt field current I_s being measured to the left from the origin O as indicated. The straight line OB shows the relation between I_s and R_sI_s, where R_sI_s is the electromotive force required to produce the current I_s through the shunt field winding. Choose a point P on OA. Then SP is the total induced electromotive force E corresponding to the shunt field current OS, and QP is the difference between total induced electromotive force E and terminal voltage E_x $(= R_sI_s)$. That is, QP is equal to R_aI_a. Therefore draw the line SC at a slope representing the known value of R_a and the point P' where the horizontal line through P cuts the line drawn through Q parallel to SC is a point on the E, I_x curve.

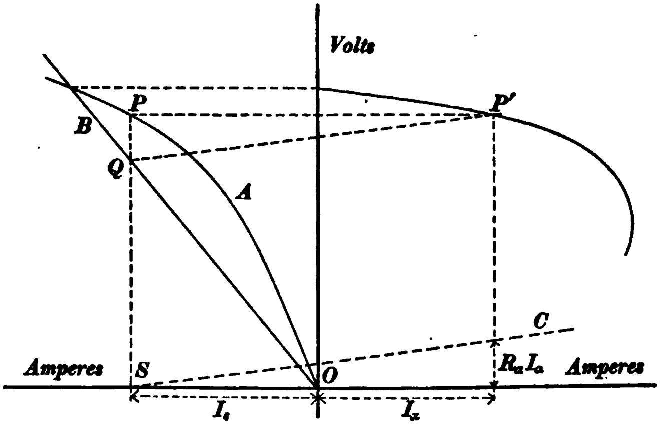

Fig. 12.

15. **Two shunt generators in parallel.**—The curves OA and OB, Fig. 13, are the external volt-ampere characteristics of two dissimilar shunt generators, the curve OB being plotted to the left of the origin for convenience. If these shunt generators are connected in parallel their terminal voltages must be the same, and if the total current delivered by the two machines is,

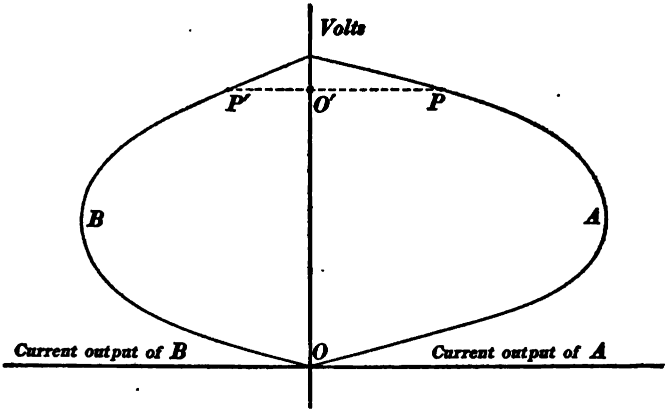

Fig. 13.

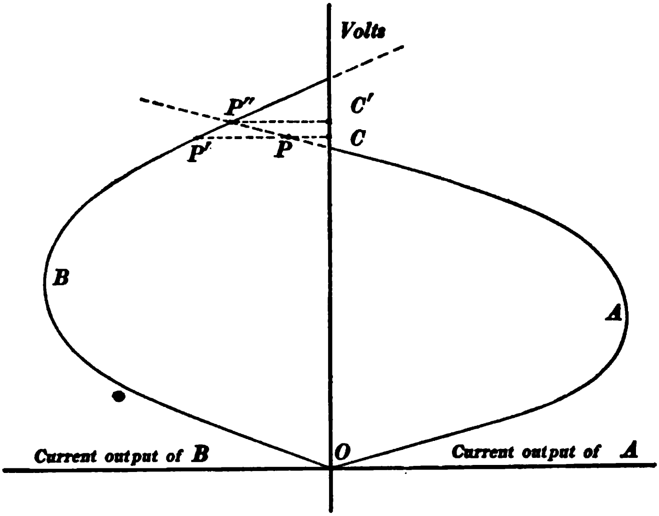

Fig. 14.

say, 100 amperes, then the operating points P' and P are at the same height above the current axis, the distance $P'P$ represents 100 amperes, the distance $O'P$ represents the current delivered by machine A and the distance $O'P'$ represents the current delivered by machine B.

Figure 13 shows the characteristics of two shunt generators of which the terminal voltages are equal at no load, and Fig. 14 shows the characteristics of two shunt generators of which the terminal voltages at no load are not equal. The dotted extensions of OA and OB represent motor action (current output negative).

16. **Two rigidly coupled shunt motors in parallel.**—The curve A, Fig. 15, shows the relation between speed and torque of a shunt motor A taking current at a given voltage, and the curve B shows the relation between speed and torque of another shunt motor B taking current from the same supply mains. If the two motors are rigidly coupled together they must run at the same speed OC, and the total torque $P'P$ developed by both machines together is shared unequally, CP being the torque developed by machine A and CP' being the torque developed by machine B.

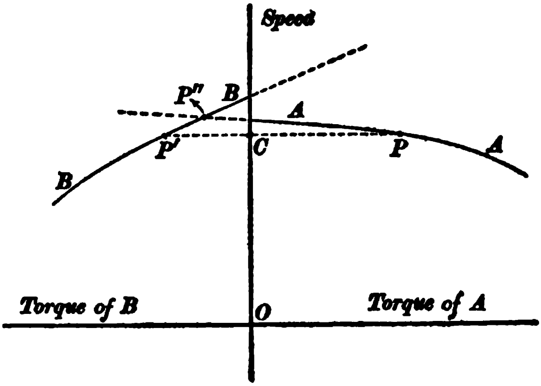

Fig. 15.

17. **Examples showing use of volt-ampere characteristic of a**

series generator.—A typical volt-ampere characteristic of a series generator is shown by the curve OKP in Fig. 18. The abscissas of this curve represent values of current output and the ordinates represent corresponding values of the terminal voltage of the machine when driven at a given speed. • The following examples depend upon the use of what may be called the *volt-ampere characteristic of the receiving circuit,* a curve of which the abscissas represent values of current in the receiving circuit and the ordinates represent corresponding values of voltage acting on the receiving circuit.

When the receiving circuit consists simply of a resistance R_r the electromotive force acting on the receiving circuit must be equal to R_rI so that the volt-ampere characteristic of the receiving circuit is a straight line AB as shown in Fig. 16.

When the receiving circuit consists, say, of a storage battery which is to be charged by the generator, then the electromotive force acting on the receiving circuit must be equal to the sum of R_rI and the counter electromotive force e of the battery; where R_r is the resistance of the battery and connecting wires. In this case the receiving circuit characteristic is the straight line AB, shown in Fig. 17.

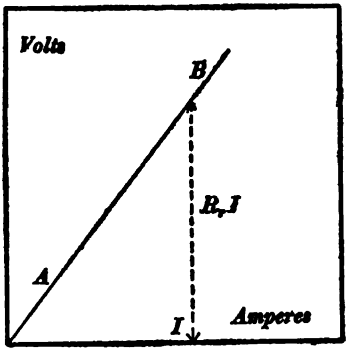

Fig. 16.

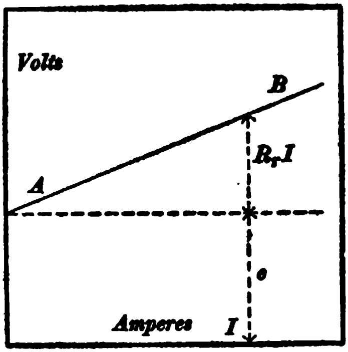

Fig. 17.

A receiving circuit containing motors has no definite characteristic in the sense in which the term is here defined, inasmuch as with a given applied voltage the current may have a great variety of values depending upon the motor load.

Now, evidently, the terminal electromotive force of a generator is identical with the electromotive force applied to the receiving circuit, and the current output of a generator is identical with the current flowing through the receiving circuit. Therefore a generator supplying a given receiving circuit operates at that point of its external volt-ampere characteristic where the receiving circuit characteristic intersects it.

The point P in Fig. 18 shows the operating conditions of a given series generator running at given speed and supplying current to a receiving circuit of given resistance. With increase of the resistance of the receiving circuit, the line OP becomes steeper, that is, the angle θ, Fig. 18, increases, inasmuch as the tangent of θ is equal to R_rI/I, which is equal to R_r. When the resistance R_r has so far increased that the point of intersection is near the knee of the curve at K, then either a very slight additional increase of R_r, or a momentary slowing down of the generator (which reduces the ordinates of the characteristic curve) will lead to a state of affairs in which the generator characteristic and the receiving circuit characteristic do not intersect at all. The resistance R_k, corresponding to the point

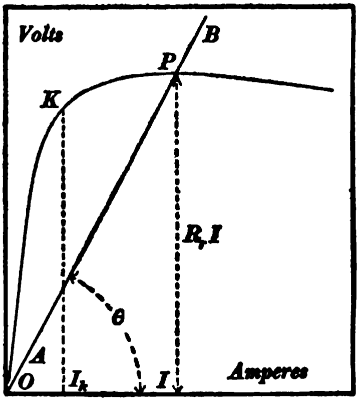

Fig. 18.

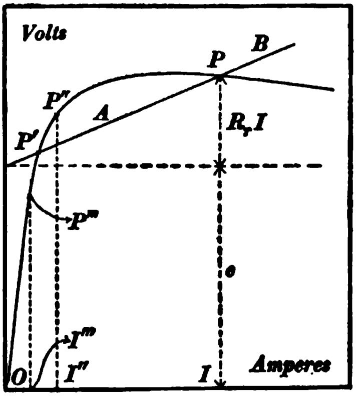

Fig. 19.

K, Fig. 18, is called the *critical resistance* for the series generator at the given speed, inasmuch as the generator when driven at the given speed cannot build up, or if it is already built up, it cannot maintain its field magnetism, if the resistance of the receiving circuit is much greater than R_k. The critical resistance is proportional to the speed at which the generator is driven. The current I_k, corresponding to the point K, is called the *critical current* of the generator. The critical current does not vary perceptibly with the speed of the generator, but has a definite value for a given generator.

The points P and P', Fig. 19, represent the two presumably possible operating conditions of a given series generator, running at a given speed, and supplying current to charge a storage battery of which the counter-electromotive force is e. In fact the point P represents a possible running condition, but it is difficult to bring about the state of affairs represented by the point P as explained below, while the point P' represents an unstable and therefore impossible condition of running. The instability of the conditions represented by the point P' may be shown as follows: Suppose that the generator is in fact operating at the point P', and that a slight momentary increase in the speed of the generator takes place. This increases the generator voltage momentarily and also the current and then, even if the generator comes quickly back to its normal speed, it will continue to operate for a very short interval of time at a point P'' a short distance above P'; but at the point P'' the generator voltage is more than sufficient to produce the current I'' (greater than $e + R_r I''$), and the excess of voltage will cause the current to go on increasing until the generator reaches the point P.

On the other hand a momentary drop in the speed of the generator will cause a momentary decrease of current and, even if the generator comes quickly back to its normal speed, it will continue to operate for a very short interval of time at a point P''' a short distance below P'; but at the point P''' the generator voltage is less than $\{e + R_r I'''\}$, so that the current will continue to fall off more and more. In this latter case (with the storage battery in circuit) the current does not merely drop to zero, but a reversed current is started by the battery. This current causes the generator to build up in the opposite direction, and then both generator and battery work together to produce current, the battery being of course thereby discharged. The point P^{IV}, Fig. 20, shows the operating conditions in this case.

The difficulty in getting a series generator to operate at the point P, Fig. 20, and so charge a storage battery, is due to the fact that when the circuit is first closed (current then equal to zero) the generator begins operating at the point O, Fig. 20, and the effect of the battery is to cause the generator always to build up in such a direction as to carry the operating point to P^{IV}, thus discharging the battery instead of charging it. Under these conditions the battery and the generator work together to produce an excessive current, and the power delivered by the battery and by the generator is all used in heating the circuit.

When a street car is coasting down hill it may drive the series dynamos, which are geared to its axles, as generators, if the con-

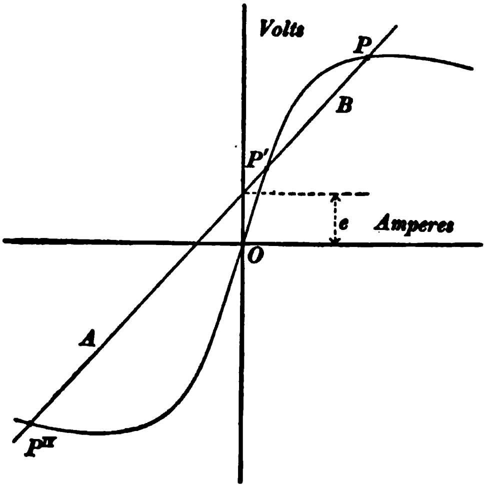

Fig. 20.

nections between field and armature are reversed. The difficulty in getting these dynamos to build up in the proper direction to charge the line, as it were, thus returning power to the system, is precisely the same as that which is described above. The line voltage, between trolley and rail, always causes the series machines to build up in the wrong direction to accomplish the desired result. The action which does take place is that the two series machines build up as generators in the direction such as to help the trolley voltage force an excessive current through the circuit including the machines and the rheostats.

Note.—The calculation of the voltage regulation of an alternator as explained in Sections 585 and 586 of the *Standardization Rules of The American Institute of Electrical Engineers* (edition of December, 1916) illustrates the use of several types of characteristic curves of an alternator.

APPENDIX C.

ARMATURE WINDINGS.*

18. Alternator armature windings and direct-current armature windings.—The windings of an alternator armature are usually of the type described and illustrated in Art. 40 of Chapter III and in Arts. 160 and 161 of Chapter XII. The stator winding and wire winding of the rotor of a polyphase induction motor are arranged exactly as the armature winding of a polyphase alternator. Thus Fig. 296 of Art. 207 shows a 4-pole 2-phase induction motor stator winding, and it also represents a 4-pole 2-phase rotor winding or a 4-pole 2-phase alternator armature winding. The frontispiece shows an 8-pole 3-phase Y-connected alternator armature winding.

The armature windings referred to above are called *open-circuit armature windings* because the wire on the armature does not form a closed circuit.†

On the other hand, in the type of winding which is almost universally‡ used on direct-current dynamo armatures the wire forms a closed circuit, as in the simple ring winding which is shown in Fig. 28 of Chapter I. Such armature windings are called *closed-circuit windings*.

In general, any direct-current dynamo may be made into an alternator (a synchronous converter) by providing it with collector rings as explained in Art. 185 of Chapter XV; but certain types of closed-circuit armature windings, the so-called multiplex windings which are described later, are not well adapted for use in the synchronous converter because it is not desirable to connect one set of collector rings to the several simplex windings which together constitute the multiplex winding.*

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* Electrical railway and mill repairs involve a great deal of re-winding of motor armatures, and therefore a knowledge of armature winding may be important to the operating engineer. For more complete treatment of this subject see E. Arnold's *Die Gleichstrommachine*, Vol. I, Berlin, 1902; and Parshall and Hobart's *Armature Windings*, New York, 1895.

† The three windings of a Δ-connected 3-phase alternator armature do form together a closed circuit, but each winding is of the open-circuit type.

‡ The obsolete direct-current arc-lighting dynamos had open-circuit armature windings.

19. **The ring winding and the drum winding.**—The remainder of this Appendix is devoted to the closed-circuit armature winding as commonly used in direct-current dynamos. The simplest type of closed-circuit winding is the simple ring winding as described in Art. 18 of Chapter I, and the relationship between the ring winding and the drum winding is explained in Art. 111 of Chapter VIII.

The simple ring winding may be used with a bipolar or multipolar field magnet as explained in Art. 110 of Chapter VIII, and each conductor on the outside of the armature may *be* a commutator bar or may *be connected to* a commutator bar; or every second, third or fourth, etc., conductor may *be* a commutator bar or may *be connected to* a commutator bar, as stated in Art. 111 of Chapter VIII.

In the drum armature winding the connections across the ends of the armature must be along chords of nearly $360°/p$, where p is the number of field magnet poles, and, at most, every alternate conductor can *be* a commutator bar or *be connected to* a commutator bar, as explained in Art. 111 of Chapter VIII.

The drum winding is almost universally used in present-day direct-current dynamos, and *the remainder of this Appendix is devoted to the drum winding.*

20. **Inductors.**—The portions of the armature wire which actually cut the magnetic lines of force (the fine radial lines in Fig. 26 of Chapter I) as the armature rotates and in which electromotive forces are induced are called the *inductors*. Thus

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* This statement, as worded, applies only to a particular type of multiplex armature winding, namely, the m-plex winding which has m reëntrances, but no multiplex winding is well suited for use in the synchronous converter. The connections of the multiplex windings to one set of collector rings introduce a large number of paths for short-circuit currents.

the straight portions *aa* in Figs. 21 and 24 are inductors. Armature inductors are often called armature conductors but it is better to use the term inductor in the following discussion to avoid confusion.

21. The lap winding.—Figure 21 shows a single loop or turn of wire laid on an armature body and connected to adjacent commutator segments No. 1 and No. 2; a precisely similar loop or turn is connected between commutator segments No. 2 and No. 3 as indicated by the short stubbs *bb*; another similar loop or turn is connected between commutator segments No. 3 and No. 4; and so on until the winding is completed as indicated in Figs. 22 and 23.

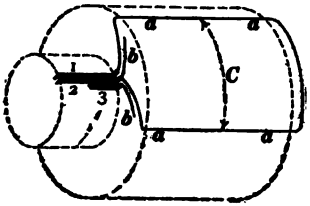

Fig. 21.

One element of a *lap winding* for a 4-pole field (the angle *C* is nearly 360 ÷ number of field poles). Bar winding.

The portion of the winding which is connected between two

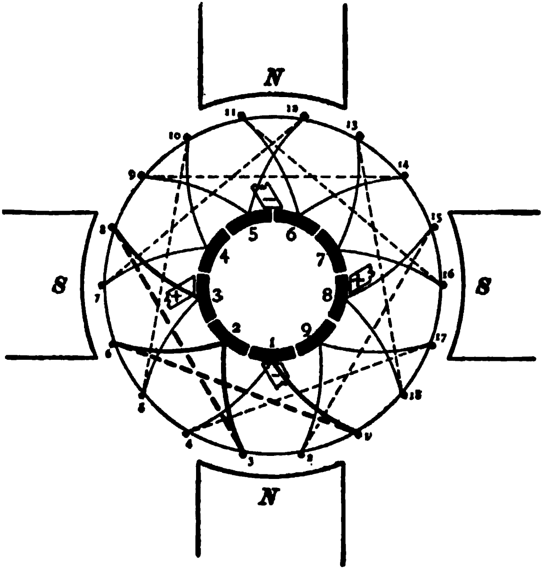

Fig. 22.
End view of 4-pole lap winding. Bar winding.

commutator bars or segments is called an *element* or *section* of the winding. In Figs. 21–23 each element of the winding consists of a single loop or turn (two inductors) and the winding is called a *bar winding*. Each element of the winding may consist of two or more loops or turns of wire as indicated in Fig. 24. In this case each element or section of the winding is called an *armature coil* and the winding is called the *coil winding*. Figure 25 is a developed diagram exactly like Fig. 23 but showing two consecutive armature coils of two turns each.

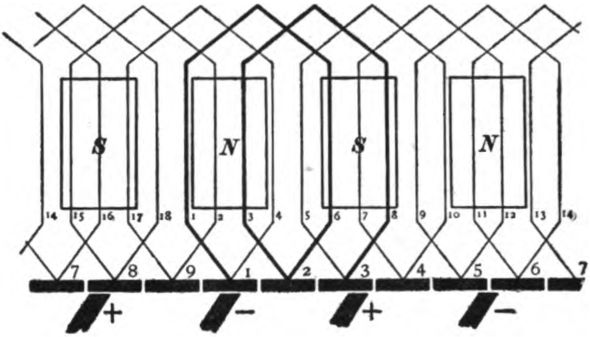

Fig. 23.

Developed diagram of 4-pole lap winding. Bar winding.

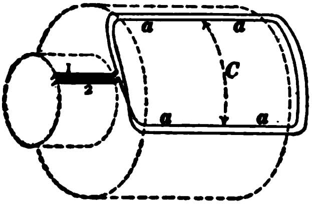

Fig. 24.

One coil of a lap winding for a 4-pole field (the angle *C* is nearly 360° ÷ number of field poles). Coil winding.

Consecutive elements of the windings shown in Figs. 21–25 overlap each other, as may be seen with especial clearness in Fig. 23. Therefore this type of drum armature winding (built up of elements like Fig. 21 or Fig. 24) is called the *lap winding*.

22. The wave winding.—Figure 26 shows a single loop or turn of wire laid on an armature body and connected to commutator segments No. 1 and No. *x* nearly 180° apart;* a precisely

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* In general nearly 360° + $2p$, where p is the number of field poles.

similar loop or turn is connected between commutator segments No. x (see stubb b) and No. 2; another similar loop or turn is connected between commutator segments No. 2 and No. $(x + 1)$; and so on until the winding is completed as indicated in Figs. 27 and 28.

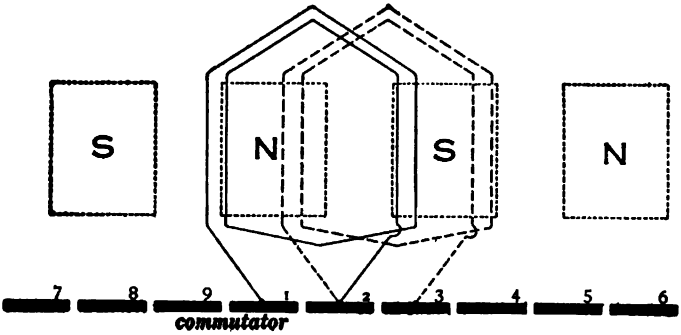

Fig. 25.

Like Fig. 23 but showing two consecutive armature coils of two turns each. Coil winding, lap.

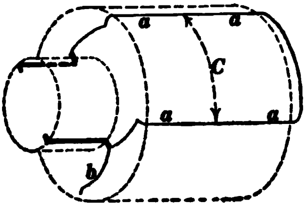

Fig. 26.

One element of a wave winding for a 4-pole field (the angle C is nearly 360° ÷ number of field poles) commutator segments 2 and $(x + 1)$ are next to segments 1 and x respectively but they are not shown. Bar winding.

The portion of the winding which is connected between two commutator bars or segments is called an *element* or *section* of the winding. In Figs. 26–28 each element of the winding consists of a single loop or turn (two inductors) and the winding is called a *bar winding*. Each element may consist of two or more loops or turns of wire as indicated in Fig. 29. In this case each element or section of the winding is called an *armature coil*, and the winding is called the *coil winding*. Figure 30 is a developed diagram showing two consecutive armature coils of two turns each.

Consecutive winding elements form a wave or zig zag line in Figs. 26–28 and 30, as may be seen with especial clearness in

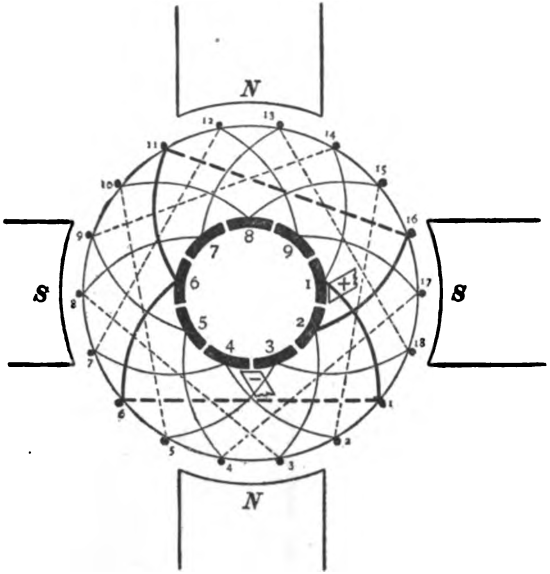

Fig. 27.

End view of a 4-pole wave winding. Bar winding.

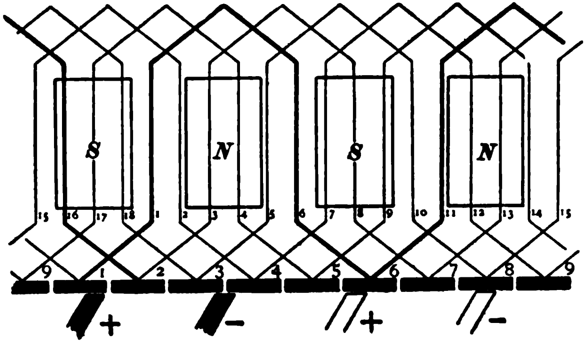

Fig. 28.

Developed diagram of a 4-pole wave winding. Bar winding.

Fig. 28. Therefore this type of drum armature winding (built up of elements like Fig. 26 or Fig. 29) is called the *wave winding.*

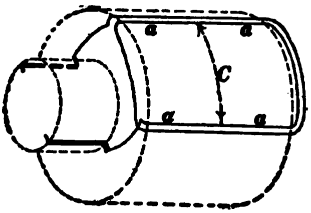

Fig. 29.

One coil of a wave winding for a 4-pole field. (The angle C is nearly 360° + number of field poles.) Coil winding.

Remark.—In any bar-wound armature the one-turn elements can be replaced by n-turn elements or coils, and in any coil-wound armature the armature coils can be replaced by one-turn elements. Consequently a complete understanding of closed-circuit drum windings may be developed by considering the bar winding only. *Therefore the following discussion is devoted chiefly to the bar winding.*

23. **Winding pitches.**—In Figs. 22 and 23 each inductor at the back or pulley end of the armature is connected to the 5th following inductor. Thus inductor No. 1 is connected at the back end to inductor No. 6. The interval between connected inductors at the back end of the armature is called the *back pitch* of the winding. The back pitch is 5 in Figs. 22 and 23.

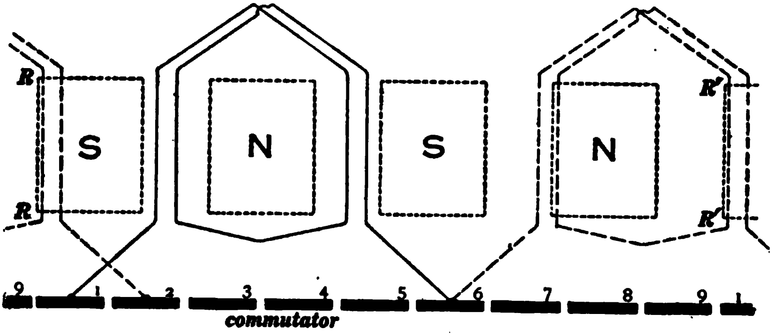

Fig. 30.

Like Fig. 28 but showing two consecutive armature coils of two turns each. Coil winding, wave; $R'R'$ is a repetition of RR.

In Figs. 22 and 23 each inductor at the front or commutator end of the armature is connected to the third preceding* inductor.

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* The winding is understood as being traversed in the direction of increasing numbers in Figs. 22 and 23.

Thus inductor No. 6 is connected to inductor No. 3 (by way of a commutator bar, of course). The interval between connected inductors at the front end of the armature is called the *front pitch* of the winding. The front pitch is -3 in Figs. 22 and 23.

In Figs. 27 and 28 front and back pitches are both equal to $+5$ (inductor No. 1 to No. 6 at back end, and No. 6 to No. 11 at front end). Front and back pitches may be unequal in value but they are always alike in sign in the wave winding, and the average pitch, y, which is equal to half the sum of front and back pitches is an important quantity in the discussion of wave windings.

Commutator pitch.—In Figs. 22 and 23 any winding element is connected from commutator bar No. x to commutator bar No. $(x+1)$ and the commutator pitch is $+1$. In general the commutator pitch is m when a winding element is connected from bar No. x to bar No. $(x+m)$. In Figs. 27 and 28 the commutator pitch is 5.

24. Simplex windings and multiplex windings.—Consider two similar simplex windings like Figs. 22 and 23, for example, and let these windings with their commutators be designated by A and B. Imagine the winding B to have its inductors sandwiched between the inductors of winding A (on the same armature body), and imagine the B commutator bars to be sandwiched between the A commutator bars. We then have what is called a *duplex lap winding*, and each brush must be thick enough to always touch two commutator bars, an A bar and a B bar, so that both constituent windings may always be connected to the brushes and both deliver or receive current steadily.

Imagine two similar simplex wave windings like Figs. 27 and 28 sandwiched together and we have a *duplex wave winding*. Three simplex windings (lap or wave) sandwiched together would give a *triplex winding*, and so on.

Remark.—An m-plex winding as here described presents m closed circuits of wire inasmuch as each constituent simplex winding is a closed-circuit winding. This is usually expressed by saying that the m-plex winding as here described has m reëntrances. An m-plex winding may, however, have any number of reëntrances from 1 to m. This may be illustrated as follows by considering a lap winding with Z inductors and K $(= Z/2)$ commutator bars, and with a commutator pitch m equal to 2:

First case: K an even number.—Beginning at commutator bar No. 1 the first element of the winding ends at bar No. 3, the next at No. 5 and so on, until half of the winding elements have been laid on the armature, and then we connect again to commutator bar No. 1 getting a complete simplex lap winding with $Z/2$ inductors and $K/2$ commutator bars. The even numbered commutator bars are used in a second complete simplex winding; and the two simplex windings together constitute a duplex, doubly reëntrant lap winding.

Second case; K an odd number.—Beginning at commutator bar No. 1 we do not connect again to bar No. 1 when half of the winding elements have been laid on the armature, *but to commutator bar No. 2*; then as the remainder of the winding elements are laid on they are connected to the even numbered bars; and finally when the last winding element is laid on the armature it is connected to bar No. 1 so that the winding has a single reëntrance. It is a duplex lap winding which does not consist of two complete simplex windings.

Remark.—The reader should be able to think out the conditions of reëntrancy for the multiplex wave winding, especially considering that the conditions are stated in the table in Art. 28.

Advantages of multiplex windings.—The advantage of multiplex over simplex windings is that sparking at the brushes is reduced by the use of multiplex windings. In a simplex winding one winding element is short-circuited when a brush touches two adjacent commutator bars as explained in Art. 135, Chapter X, and a brush must at certain instants touch two adjacent commutator bars. In a duplex winding no portion* of the winding is short-circuited when a brush touches two adjacent commutator bars; short-circuit only occurs when a brush touches three adjacent bars, and, although a brush must at certain instants touch three adjacent bars in a duplex winding, still the short-circuit includes a greater length of path in the material of the brush than in case of a simplex winding, and the resistance of the brush material is more effective in the prevention of sparking.

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* If two positive (or two negative) brushes each touch two commutator bars there are short-circuit paths through portions of the armature winding and through the brush leads from brush to brush.

The multiplex winding is not well adapted for use in the synchronous converter.

25. Number of current paths between brushes or brush sets.—A simplex lap winding always has p current paths between brush sets, where p is the number of field magnet poles; and an m-plex lap winding always has mp current paths between brush sets.*

A simplex wave winding always has 2 current paths between brush sets; and an m-plex wave winding always has $2m$ current paths between brush sets.*

26. Number of inductors in series in each current path.—Dividing the total number of inductors Z by the number of current paths p' gives the number of inductors in series in each path.

In a multipolar machine, with a given total number of inductors Z, the lap winding gives many more paths in parallel between brushes, and fewer inductors in series in each path, than the wave winding. Therefore the use of lap winding is sometimes called *parallel grouping* of the inductors, and the use of wave winding is sometimes called *series grouping* of the inductors.

Furthermore, on account of the fact that the simplex lap winding has p paths in parallel between brushes, this winding is sometimes called a *multiple-path winding*, and on account of the fact that the simplex wave winding has only two paths in parallel between brushes (independently of the number of poles) this winding is sometimes called a *two-path winding*.

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* This is true whatever the degree of reëntrancy of the winding.

27. Number of brush sets required.—In any closed circuit winding there are p neutral points* around the armature or commutator, and p brush sets may be used in any case, where p is the number of field magnet poles.

In the lap winding p brush sets must be used if all the available current paths in the armature are to be utilized so as to make the armature resistance as low as possible.

In the wave winding all of the current paths are utilized if two brush sets are used. This is shown in Figs. 27 and 28, and is evident from the following considerations: Suppose first that there are four brush sets, one at each neutral point. The commutator segments 1 and 6, under the two positive brushes are electrically connected through the two inductors Nos. 1 and 6, and their end connections. The figure shows that these two inductors are situated between the pole pieces and have no electromotive forces induced in them, hence they may be considered simply as cross connections between the two positive neutral points. The negative neutral points are similarly connected by the idle inductors Nos. 5 and 10. In consequence of these cross connections the only function of the second pair of brush sets is to aid the first set in collecting current from and delivering current to the commutator segments by the increased area of contact of brushes with commutator; but the distribution of current in the windings is in no way altered by the addition or removal of the second pair of brush sets.

When, however, two brushes only are used for a p-pole wave-wound armature then $p/2$ winding elements, all in series, are short-circuited under a brush when the brush touches two commutator bars (in a simplex winding). Therefore the wave winding using two brushes has a greater tendency to spark at the brushes than the lap winding with p brushes. This disadvantage of the wave winding is obviated by using p brushes instead of two.†

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* Like the points a and b in Figs. 189, 190 and 191.

† See Hobart's *Electric Motors*, page 48, Whitaker & Co., London, 1904.

28. Conditions which must be satisfied by a closed circuit drum winding.—*Lap and wave windings.*—No drum winding can have an odd number of inductors. Both front and back pitches must be odd in a simplex winding. This is due to the fact that odd numbered inductors are to be thought of as returns for the even numbered inductors, and therefore even numbered inductors must be connected at both ends to odd numbered inductors. This requires an odd pitch at both ends.

Both front and back pitches must be approximately equal to Z/p in order that the two inductors which constitute a winding element may move simultaneously under field poles of opposite polarity. Under these conditions electromotive forces induced in the two inductors will be additive. The smallest value of front or back pitch which meets this requirement is a pitch which stretches completely across a pole face, and the largest value of front or back pitch which meets this requirement is a pitch which barely stretches across from a given pole tip to the nearest pole tip of like polarity. When front and back pitches differ considerably from Z/p the winding is called a *chord winding.*

For a given number of inductors the front and back pitches must be chosen to comply with the following requirements:

(*a*) All the winding elements must be similar and they must be placed symmetrically on the armature core.

(*b*) In tracing through a simplex winding every inductor must be passed over once and once only, and the winding must close on itself or be reëntrant.

(*c*) In a multiplex winding which consists of a combination of complete and independent simplex windings, each simplex winding must satisfy condition (*b*).

(*d*) In an m-plex winding having n reëntrances (forming n closed circuits), each nth of the winding must satisfy (*b*).

(*e*) In a two layer winding (in slotted armatures, two inductors per slot) the inductors in the lower layer should be considered even numbered, and those in the upper layer odd numbered.

Lap windings.—(*a*) Front and back pitches are opposite in sign.

(*b*) Front and back pitches cannot be equal. Equality of front and back pitches would make a closed circuit of each winding element.

(*c*) In simplex lap windings front and back pitches differ ***numerically*** by 2, and the commutator pitch is ± 1.

(*d*) In an m-plex lap winding front and back pitches differ ***numerically*** by $2m$, and the commutator pitch is $\pm m$.

(*e*) The total number of inductors Z may be any even number, except in case of slotted armature cores, in which case Z must be a multiple of the number of slots. (The number of slots may be even or odd.)

Wave windings.—(*a*) Front and back pitches are alike in sign.

(*b*) Front and back pitches may be equal or they may differ by any multiple of 2. When one pitch, the front pitch for example, is made considerably greater than Z/p then the other pitch must be made considerably smaller than Z/p. The average pitch of a wave winding is always ***very nearly*** equal to Z/p according to the next paragraph.

TABLE.

SUMMARY FOR DRUM WINDINGS.

	Lap or Parallel Grouping.			Wave or Series Grouping.		
	Simplex.	Duplex.	Triplex.	Simplex.	Duplex	Triplex.
No. of Paths in Parallel	p	$2p$	$3p$	2	4	6
No. of Inductors in Series per Path	Z/p	$Z/2p$	$Z/3p$	$Z/2$	$Z/4$	$Z/6$
No. of Brush Sets	p	p	p	Min. 2 Max. p	Min. 2 Max. p	Min. 2 Max. p
Angular Distance between Brush Sets	$360/p$	$360/p$	$360/p$	$360/p$ or an odd multiple thereof.	$360/p$	$360/p$
No. of Inductors Possible	Any even number			$py \pm 2$	$py \pm 4$	$py \pm 6$
Reëntrancy	Single.	Single if K is odd. Double if K is even.	Single if $K/3$ is not an integer. Triple if it is.	Single.	Single if y is odd. Double if y is even.	Single if $y/3$ is not an integer. Triple if it is.

(*c*) In a simplex wave winding $p \times$ average pitch must be equal to $Z \pm 2$. Therefore $Z = py \pm 2$.

(*d*) In an m-plex wave winding $p \times$ average pitch must be equal to $Z \pm 2m$. Therefore $Z = py \pm 2m$.

p = number of field magnet poles.
Z = number of armature inductors.
K = number of commutator segments.
y = average winding pitch (half the sum of front and back pitches).

29. **Examples of drum windings.**—The following figures, namely, Figs. 31–35, show typical examples of drum windings. In these figures the short radial numbered lines represent the inductors, the crossed lines outside of the circle of inductors represent the connections at the back end of the armature, and the crossed lines between the circle of inductors and the commutator at the center represent the connections at the front end of the armature.

In these figures a small number of armature inductors is chosen for the sake of simplicity, and it is to be noted that for a given number of armature inductors, Z, several different combinations of front and back pitches are permissible for a given type of winding, especially when Z is a large number as it usually is in actual machines. Furthermore a change of a few units in the value of Z opens up a greatly increased choice of pitches for a given type of winding. Therefore the examples of windings shown in Figs. 31–35 should be looked upon as chosen from among a great number of possible cases.

Fig. 31 shows a six-pole drum simplex lap winding with 60 inductors, having a back pitch of $+9$, a front pitch of -11, and a commutator pitch of -1. This winding presents six paths in parallel between the brushes with one sixth of the inductors in series in each path, and it must have six brush sets as shown in Fig. 31. This winding is singly reëntrant.

Fig. 32 shows a six-pole drum simplex wave winding with 62 inductors, having a back pitch of + 9, a front pitch of + 11, an average pitch of 10, and a commutator pitch of + 10. This

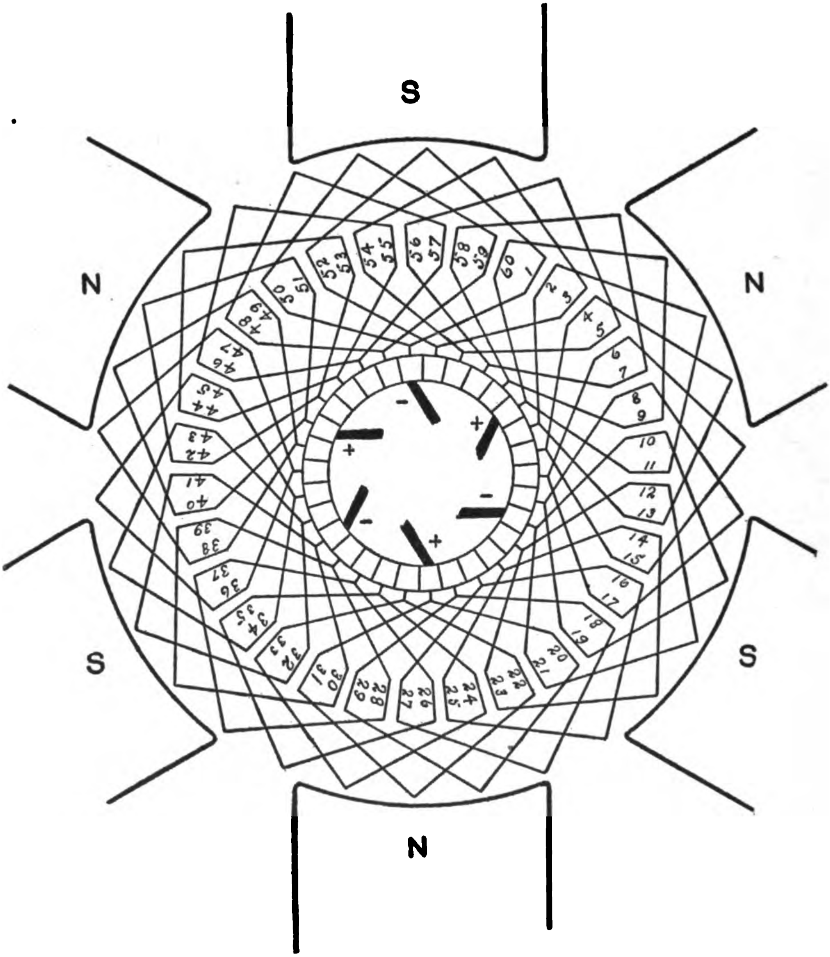

Fig. 31.

Six-pole drum simplex lap winding. $Z = 60$, front pitch = − 11, back pitch = + 9, commutator pitch = − 1, number of paths = 6.

winding presents two paths in parallel between the brushes with one half of the inductors in series in each path, and it requires only two brush sets as shown in Fig. 32. This winding is singly reëntrant.

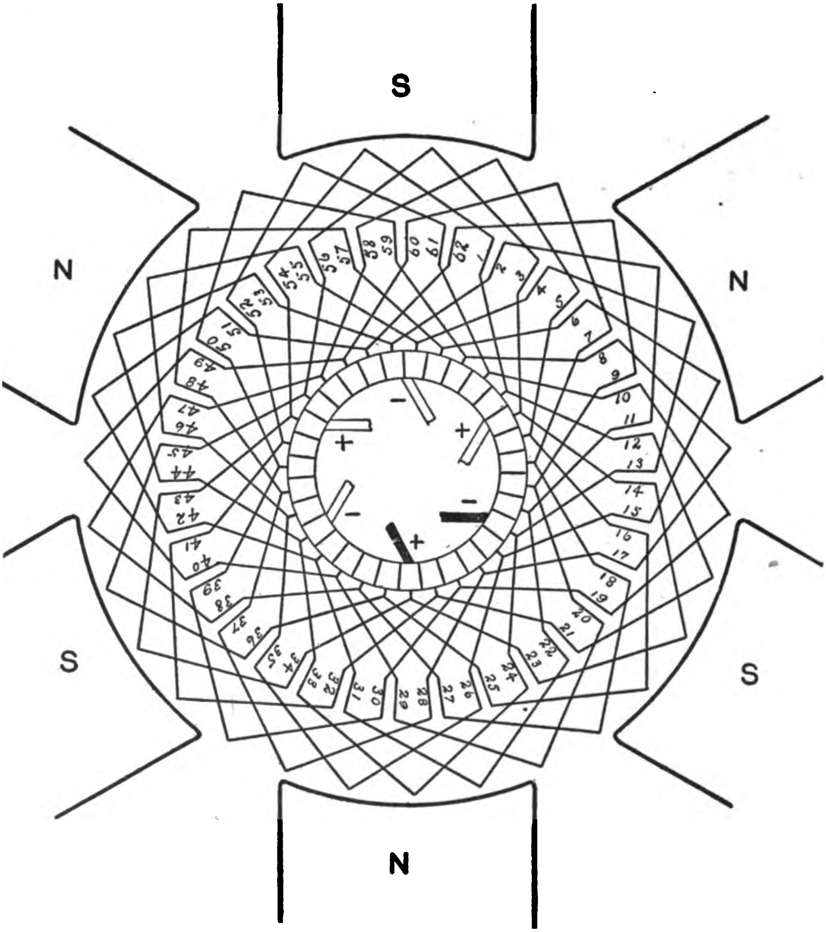

Fig. 32.

Six-pole drum simplex wave winding. $Z = 62$, front pitch $= +11$, back pitch $= +9$, average pitch 10, commutator pitch $+10$, number of paths $= 2$.

Fig. 33 shows a six-pole drum duplex lap winding having 60 inductors. The dotted lines and shaded commutator segments represent a complete simplex lap winding having 30 inductors, and the full lines and blank commutator segments represent another identically similar simplex winding. These two simplex windings when sandwiched together on one armature core, as

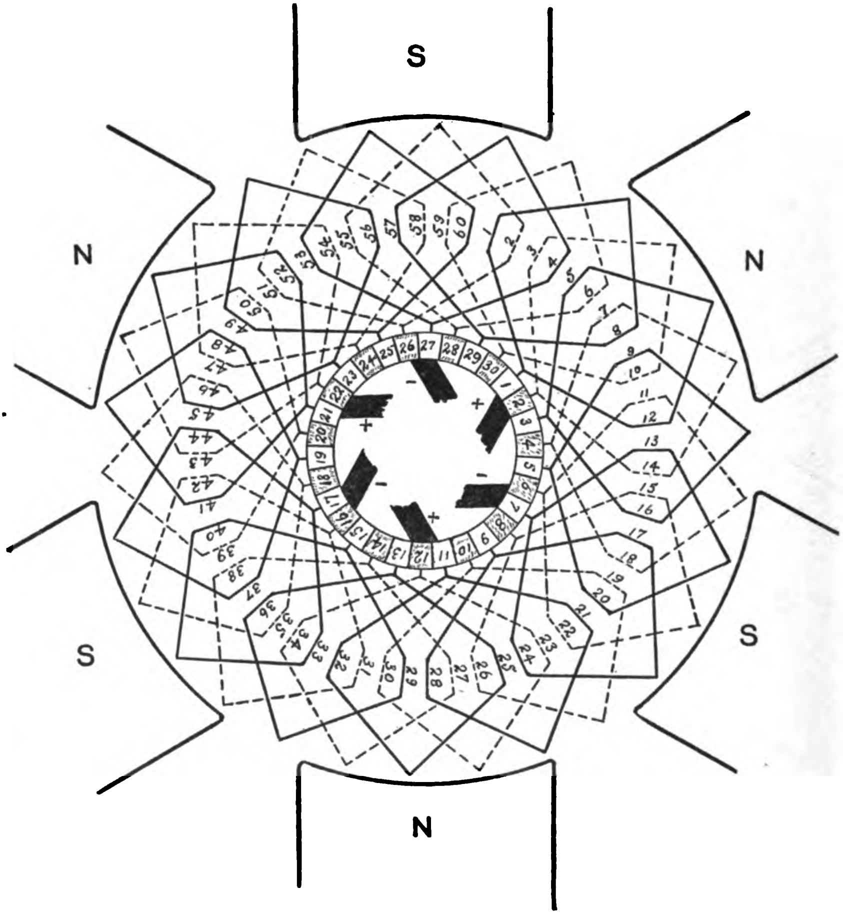

Fig. 33.

Six-pole drum duplex lap winding, doubly reëntrant. $Z = 60$, front pitch $= -11$, back pitch $= +7$, commutator pitch $= -2$, number of paths $= 12$.

shown in Fig. 33, form a duplex lap winding. This duplex winding is doubly reëntrant inasmuch as each of the constituent simplex windings reënters itself. In this winding the back pitch is + 7, the front pitch is − 11, and the commutator pitch is − 2; and all of the conditions necessary for duplex lap windings

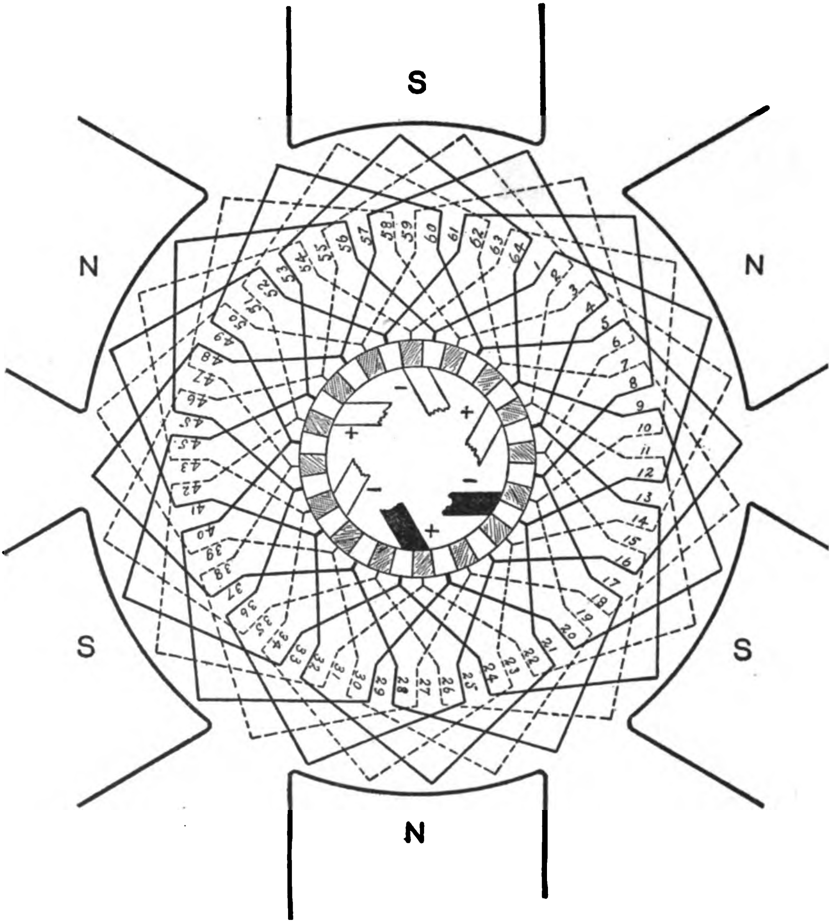

Fig. 34.

Six-pole drum duplex wave winding, doubly reëntrant. $Z = 64$, front pitch $= + 9$, back pitch $= + 11$, average pitch 10, commutator pitch 10, number of paths $= 4$.

as given in Art. 28 are satisfied. This winding presents 12 paths in parallel between the brushes, 6 paths through each constituent simplex winding, and it requires 6 brush sets, each brush being thick enough to always touch at least two adjacent commutator segments.

Fig. 34 shows a six-pole drum duplex wave winding having 64 inductors. The dotted lines and shaded commutator segments represent a complete simplex wave winding having 32 inductors, and the full lines and blank commutator segments represent another identically similar simplex wave winding. These two simplex windings when sandwiched together, as shown in Fig. 34, form a duplex wave winding. This duplex winding is doubly reëntrant inasmuch as each of the constituent simplex windings reënters itself. In this winding the back pitch is + 11, the front pitch is + 9, the average pitch is 10, and the commutator pitch is 10; and all the conditions necessary for duplex wave windings as given in Art. 28 are satisfied. This winding presents four paths in parallel between the brushes, two paths through each constituent simplex winding, and two brush sets only are required as shown in Fig. 34.

Fig. 35 shows a six-pole drum duplex wave winding which is singly reëntrant. This duplex winding cannot be looked upon as two simplex windings sandwiched together. In this winding the back pitch is + 11, the front pitch is + 11, the average pitch is + 11 and the commutator pitch is + 11. This winding presents four paths in parallel between the brushes, two paths each way from each of the two broad brushes, as may be seen by tracing through the windings from brush to brush in Fig. 35. Two brush sets are sufficient for this winding as shown in Fig. 35.

It will be noticed that the inductors of the simplex windings shown in Figs. 31 and 32 are grouped in pairs which are drawn very close together. The same is true of the constituent simplex windings in Figs. 33 and 34. This is intended to show the common arrangement in slotted armatures where the even numbered inductors lie in the bottoms of the slots, and the odd numbered inductors lie in the tops of the slots, as shown in Fig. 36. In this case the front or back pitch is reckoned by counting consecutive inductors exactly as if they were arranged in one layer.

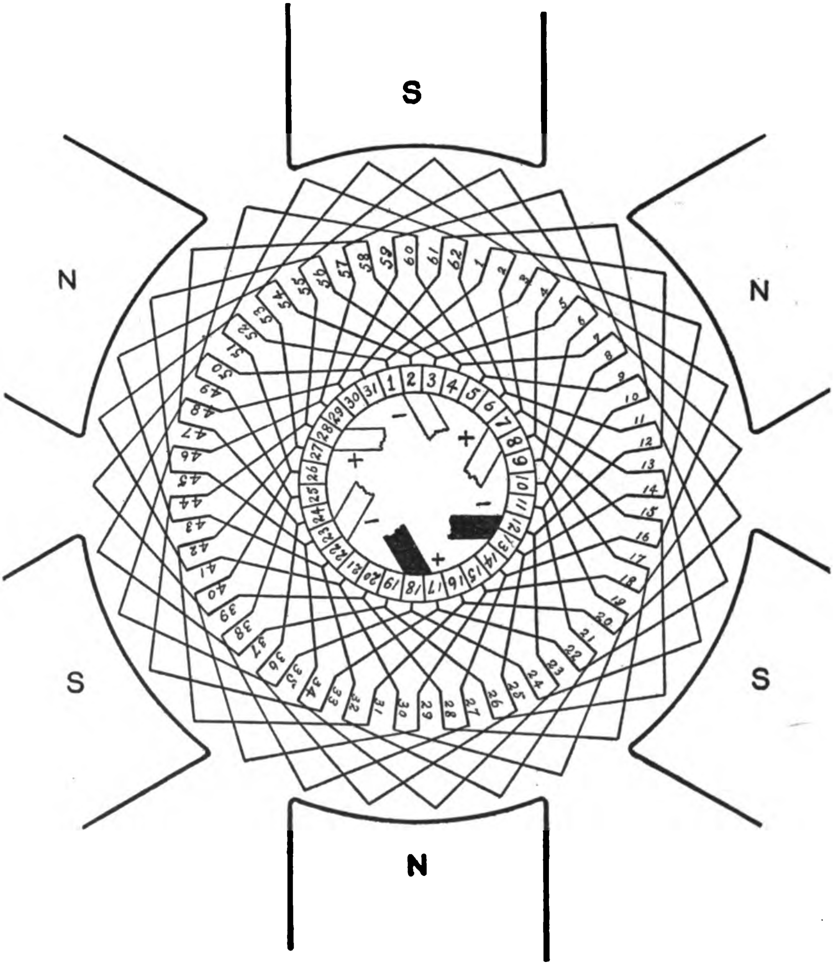

Fig. 35.

Six-pole drum duplex wave winding, singly reëntrant. $Z = 62$, front pitch $= +11$, back pitch $= +11$, average pitch $= 11$, commutator pitch $= +11$, number of paths $= 4$.

If a winding element consists of a coil of n turns of wire instead of two inductors, as explained in the remark at the end of Art. 22, then it is convenient to assign numbers to the consecutive *bundles* of *n* inductors each, that is, to the consecutive *half-coils*, instead of assigning numbers to the individual inductors. In this case the pitches, front and back, are reckoned by counting consecutive bundles or half-coils. In the case of slotted armatures wound with formed coils it is convenient to express the back pitch in terms of the number of slots spanned by the coil, one side of the coil always being in the top of one slot, and the other side being in the bottom of the *n*th following slot, where *n* is the slot pitch. The number of slots spanned by the coils is called the *slot pitch* or the *throw* of the coils. When the throw of the coils is given, the only additional information needed for properly connecting the coils (of a simplex winding) is

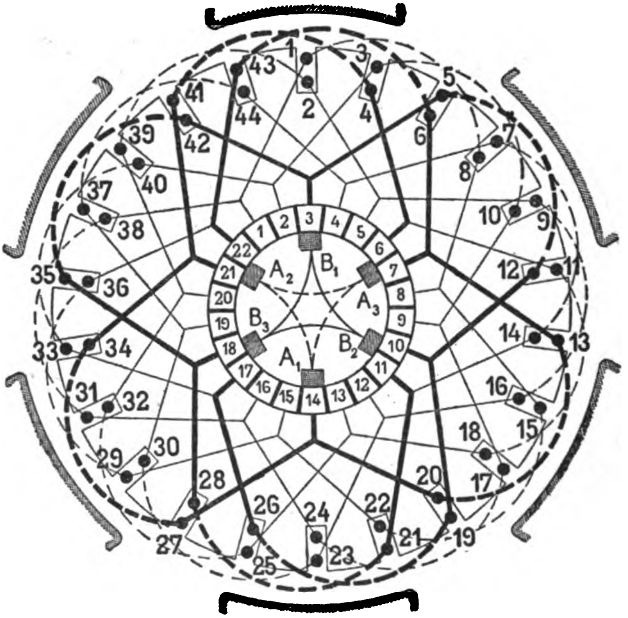

Fig. 36.

Six-pole drum simplex wave winding. Number of half-coils (which takes the place of Z in the theory of the winding) = 44, front pitch = + 7, back pitch = + 7, commutator pitch = + 7, throw of coils = 3, number of paths = 2.

the numbers of the commutator segments to which the terminals of any given coil are to be connected.

The matters above mentioned are illustrated in Fig. 36, which represents a six-pole drum simplex wave winding having 22 coils of any given number of turns each, the half-coils being numbered from 1 to 44 as shown in the figure. The back pitch is 7, inasmuch as half-coil No. 5 is connected across the back, as shown by the dotted line, to half-coil No. 12, and half-coil No. 7 is connected at the back to No. 14, and so on. The front pitch is also 7, inasmuch as half-coil No. 2 is connected at the front end to half-coil No. 9, as shown by the full line and so on. The commutator pitch is also 7, inasmuch as the terminals of the complete coil 1–8 are connected to commutator bars No. 1 and No. 8. The slot pitch or throw of the coils is 3.

Any one pair of brushes is sufficient for the wave winding shown in Fig. 36. The figure, however, shows three positive brushes A_1, A_2 and A_3 and three negative brushes B_1, B_2 and B_3.

30. Further comparison of lap and wave windings. Equipotential connections.—In practice the armature of a dynamo never is exactly centered in the field, but it is usually nearer to one pole or poles than to others, especially after the bearings have become worn. Inaccurate centering of the armature causes more flux to enter or leave the armature at certain poles than at others, so that greater electromotive forces are induced in the inductors under certain poles than in the inductors under other poles. These inequalities of electromotive force exactly balance each other insofar as the closed circuit of the whole winding is concerned, but if there are two or more positive brush sets and two or more negative brush sets these inequalities of electromotive force produce local currents through certain paths of the winding, and through the wires which connect the positive (or negative) brush sets together. These local currents increase the heating of the armature and they tend also to increase the sparking.

Now the various inductors which constitute a given current path through a wave wound armature are distributed under all of the field magnet poles, so that, in the wave winding, equal electromotive forces are induced in all of the armature paths and there is no perceptible tendency for the production of local currents through certain armature paths, and through the wires which connect the brush sets. On the other hand the various inductors which constitute a current path in a lap wound armature lie under two adjacent field magnet poles and, therefore, an inaccurate centering of the armature may lead to the production of very considerable local currents through the current paths and brush connections of a multipolar lap-wound armature.

These local currents may be prevented from flowing out through the brushes and across the brush connections, by making permanent low resistance cross-connections in the commutator (or in the armature winding) between each commutator bar, and every other bar which passes at the same instant under a positive brush. These cross-connections are called ***equipotential connections*** for the reason that every set of commutator bars so connected would be at the same potential (no electromotive force between them) if the magnetic field were symmetrical. Equipotential connections obviate the increased tendency to sparking due to the local currents.*

31. **Modified multiplex wave winding.**—The above mentioned advantage of the wave winding over the lap winding, namely, that the electromotive forces induced in the various paths of a wave winding are always balanced and do not tend to produce local currents, applies to multiplex wave windings as well as to simplex wave windings. Now, one advantage of the lap winding over the wave winding, for low electromotive force machines, is that a multi-path lap winding does not require an excessive number of commutator bars like the multi-path wave winding.

*** The short-circuit currents in the armature due to lack of symmetry of field magnet have a magnetizing action which tends to correct the lack of symmetry in the distribution of the magnetic flux.**

The following modification of the singly reëntrant *m*-plex multipolar wave winding gives a winding having $2m$ balanced paths and having only $1/m$th as many commutator bars as the original *m*-plex winding.

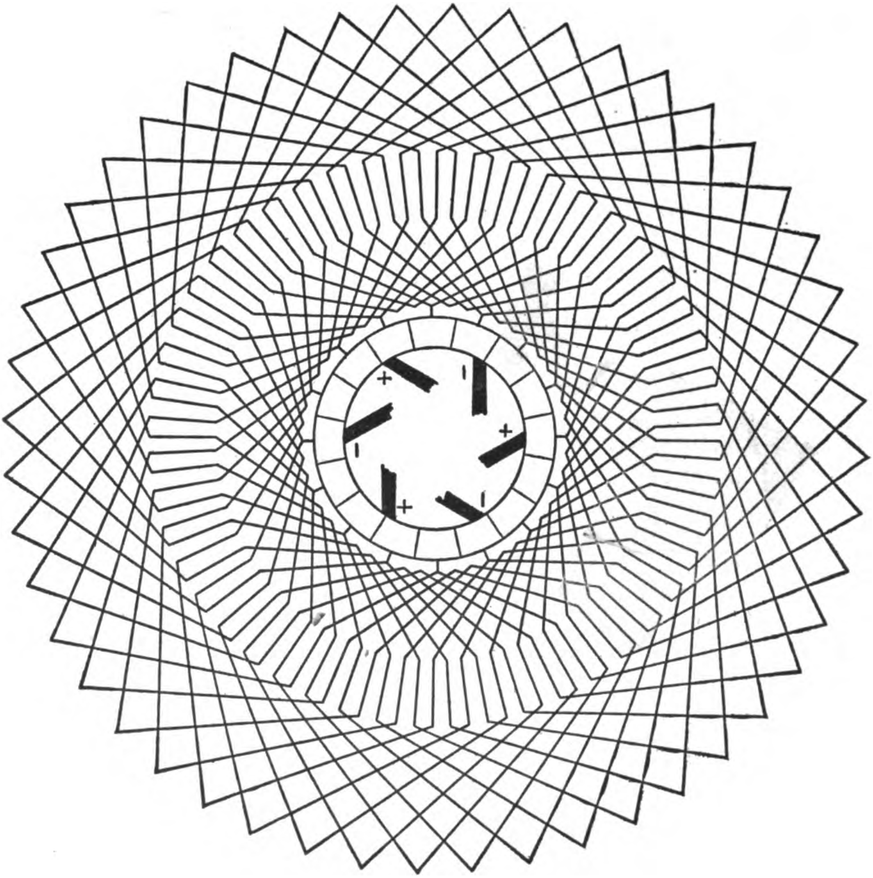

Fig. 37.
Modified singly reëntrant duplex six-pole wave winding.

Consider a singly reëntrant *m*-plex multipolar wave winding (Fig. 35 shows a singly reëntrant duplex six-pole wave winding). In such a winding a commutator segment is of course inserted at the terminus of each winding element. Suppose, however, that, starting from a given commutator segment, m of these winding elements are wound as one extended winding element before a commutator segment is inserted, then another group of m elements are wound as another extended element, and so on. The result will be a modified type of singly reëntrant wave winding having as many paths as the original m-plex winding, namely $2m$ paths, but having only $1/m$th as many commutator segments, each segment being made m times as broad as before.

Fig. 37 shows one of these modified six-pole wave windings with 16 commutator bars. This winding may be thought of as derived from a singly reëntrant triplex wave winding with a front pitch of 17, and a back pitch of 17. In this case it is necessary to use all six of the narrow brushes in order to utilize the six paths in the armature.

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